**ECE 534: Elements of Information Theory, Fall 2013**
Homework 9
Out: October 30, due November 4, 2013 (BONUS)

**Problem 9.2** Consider the ordinary Gaussian channel with two correlated looks at $X$, that is, $Y = (Y_1, Y_2)$, where

$$Y_1 = X + Z_1$$
$$Y_2 = X + Z_2$$

with a power constraint $P$ on $X$, and $(Z_1, Z_2) \sim \mathcal{N}_2(0, K)$, where

$$K = \begin{bmatrix} N & N\rho \\ N\rho & N \end{bmatrix}.$$

Find the capacity $C$ for

1. $\rho = 1$
2. $\rho = 0$
3. $\rho = -1$

**Problem 9.4** *Exponential noise channel.* $Y_i = X_i + Z_i$, where $Z_i$ is i.i.d. exponentially distributed noise with mean $\mu$. Assume that we have a mean constraint on the signal (i.e. $E[X_i] \leq \lambda$). Show that the capacity of such a channel is $C = log\left(1 + \frac{\lambda}{\mu}\right)$.

**Problem 9.5** Consider an additive noise fading channel $Y = XV + Z$, where $Z$ is additive noise, $V$ is a random variable representing fading, and $Z$ and $V$ are independent of each other and of $X$. Argue that knowledge of the fading factor $V$ improves capacity by showing that $I(X;Y|V) \geq I(X;Y)$.

**Problem 9.9** *Vector Gaussian channel.* Consider the vector Gaussian noise channel

$$Y = X + Z$$

where $X = (X_1, X_2, X_3)$, $Z = (Z_1, Z_2, Z_3)$, $Y = (Y_1, Y_2, Y_3)$, $E||X||^2 \leq P$, and

$$Z \sim \mathcal{N}\left(0, \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}\right).$$

Find the capacity. The answer may be surprising.