1. **Dijkstra’s all-pairs shortest path algorithm (2+2+2+2=8 points)** For Dijkstra’s all-pairs shortest path algorithm, \( W = \Theta(n^3) \). For \( p \) processors and \( n \) partitions, with \( p > n \), we have \( n \) groups of processors of size \( p/n \) each. Therefore, on a hypercube, we have

\[
T_p = \Theta(n^3/p) + \Theta(n \log p)
\]

The first term represents the computation complexity, the second term, the communication complexity.

(a) What is the efficiency?
(b) What is the condition for cost-optimality?
(c) What is the isoefficiency function due to communication?
(d) What is the isoefficiency function due to concurrency?

2. **Load balancing (4+5=9 points)** To analyze load balancing for a parallel depth-first search, it is useful to define \( V_p \) as the number of work requests such that, after every \( V_p \) work requests, each processor receives at least one work request. Consider the random polling scheme for load balancing.

(a) Observe that for load balancing using random polling to request for more work, \( V_p \) is unbounded. Show the steps to calculate the *average-case* value of \( V_p \).

(b) What are the different isoefficiency functions that come into play when random polling is implemented on a Network-of-Workstations (Ethernet) topology? Calculate the overall isoefficiency function for this topology.

3. **Bitonic sorting (2+3+1+2=8 points)**

(a) Formulate the recurrence relation to compute the depth of a bitonic sorting network. Solve it.

(b) For a hypercube of dimension \( d \) having \( 2^d = p \) processors, and for an input of \( n \) elements to be sorted (assume \( n > p \)), derive the parallel run-time \( T_p \)

(c) Continuing part (b), derive the efficiency.

(d) Continuing (b) and (c), derive the isoefficiency function due to communication.

4. **Hypercube broadcast (9 points)** Give the pseudo-code for the *one-to-all broadcast* on a \( d \)-dimensional hypercube. You may assume the source is node 0. Add comments as necessary to explain the code.