CS 505 Spring 2025 — Homework 2

YOUR NAME HERE (FIRST AND LAST) (UID: YOUR UID HERE)

Due Date: February 27, 2025, no later than 2:00pm Central Time.

Collaboration Policy

Collaboration between students is encouraged. However, **all collaborations** need to be acknowledged (whether they are in this class or not in this class). You **MUST** list all collaborators for homework assignments. Moreover, collaborating **does not mean** you can copy-paste work from each other. Each submission needs to be in the students own words, otherwise it will be considered plagiarism.

Moreover, you are allowed to look to other resources for help with the homework. Please correctly cite such resources by using the `\cite` command, and including the correct citations in `local.bib`.

Finally, please acknowledge any other discussions that helped you complete this assignment. This can include "office hours," "Piazza," or other discussions where a direct collaboration did not happen.

Collaborator and Discussion Acknowledgements

Please list your collaborators below. Include their First and Last names, along with their UID if they are a UIC student. If you did not collaborate with others for this assignment, please copy-paste the following line into the first item of the `itemize` below.

I worked on this assignment individually and did not collaborate with others.

- Collaborator 1...

1 Diagonalization (25 Points)

1.1 Part 1 (10 Points)

Prove that if **NEXP** $\neq$ **EXP** then **NP** $\neq$ **P**.

Hint: You can use Problem 3 Part 2 from Homework 1 to help. We recall it here. Define a function $\text{pad}\colon \{0,1\}^* \times \mathbb{N} \to \{0,1\}^* \times \{\#\}^*$ as follows. For $x \in \{0,1\}^*$ and $k \in \mathbb{N}$, $\text{pad}(x,k) = (x, \#^j)$, where $j = \max\{0, (k - |x|)\}$ and $\#^j$ denotes writing the symbol $\#$ j times (e.g., $\#^3 = \#\#\#$).

Let $A \subseteq \{0,1\}^*$ be any language and let $T\colon \mathbb{N} \to \mathbb{N}$ be a function. Let $\mathbf{pad}(A,T)$ denote the following language:

$$\mathbf{pad}(A,T) = \{\text{pad}(x, T(|x|)) \mid x \in A\}.$$

If $A \in \mathbf{DTIME}(n^6)$ then $\mathbf{pad}(A, n^2) \in \mathbf{DTIME}(n^3)$.

Proof of Problem 1 Part 1. Your answer here... □

1.2 Part 2 (10 Points)

Define the *unique-SAT* problem as

$$\text{USAT} = \{\phi : \phi \text{ is a Boolean formula with exactly 1 satisfying assignment}\}.$$

Prove that USAT $\in \mathbf{P}^{\text{SAT}}$.

Proof of Problem 1 Part 2. Your answer here... □

1.3 Part 3 (5 Points)

Prove that $\mathbf{DSPACE}(n) \neq \mathbf{NP}$.

Hint: Use the Space Hierarchy Theorem and the fact that **NP** *is closed under polynomial-time reductions.*

Proof of Problem 1 Part 3. Your answer here... □

2 Space Complexity (25 points)

2.1 Part 1 (10 Points)

Let UCYCLE $= \{G : G$ is an undirected graph that contains a simple cycle.$\}$. Recall that in an undirected graph, a simple cycle is a path of vertices $(v_1, \ldots, v_k)$ such that $v_1 = v_k$ and for all $i, j \in \{1, \ldots, k-1\}$, we have $v_i \neq v_j$ whenever $i \neq j$ (i.e., all nodes are unique except the first and last). Prove that UCYCLE $\in \mathbf{L}$.

Note: G may not be a connected graph.

Proof of Problem 2 Part 1. Your answer here... □

2.2 Part 2 (15 Points)

Recall the language 2SAT $= \{\phi : \phi$ is a satisfiable 2CNF formula.$\}$. Show that 2SAT is **NL**-complete.

Proof of Problem 2 Part 2. Your answer here... □