Homework 4

Due: 8:00am Monday April 1 in Gradescope.

Only if you have a valid excuse (hospitalization, etc), you may submit by 8:00am Wed April 3 in Gradescope.

1. Exercise 2.12
2. Problem 2.31
3. Show using the pumping lemma that the language

   $$L_1 = \{ww \mid w \in \{0,1\}^*\}$$

   is not context-free.
4. Show that the following langauge is context-free.

   $$L_2 = \{a^n b^n \mid n \neq 100, n \geq 0\}$$

   Assume the alphabet $\Sigma = \{a, b\}$.
5. Give an implementation-level description of a Turing Machine that decides the following language. Assume the alphabet $\Sigma = \{0, 1\}$.

   $$L_3 = \{w \mid w \text{ contains twice as many } 0s \text{ as } 1s\}$$

6. Give a state transition diagram for the language E described on page 175 in Example 3.12. (An implementation-level description of the Turing machine is given on page 175. You need to fill in additional details, and create the state transition diagram.)
7. Show that the collection of Turing-recognizable languages is closed under:
   (a) union
   (b) intersection

   Hint: you can use proof by contruction.