Th: The GS algo is man-optimal, ie, each man gets his best valid partner.

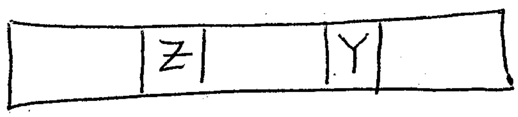
Proof: By contradiction.

Let $S^* \equiv$ stable assignment that is not man-optimal.

Let $Y \equiv 1^{st}$ man rejected by his best valid partner (say, A) who prefers someone else (say, Z)

// rejection happens when Y proposes to A OR when Z proposes to A

A's list

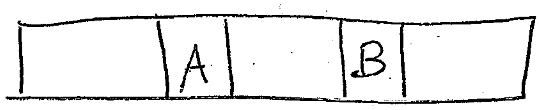


As S^* is Not man optimal, must exist some stable assignment, say S, in which Y–A paired

In S, Z–B (say), paired

$\therefore$ B is valid partner of Z

Z's list



because Y's rejection at/before $Z \to A$ proposal

$\wedge$ Z not yet rejected by valid partner

In S, Z & A prefer each other over assigned partners (B, Y, resp.)

$\therefore$ Z–A unstable in S

$\therefore$ Stable S (w/ Y–A pair) cannot exist.

$\therefore$ S^* is NOT not man-optimal.

ie, S^* cannot exist. $\boxed{\text{Qed}}$

Th: The GS algo is woman-pessimal, ie, each woman gets her <u>worst</u> valid partner.

Proof: By contradiction.

Let $S^* \equiv$ stable assignment that is not woman-pessimal.

In S^*, Z–A paired but A's worst valid partner is someone else, say Y

A's list

	Z		Y	

$\therefore$ Must exist some <u>stable</u> assignment, say S, in which Y–A paired.

In S, Z–B (say) paired

$\therefore$ B is valid partner of Z

Man-optimality $\Rightarrow$

Z's list

	A		B	

In S, Z & A prefer each other over assigned partners (B, Y, resp.)

$\therefore$ Z–A unstable in S

$\therefore$ Stable S (w/ Y–A pair) cannot exist.

$\therefore$ A's worst valid partner is Z

$\therefore$ S^* cannot exist.

$\boxed{\text{QED}}$