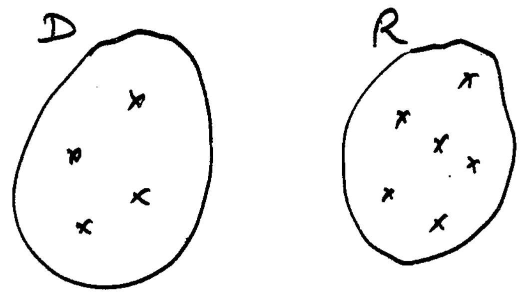
Computability

Complexity

Automata

- Sets : $\{\_, \_, \_\}$
  multi-set
  $\cap$ $\cup$
  Venn Diagrams
  $\overline{S} = \mathcal{U} - S$
  DeMorgan's Laws : $\overline{A \cup B} = \overline{A} \cap \overline{B}$
  $\overline{A \cap B} = \overline{A} \cup \overline{B}$
  $|S|$
  cross-product
- Sequences : $(\_, \_, \dots)$ $\langle \_, \_, \dots \rangle$
  (a.k.a. tuple)
- Function $f : D \to R$
  $f(a_1, a_2 \dots a_k) \to R.$
  $D_1 \times D_2 \times \dots D_k$



'onto' vs 'into'

- Predicate : T F
  (property)
- Relation $\approx$ Predicate
  → Equivalence — Reflexivity, Symmetry, Transitivity
  ↳ often expressed as set of Tuples that map to 'T'

Graphs : (V, E)

- undirected
- directed

→ paths

→ fully connected graph : each pair of nodes has a path

→ degree of a node

→ strongly connected graph :
(for directed graphs)

→ k-regular graph : Every node has degree 'k'

Boolean Logic :

- literals
- $\wedge \ \vee \ \neg \ \rightarrow \ \oplus$

(*) (essential operators are $\wedge, \neg$)

- $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$
- $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

} distributive laws.

(*) egs: $p \vee q = \neg(\neg p \wedge \neg q)$

$p \rightarrow q = \neg p \vee q$

Languages :

Symbols

Alphabet : set of symbols

Strings : a finite sequence of symbols from alphabets

Language : a set of "valid or legal" strings

- "generated" by a grammar (rule set
- by definition

Strings $\longrightarrow$ ϵ (ε) "empty string"

concat

w, $|w|$

w^R

lexicographic

<u>Proofs:</u> Theorem

Lemma

Corollary

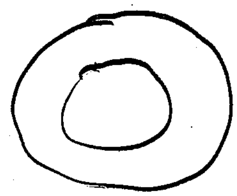
Axioms / Definition

<u>Proof techniques:</u> Induction, Contradiction, Construction

Counter-example

LHS iff RHS

$\overline{A \cup B} = \bar{A} \cap \bar{B}$



<u>Theorem</u>: $\forall n \mid n$ is even and > 2, there exists 3-regular graph,

Graph $= (V, E)$; $E = \{(i, i+1), (i, i-1), (i, i+\frac{n}{2})\}, \forall i$

// mod n

<u>Theorem</u>: n is even $\overset{\Longrightarrow}{\underset{\Longleftarrow}{\text{iff}}}$ n^2 is even

($\Longrightarrow$) Let $n = 2k$

$n^2 = 4k^2$

$\Longleftarrow$ (alternate) if n is odd, n^2 is od.

Theorem : $\sqrt{2}$ is ir-rational // rational # is $\frac{a}{b}$ of irreducibl

Pf : (contradiction)

