

## Homework 5

**Due: 8:00am Monday April 15 in Gradescope.**

**Only if you have a valid excuse (hospitalization, etc), you may submit by 8:00am Wed April 17 in Gradescope.**

1. Show that the collection of decidable languages is closed under:

- (a) union
- (b) intersection
- (c) concatenation
- (d) star
- (e) complementation

Hint: you can use proof by construction.

2. Show that the collection of Turing-recognizable languages is closed under:

- (a) concatenation
- (b) star

Hint: you can use proof by construction.

3. Let  $ALL_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$ . Show that  $ALL_{DFA}$  is decidable.

4. Let  $E_{TM} = \{\langle M \rangle \mid M \text{ is a Turing Machine and } L(M) = \emptyset\}$ . Show that  $\overline{E_{TM}}$ , the complement of  $E_{TM}$ , is Turing-recognizable.

5. Let  $\mathcal{B}$  be the set of all infinite sequences over  $\{0, 1\}$ . Show that  $\mathcal{B}$  is uncountable using a proof by diagonalization.