ECE 534: Elements of Information Theory, Fall 2013
Homework 1
Out: August 28, 2013
Due: September 4, 2013 in class Solutions

(Problem 2.1) *Coin Flips.* A fair coin is flipped until the first head occurs. Let X denote the number of flips required.

(a) Find the Entropy H(X) in bits

(b) A random variable X is drawn according to this distribution. Find an "efficient" sequence of yes-no questions of the form, "Is X contained in the set S?". Compare H(X) to the expected number of questions required to determine X.

(Problem 2.4) *Entropy of functions of a random variable.* Let X be a discrete random variable. Show that the entropy of a function of X is less than or equal to the entropy of X by justifying the following steps:

$$H(X, g(X)) \stackrel{(a)}{=} H(X) + H(g(X)|X) \stackrel{(b)}{=} H(X) \tag{1}$$

$$H(X, g(X)) \stackrel{(c)}{=} H(g(X)) + H(X|g(X)) \stackrel{(d)}{\geq} H(g(X)) \tag{2}$$

Thus, $H(g(X)) \leq H(X)$.

(Problem 2.12) Consider the table below:

X Y	0	1
0	1/3	1/3
1	0	1/3

Table 1: p(x,y) for problem 2.12.

Find:
(a) $H(X), H(Y)$.
(b) $H(X|Y), H(Y|X)$.
(c) $H(X, Y)$.
(d) $H(Y) - H(Y|X)$.
(e) $I(X; Y)$.
(f) Draw a Venn diagram for the quantities in parts (a) through (e).

(Problem 2.20) *Run-length coding.* Let X_1,X_2, . . . , X_n be (possibly dependent) binary random variables. Suppose that one calculates the run lengths $\mathbf{R} = (R_1, R_2, \ldots)$ of this sequence (in order as they occur). For example, the sequence X = 0001100100 yields run lengths R = (3, 2, 2, 1, 2). Compare $H(X_1, X_2, ..., X_n)$, $H(R)$, and $H(Xn, R)$. Show all equalities and inequalities, and bound all the differences.