1. (3 + 3 + 3 = 9 points) For the graph below, show the steps (i.e., you may draw graphs and/or list the changes) to find the minimum spanning tree using:
   
   (a) Kruskal’s algorithm.
   (b) Prim’s algorithm. Use D as the starting node.
   (c) Reverse-delete algorithm

2. (5 + 2 = 7 points) You are given an array \( A[1 \ldots n] \) of distinct numbers. The array is bitonic, which we define as follows for the purposes of this problem.

   There is some index \( i \) between 1 and \( n \) such that the values in the array increase from \( A[1] \) to \( A[i] \), and then decrease from \( A[i] \) to \( A[n] \).

   Your task is to identify the element \( A[i] \) (the maximum value in this array) in \( O(\log n) \) time.

   (a) Design and present the algorithm.
   (b) Derive the time complexity from first principles, i.e., analyze the algorithm and/or the recurrence relation used (if any), to prove that the time complexity is \( O(\log n) \).

3. (0.5 + 0.5 + 0.5 + 2.5 = 4 points) For the following recurrence relation,
   
   \[
   T(n) = 4T\left(\frac{n}{4}\right) + 3n
   \]

give the:

   (a) number of levels in the recursion tree
   (b) size of the subproblem at level \( i \) of the recursion tree
   (c) number of subproblems at level \( i \) of the recursion tree
   (d) solution to the recurrence relation (show the steps or reasoning used)