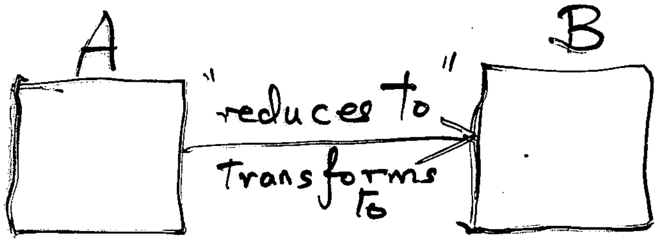
REDUCIBILITY

- Technique to show problems are not solvable.
  // "relating" the new problem to another problem that we know is (not) solvable



eg $A_{TM} \xrightarrow{\text{reduce}} HALT_{TM}$

- a soln to B can be used as soln to A (// show how)

- solving A cannot be harder than solving B

$\therefore$ If B is decidable, A is decidable

&

$\therefore$ || If A is undecidable, then B is also undecidable.
(eg A_{TM})

[contrapositive]

KEY/MAIN BURDEN: show how soln to B [ie. decider of B] can be used as a soln to A [ie. decider of A].

HALTING PROBLEM. // $A_{TM} = \{ \langle M, w \rangle \mid M \text{ accepts } w \}$

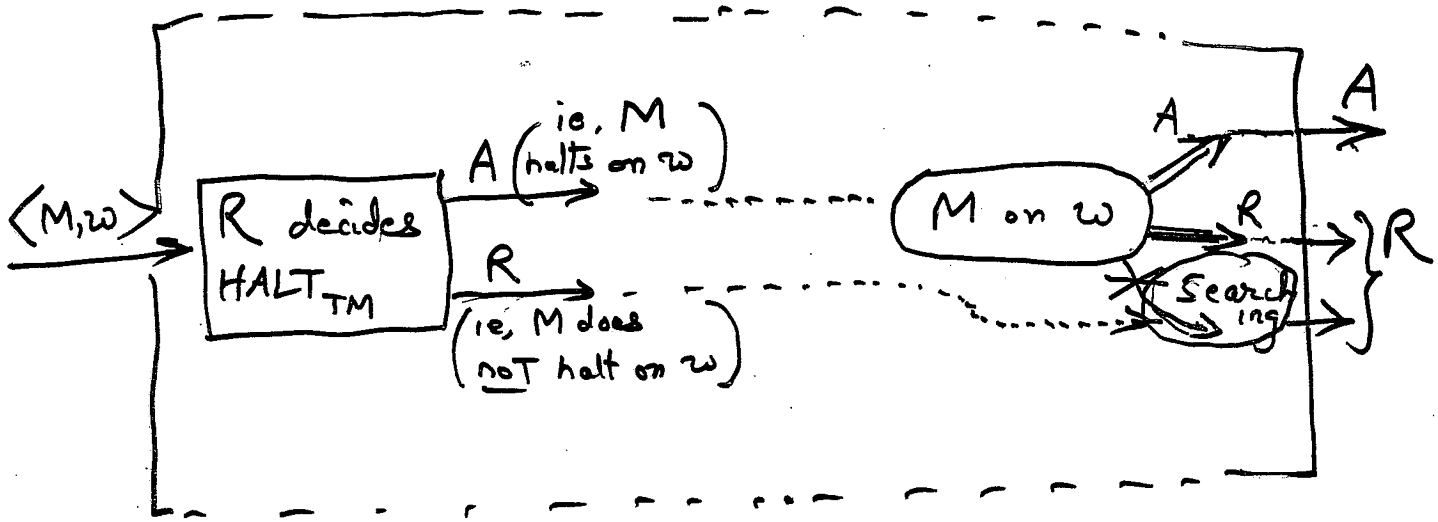
$HALT_{TM} = \{ \langle M, w \rangle \mid M \text{ halts on } w \}$

Th: $HALT_{TM}$ is undecidable.

Proof: Reduce A_{TM} to $HALT_{TM}$.

(Contradiction:) If 'R' decided $HALT_{TM}$, then A_{TM} also be decidable.

SHOW how soln (decider) of $HALT_{TM}$ can be used as a soln (decider) to A_{TM}.



