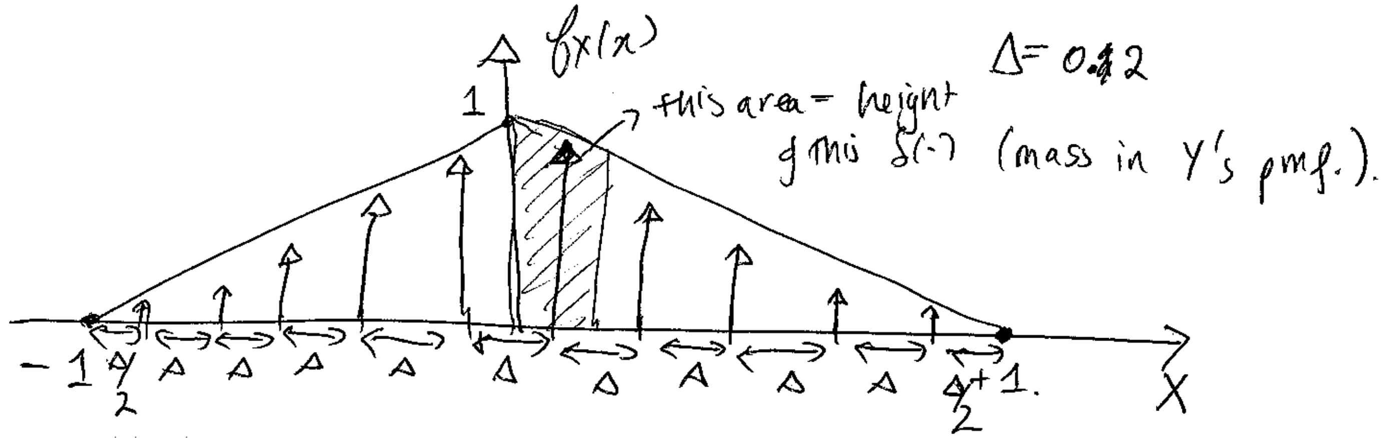
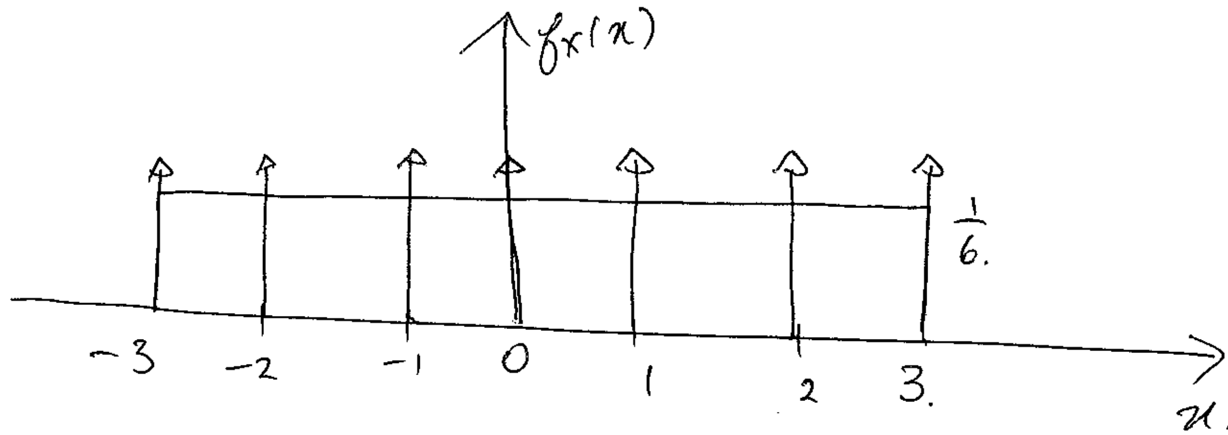
Quantization:

$X \longrightarrow$ [Uniform quantizer, step size Δ] $\longrightarrow Y$

continuous $\qquad\qquad\qquad\qquad\qquad\qquad$ discrete.

Before, $\Delta = 1$ (round to nearest integer).





How many levels do we need to quantize to, to achieve a certain quality? Often "quality" is measured in "Signal to Noise Ratio" (SNR)

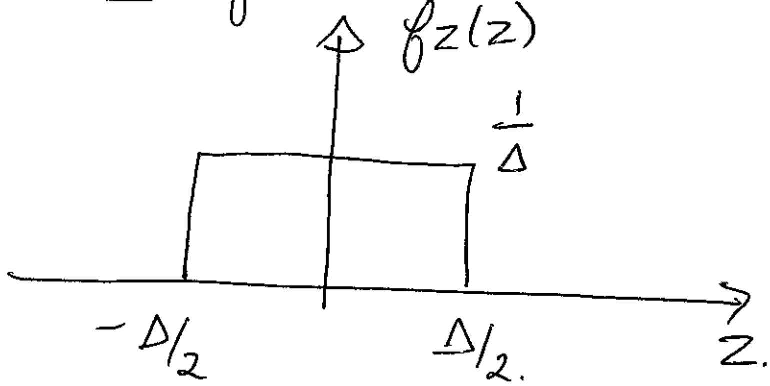
Here, noise $\equiv$ quantization. (signal-to-quantization-noise ratio)

To get SNR, need "power" of signal $E[X^2]$ if $\mu_X = 0$, and the "power" of the quantization noise $E[(X-Y)^2]$

In general $X \rightarrow \boxed{\text{Quantizer, step size } \Delta} \rightarrow Y$.

$$|X-Y|_{max} = \frac{\Delta}{2}$$

Let $Z = X - Y$ = quantization error, then Z has PDF <u>approximately</u> equal to



Then $E[Z^2] = \text{var}(Z) + \mu_Z^2$ = "power" of the error signal.

$$= \frac{(\text{span})^2}{12} + 0^2 = \frac{\Delta^2}{12}.$$

Signal (power) to quantization "noise" power

$$SNR_Q \triangleq \frac{E[X^2]}{E[Z^2]} \qquad SNR_Q(dB) \triangleq 10\log_{10}\left(\frac{E[X^2]}{E[Z^2]}\right)$$

EX: RV X is uniform, continuous over $[-10,+10]$. How many output levels are needed to ensure that

$$SNR_Q \text{ (dB)} \geq 60 \text{ dB}.$$

Assume the quantization is uniform, of length Δ.

Let L = # output levels. Then, since they are uniformly spaced over $[-10,+10]$, $\Delta = \frac{\text{span}}{L} = \frac{20}{L}$.

Therefore, $E[Z^2 = (X-Y)^2] \cong \frac{\Delta^2}{12} = \frac{\left(\frac{20}{L}\right)^2}{12} = \frac{20^2}{12L^2}$.

Also, $E[X^2] = Var(X) + \mu_X^2 = \frac{(\text{span } x)^2}{12} = \frac{20^2}{12}$

Thus, $SNR_Q = \frac{20^2/12}{20^2/12L^2} = L^2$

$$\Rightarrow \quad 10\log_{10}(SNR_Q) \geq 60 \quad \text{(dB)}.$$

$$10\log_{10}(L^2) \geq 60$$

$$20\log_{10}(L) \geq 60$$

$$\Rightarrow \quad \log_{10}(L) \geq 3.$$

$$\Rightarrow \quad \boxed{L \geq 1000}$$