$$\sqrt{2} = \frac{a}{b}$$

$$\sqrt{2}\,b = a$$

$$2b^2 = a^2$$ // a is even $= 2\alpha$

$$2b^2 = (2\alpha)^2$$

$$2b^2 = 4\alpha^2$$

$$b^2 = 2\alpha^2$$ // b is even $= 2\beta$

$$(2\beta)^2 = 2\alpha^2$$

$$4\beta^2 = 2\alpha^2$$

$$2\beta^2 = \alpha^2$$

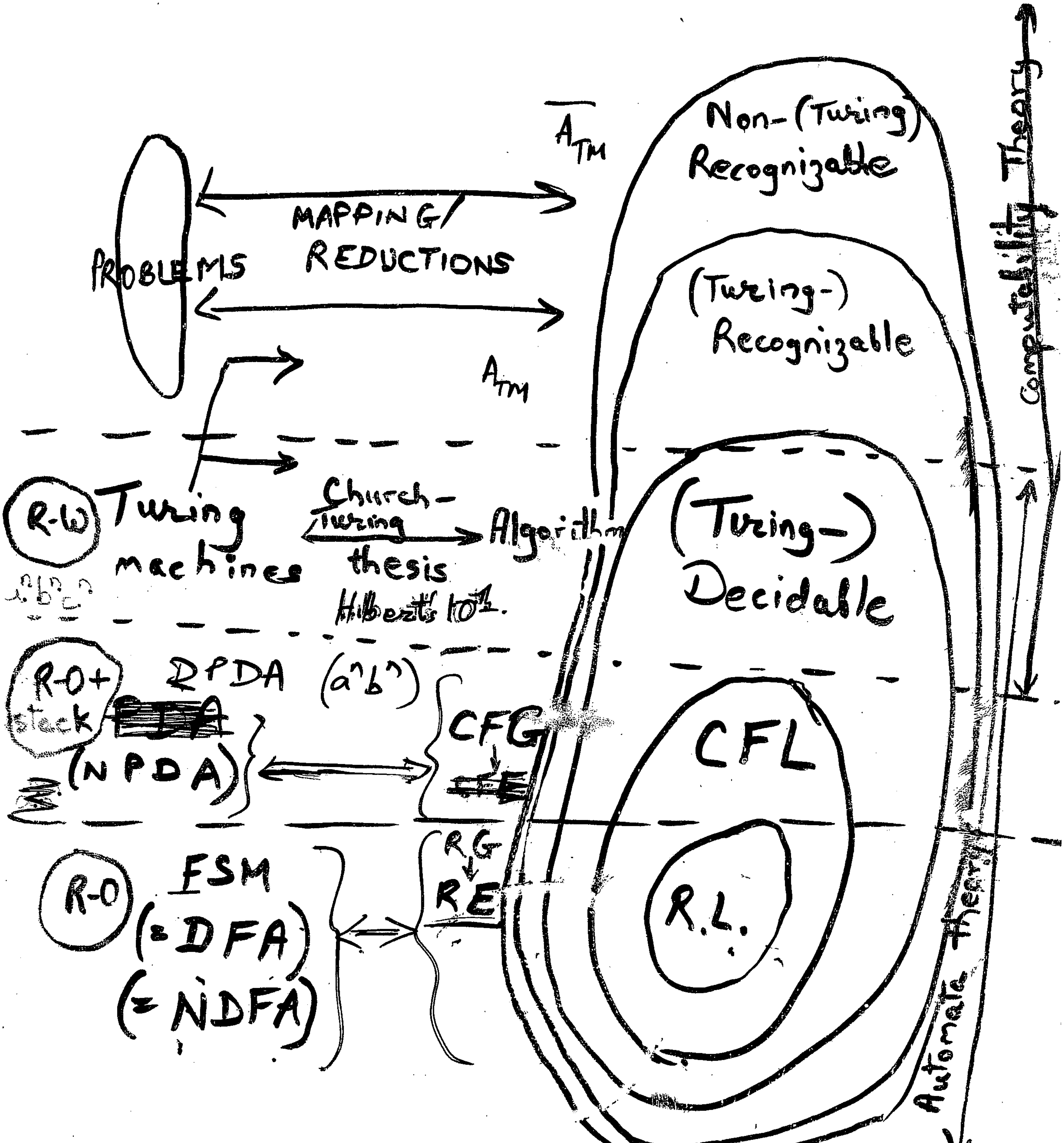
Kurt Godel : 1906 – 1978

Alonzo Church : 1903 – 1995 (λ-calculus)

Alan Turing : 1912 – 1954 (Turing machine)

Church-Turing thesis.

Noam Chomsky : 1928 — .


Non-(Turing) Recognizable
(Turing-) Recognizable
(Turing-) Decidable
CFL
R.L.
Computability Theory
Automata Theory
$\overline{A_{TM}}$
MAPPING/ REDUCTIONS
PROBLEMS
A_{TM}
R-W
Turing machines
$a^n b^n c^n$
Church-Turing thesis
Algorithm
Hilbert's 10th
R-O+ stack
DPDA ($a^n b^n$)
~~PDA~~
(NPDA)
CFG
~~CFL~~
R-O
FSM
(= DFA)
(= NDFA)
RG
RE

R.G.	R.L.	DFA = NDFA	
			$\subset$
DCFG	DCFL	DPDA	
			$\subset$
NCFG	NCFL	NDPA	
			$\subset$
(D) CSG	(D) CSL	(D) Lin. Bounded TM	
			$\subset / = ?$
(N) CSG	(N) CSL	(N) Lin. Bounded TM	
			$\subset$
?	Turing Decidable	?	
			$\subset$
Unrestricted grammar	Turing-Recognizable	(D=N) TM	
			$\subset$
	Turing-Unrecognizable		