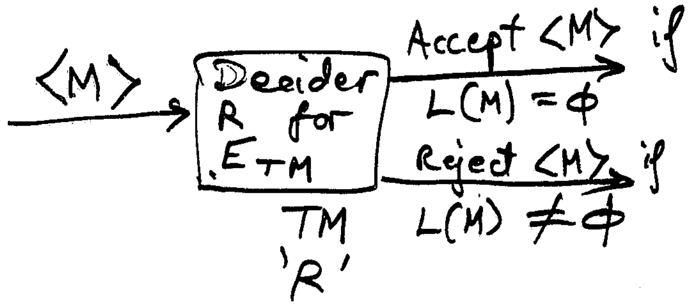
'Decider' for A_{TM}

Proof: (construct a TM S to decide A_{TM}, using TM R that decides $HALT_{TM}$

S: On input $\langle M, w \rangle$:

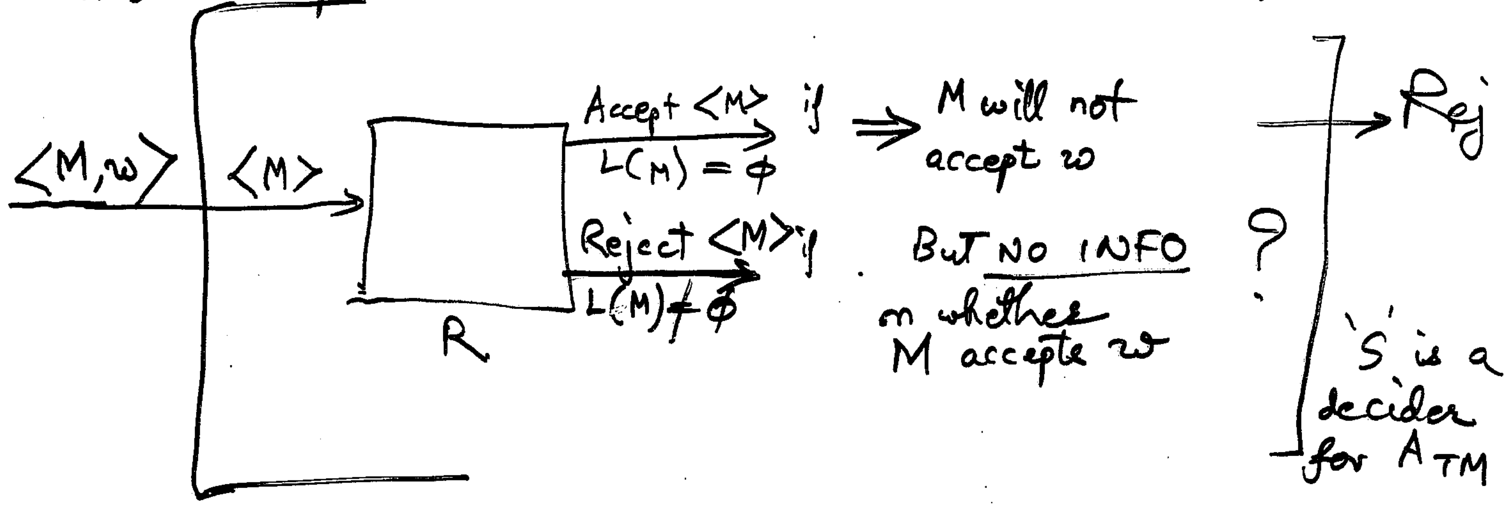
1) Run R on input $\langle M, w \rangle$
2) If R rejects $\langle M, w \rangle$, then REJECT
Else R accepts $\langle M, w \rangle$, then simulate M on w //must halt
— If M accepts w, then ACCEPT
— else (M rejects w), then REJECT

$E_{TM} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \phi\}$



Is E_{TM} decidable? i.e., is it possible to build such a TM as in box above?

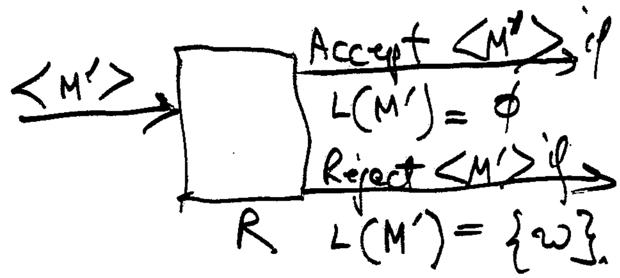
Initial attempt



Show how a decider TM S for A_{TM} can be built using a decider TM R for E_{TM}.

