**Th.** The GS algo is man-optimal, i.e., each man gets his best valid partner.

**Proof:** By contradiction.

Let $S^*$ = stable assignment that is not man-optimal.

Let $Y$ = 1st man rejected by his best valid partner (say, $A$) who prefers someone else (say, $Z$)

// rejection happens when $Y$ proposes to $A$ or when $Z$ proposes to $A$

<table>
<thead>
<tr>
<th>A's list</th>
<th>Z</th>
<th>Y</th>
</tr>
</thead>
</table>

As $S^*$ is not man-optimal, must exist some stable assignment, say $S$, in which $Y$-$A$ paired

In $S$, $Z$-$B$ (say), paired

$\therefore B$ is valid partner of $Z$

<table>
<thead>
<tr>
<th>Z's list</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
</table>

because $Y$'s rejection at/before $Z$-$A$ proposal

$\land Z$ not yet rejected by valid partner

In $S$, $Z$ & $A$ prefer each other over assigned partners ($B$, $Y$, resp.)

$\therefore Z$-$A$ unstable in $S$

$\therefore$ Stable $S$ (w/$Y$-$A$ pair) cannot exist.

$\therefore S^*$ is not not man-optimal. i.e., $S^*$ cannot exist. \(\Box\)
Th: The GS algo is woman-pessimal, i.e., each woman gets her worst valid partner.

Proof: By contradiction.

Let $S^*$ = stable assignment that is not woman-pessimal.

In $S^*$, Z-A paired but A's worst valid partner is someone else, say Y

A's list: 

```
Z  Y
```

:: Must exist some stable assignment, say $S$, in which Y-A paired.

In $S$, Z-B (say) paired

:: B is valid partner of Z

Man-optimality $\Rightarrow$

Z's list: 

```
A  B
```

In $S$, Z & A prefer each other over assigned partners (B, Y, resp.)

:: Z-A unstable in $S$

:: Stable $S$ (w/ Y-A pair) cannot exist.

:: A's worst valid partner is Z

:: $S^*$ cannot exist.

QED