t-BNE: Tensor-based Brain Network Embedding

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Brain Network

Neuroimaging

Brain network
Graph Classification

• Existing work
  • Computing graph-theoretical measures.
  • Extracting subgraph patterns.

• This work
  • Learning latent representations via tensor factorization.
  • Embedding the graph in addition to embedding nodes.
Problems & Solutions

- (P1) How can we preserve the graph property in the tensor factorization process?
Problems & Solutions

- (P2) How can we leverage the side information associated with brain networks?
(P3) How can we fuse the classifier training and the representation learning procedures?
Problems & Solutions

Partially coupled matrix and tensor factorization
Tensor-based Framework

(S1) Partially symmetric tensor factorization

\[
\min_{B, S, W} \left\| \mathcal{X} - C \times_1 B \times_2 B \times_3 S \right\|_F^2
\]

\[
\begin{align*}
&+ \alpha \text{tr}(S^T L Z S) \\
&+ \beta \|DSW - Y\|_F^2 \\
&+ \gamma \|W\|_F^2
\end{align*}
\]

(S2) Side information guidance

\[
S^TS = I
\]

orthogonality

(S3) Partially coupled matrix and tensor factorization

\[
(3.7)
\]
Optimization: ADMM

Algorithm 1 t-BNE

Input: $\mathcal{X}, Z, Y, \alpha, \beta, \gamma$

Output: $B, S, W$

1: Set $\mu_{max} = 10^6$, $\rho = 1.15$
2: Initialize $B, S, W \sim \mathcal{N}(0, 1)$, $U = 0$, $\mu = 10^{-6}$
3: repeat
4: Update $B$ and $P$ by Eq. (3.11) and Eq. (3.13)
5: Update $U$ by Eq. (3.14)
6: Update $\mu$ by $\mu \leftarrow \min(\rho \mu, \mu_{max})$
7: Update $S$ by Eq. (3.16) with the curvilinear search
8: Update $W$ by Eq. (3.18)
9: until convergence

code available at: https://www.cs.uic.edu/~bcao1/code/t-BNE.zip
Experiments

Datasets: neutral, maintain, reappraise

37 patients with anxiety disorder (positive samples), and 32 healthy participants (negative samples)

n = 69 samples
m = 34 scalp channels
Classification Performance

<table>
<thead>
<tr>
<th>Methods</th>
<th>Neutral</th>
<th>Maintain</th>
<th>Reappraise</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-BNE</td>
<td>0.7833</td>
<td>0.7548</td>
<td>0.7524</td>
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<tr>
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<td>Rubik</td>
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<tr>
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<tr>
<td>gMSV</td>
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<td>0.6548</td>
<td>0.5952</td>
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<tr>
<td>CC</td>
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<td>0.6667</td>
<td>0.5357</td>
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</tbody>
</table>

- t-BNE: The proposed tensor factorization model for brain network embedding.
- CMTF: Coupled matrix and tensor factorization where brain networks and side information are coupled in the subject mode.
- Rubik: Tensor factorization with orthogonality and sparsity constraints.
- ALS: Tensor factorization using alternating least squares without any constraint.
- gMSV: A discriminative subgraph selection approach using side information.
- CC: Extracting local clustering coefficients as features.
Parameter Sensitivity

![Graphs showing parameter sensitivity with different scales for k, γ, α, and β.](Image)
Factor Analysis

(a) **NEUTRAL.**

(b) **MAINTAIN.**

(c) **REAPPRAISE.**
Summary

• Preserved the symmetric graph property in tensor factorization

• Incorporated side information guidance and orthogonal constraint to obtain informative and distinct latent factors

• Fused the classifier learning procedure and tensor factorization

• Facilitated better understanding of brain mechanism with anxiety disorder under different emotion regulations
Extensions

• Guidance
  • Column-wise guidance from community information
• Supervision
  • Must-link, cannot-link, separability
• Multimodality
  • Joint tensor factorization to capture consensus information between fMRI and DTI
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Q & A

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