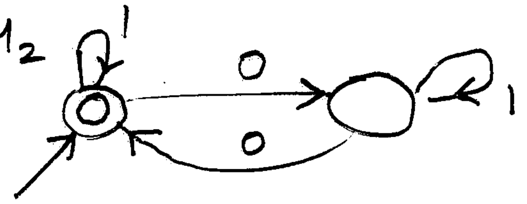
$L_1 = \{ x \mid x$ has equal number of '0's and '1's $\}$

$L_2 = \{ x \mid$ ______________ '01' and '10' substrings $\}$

DFA $\longleftrightarrow$ NDFA $\longleftrightarrow$ GNFA $\longleftrightarrow$ reg expr. $\longleftrightarrow$ reg lang

($\longleftrightarrow$ reg grammar)

M_2



PUMPING LEMMA: For <u>any</u> regular lang. A,

$\exists$ integer 'p' (pumping length) $\mid$

$\forall s \in A$ and $|s| \geq p$,

$s = x \circ y \circ z$ and 1) $x \circ y^i \circ z \in A \ (\forall i \geq 0)$

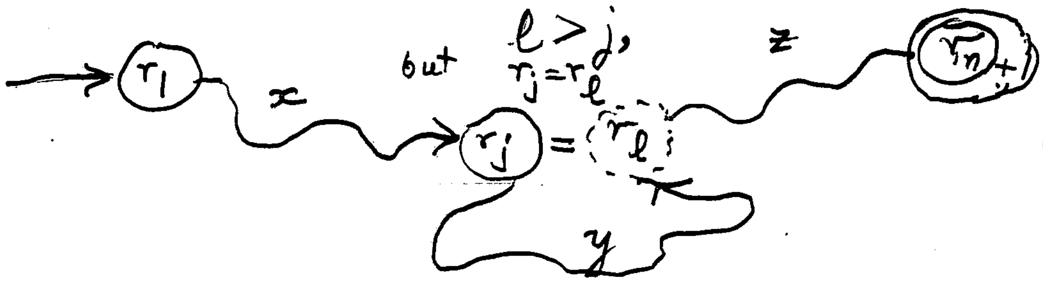
2) $|y| > 0$ // y^*?

3) $|x \circ y| \leq p$

Let $p = |Q|$ for a DFA:

$S = s_1 \quad s_2 \quad s_3 \quad \ldots \quad s_{p-1} \quad s_p] \quad s_{p+1} \ldots s_n]$

$[r_1 \quad r_2 \quad r_3 \quad r_4 \quad \ldots \quad r_p \quad r_{p+1}] \ldots \quad r_{n+1}$



(pigeon hole principle)

Use <u>contra-positive</u> to show Language is NOT regular
$\left(\begin{array}{l} a \rightarrow b \\ \neg b \rightarrow \neg a \end{array} \right)$

Identify some one $s \in A \mid (|s| \geq p)$
- no way to decompose $s = xyz$ while satisfying (1, 2, 3) together

Essentially, proof by contradiction.

<u>eg.</u> $L_1 = \{ 0^n 1^n \mid n \geq 0 \}$

Let L_1 be regular. $\Rightarrow$ 'p' exists // $\geq |Q|$ in <u>DFA</u>

Identify some $s \in A$ & show s cannot be pumped.

Let $s = 0^p 1^p$

Show: <u>no</u> way to express $s = x \circ y \circ z \mid$
- $x \circ y^i \circ z \in L_1$
- $|y| > 0$
- $|x \circ y| \leq p$

$$\underbrace{00 \quad\cdots\quad 00}_{p}\ \underbrace{11 \quad\cdots\quad 11}_{p}$$

Some non-regular languages

L_1 $\{0^n 1^n \mid n \geq 0\}$ $s = 0^p 1^p$

L_2 $\{w \mid w$ has equal # of 0^s & $1^s\}$ $s = 0^p 1^p$

L_3 $\{ww \mid w \in \{0,1\}^*\}$ $s = 0^p 1 0^p 1$

L_4 $\{0^i 1^j \mid i < j\}$ $s = 0^p 1^{p+1}$

L_5 $\{0^i 1^j \mid i > j\}$ $s = 0^{p+1} 1^p$

L_6 $\{0^n 1^n 2^n \mid n \geq 0\}$ $s = 0^p 1^p 2^p$

L_7 $\{0^m 1^n \mid m \neq n\}$ $s = 0^p 1^{p+p!}$

L_8 $\{a^{n^2} \mid n \geq 0\}$ $s = a^{p^2}$ // $p^2 + p < (p+1)^2$

L_9 $\{a^{2^n} \mid n \geq 0\}$ $s = a^{2^p}$ // $2^p + p < 2^{p+1}$

$L_2 = \{ w \mid w \text{ has equal \# of } 0^s \text{ \& } 1^s \}$

1] Let $s = 0^p 1^p$

Observe, $s = xyz$, y must have $\le p$ 0^s

$$\underbrace{000 \qquad\qquad 00}, \underbrace{111 \qquad\qquad 11}$$

2] $L_2 \cap 0^* 1^* = \underbrace{0^n 1^n}_{L_1 \text{ (not regular)}}$

$L_3 = \{ ww \mid w \in \{0,1\}^* \}$

Let $s = 0^p 1 0^p 1$

$L_7 = \{ 0^m 1^n \mid m \neq n \}$

$\cancel{\overline{L_7} = L_1}$

$(\overline{L_7} \cancel{= L_1}) \cap 0^* 1^* = \{ 0^k 1^k \mid k \geq 0 \}$

$= L_1$ (not regular)

OR pumping Lemma. Let $s = 0^p 1^{p+p!}$ eg: $p = 5$

$$\overbrace{000 \cdots 0}^{p}\ \overbrace{111 \qquad\qquad 11}^{5 \times 4 \times 3 \times 2 \times 1 = 120}\ \overbrace{11111}^{p=5}$$

pump $\frac{120}{x}$ times, where $x = \#$ states in loop

$L_8 = \{ a^{n^2} \mid n \geq 0 \}$, use $s = a^{p^2}$ then $p^2 + p < (p+1)^2$

Seq has $1, 4, 9, 16 \ldots n^2, (n+1)^2 \ldots\ldots$

Pick 's' long enough | ?