CS 301

Lecture 21 – Review

Exam topics

Broadly speaking: Everything through decidable languages (Sipser §4.1)

- CFGs, both the mathematical definition as a 4-tuple $G = (V, \Sigma, R, S)$ and as lists of rules
- Converting a CFG to CNF
- PDAs, both the mathematical definition $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ and diagrams
- Closure properties of CFLs
- TMs, both the mathematical definition $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ and diagrams
- Turing-recognizable (RE), co-Turing-recognizable (coRE), and decidable languages
- Closure properties of RE and decidable languages
- Decision problems from language theory (red are undecidable)
  - Acceptance problems: A_{DFA}, A_{NFA}, A_{REX}, A_{CFG}, A_{TM}
  - Emptiness problems: E_{DFA}, E_{CFG}, E_{TM}
  - Equivalence problems: EQ_{DFA}, EQ_{CFG}, EQ_{TM}

Types of exam questions

The questions from the exam fall into these types

- True/false questions with explanation
- Constructions
  - Construct a CFG/PDA for a context-free language
  - Convert a CFG to a PDA
  - Construct a TM diagram to recognize/decide a language
  - Give an implementation-level description of a TM that recognizes/decides a language
  - Give a high-level description of a TM that recognizes/decides a language
- Proofs
  - Prove that a language is/is not context-free using closure properties
  - Prove that a language is decidable
  - Prove that Turing-recognizable/decidable languages are closed under some operation

Exam question break down

- Five true/false questions (4 points each)
- Two constructions (20 points each)
- Two proofs (20 points each)

No pumping lemma for context-free languages questions for this exam (but possibly on the final)

Examples

1. Give a CFG that generates the language
   $A = \{w \mid w \in \{\mathtt{a}, \mathtt{b}\}^* \text{ contains at least 3 as}\}$
2. Give a PDA that recognizes the language
   $B = \{w \mid w \in \{\mathtt{a}, \mathtt{b}\}^* \text{ has odd length and the middle symbol is } \mathtt{b}\}$
3. Give an implementation-level description of a TM that decides the language
   $C = \{w \mid w \in \{\mathtt{a}, \mathtt{b}\}^* \text{ does not contain twice as many as as bs}\}$
4. Prove that decidable languages are closed under union
5. Prove that Turing-recognizable languages are closed under union
6. Prove that
   $\textsc{Infinite}_{\text{DFA}} = \{\langle M \rangle \mid M \text{ is a DFA and } L(M) \text{ is an infinite language}\}$ is decidable
7. Prove that $D = \{\langle G \rangle \mid G \text{ is a CFG over } \{0, 1\} \text{ and } \underline{1^*} \cap L(G) \neq \varnothing\}$ is decidable