Hint: Modify M to M′ such that $L(M') = \begin{cases} \{w\} & \text{if } M \text{ accepts } w \\ \phi & \text{if } M \text{ rejects } w \text{ or keeps searching.} \end{cases}$

Now, if



M' (built using input $\langle M, w\rangle$): On any input x:

1) If $x \neq w$, then reject // $x \notin L(M')$

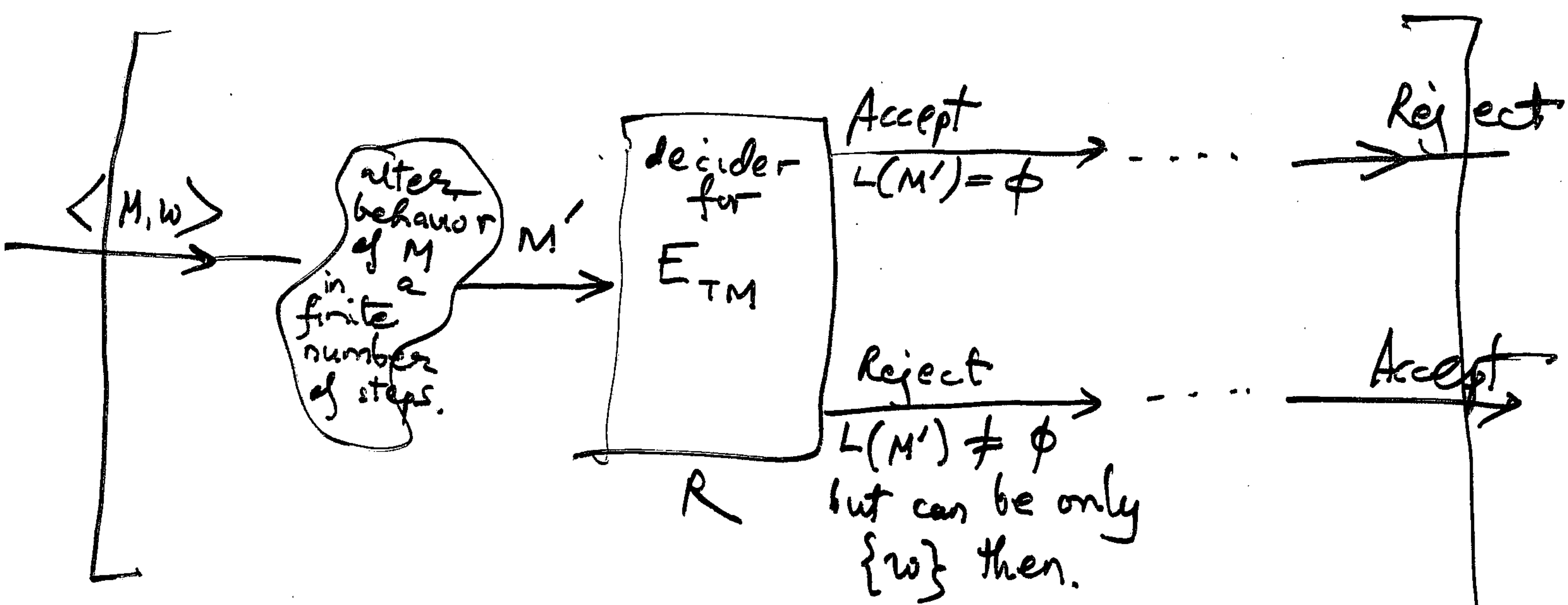
(M' behaves as M would)

2) Else $x = w$, then ~~run M~~ on w and accept if M accepts w.

