CS 505 Spring 2025 — Homework 3

YOUR NAME HERE (FIRST AND LAST) (UID: YOUR UID HERE)

Due Date: March 20, 2025, no later than 2:00pm Central Time.

Collaboration Policy

Collaboration between students is encouraged. However, **all collaborations** need to be acknowledged (whether they are in this class or not in this class). You **MUST** list all collaborators for homework assignments. Moreover, collaborating **does not mean** you can copy-paste work from each other. Each submission needs to be in the students own words, otherwise it will be considered plagiarism.

Moreover, you are allowed to look to other resources for help with the homework. Please correctly cite such resources by using the `\cite` command, and including the correct citations in `local.bib`.

Finally, please acknowledge any other discussions that helped you complete this assignment. This can include "office hours," "Piazza," or other discussions where a direct collaboration did not happen.

Collaborator and Discussion Acknowledgements

Please list your collaborators below. Include their First and Last names, along with their UID if they are a UIC student. If you did not collaborate with others for this assignment, please copy-paste the following line into the first item of the `itemize` below.

I worked on this assignment individually and did not collaborate with others.

- Collaborator 1...

1 The Polynomial Hierarchy and Alternations (25 Points)

1.1 Part 1 (5 Points)

Show that if $3\text{SAT} \leq_p \overline{3\text{SAT}}$, then $\mathbf{PH} = \mathbf{NP}$.

Proof of Problem 1 Part 1. Your answer here... $\square$

1.2 Part 2 (10 Points)

Prove that $\mathbf{APSPACE} = \mathbf{EXP}$.

Hint: the non-trivial direction is $\mathbf{EXP} \subseteq \mathbf{APSPACE}$. *This proof will use ideas that were explored in the proof that* $\text{SAT} \notin \mathbf{TISP}(n^{1.1}, n^{0.1})$.

Proof of Problem 1 Part 2. Your answer here... $\square$

1.3 Part 3 (10 Points)

Suppose that there exists a language/oracle A such that $\mathbf{P}^A = \mathbf{NP}^A$. Prove that $\mathbf{PH}^A \subseteq \mathbf{P}^A$. (I.e., the proof that $\mathbf{P} = \mathbf{NP}$ implies $\mathbf{PH} = \mathbf{P}$ relativizes).

Proof of Problem 1 Part 3. Your answer here... $\square$

2 Randomized Computations (25 points)

2.1 Part 1 (10 Points)

Prove that **BPL** $\subseteq$ **P**, where **BPL** is the set of all languages in **BPP** that are decidable in on a PTM using at most $O(\log(n))$ additional space on the read/write tapes.

Hint: Try to compute the probability that the machine ends up in an accepting configuration using either dynamic programming or matrix multiplication.

Proof of Problem 2 Part 1. Your answer here... □

2.2 Part 2 (10 Points)

Prove that a language L is in **ZPP** if and only if there exists a polynomial-time PTM M with outputs $\{0, 1, \perp\}$ such that for every $x \in \{0,1\}^*$, the following holds:

$$\Pr[M(x) \in \{L(x), \perp\}] = 1 \quad \Pr[M(x) = \perp] \leq \frac{1}{2}.$$

Hint: recall that **ZPP** *is defined with respect to expected polynomial time. You will need to argue that if you require strict polynomial time (related to the expected runtime), the probability your computation hasn't halted when you reach the hard polynomial-time cutoff is upper bounded by* $1/2$.

Proof of Problem 2 Part 2. Your answer here... □

2.3 Part 3 (5 Points)

Prove that if **NP** $\subseteq$ **BPP**, then **NP** $=$ **RP**.

Proof of Problem 2 Part 3. Your answer here... □