// $x = w$; then $x \in L(M')$ if M accepts w

S: On input $\langle M, w\rangle$

1) Construct M' using description of M, and w
2) Run R, the decider for E_{TM}, on $\langle M'\rangle$
3) If R accepts then S rejects
else R rejects then S accept

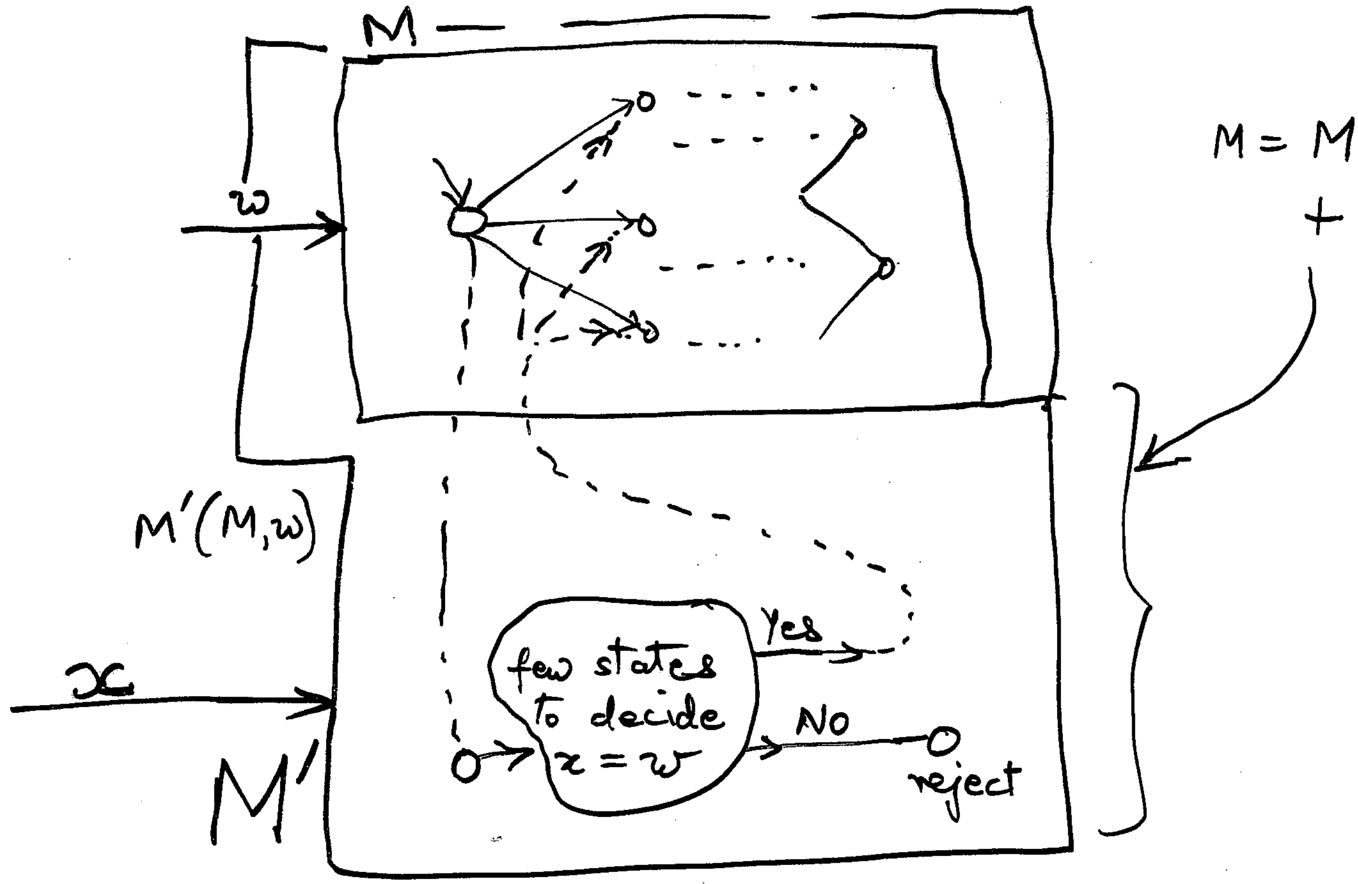


<u>Note</u>: M' need not be a decider!

M' is a recognizer of a single-string language $= \{w\}$

~~M' behaves like M~~ when input x to M' is equal to w

$M = \langle Q, ---, \delta, -,- \rangle$



eg let $L(M) = \{w, w', w''\}$.

$L(M') = ?$

if $x = w'''$, w'', w } reject

$x = w$ } accept if M accepts w