CS 594 – Advanced Crypto (Spring 2026)

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Lecture 10

February 18, 2026

Function Secret Sharing

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- Function being shared: $f(X)\colon \{0,1\}^\ell \to \mathbb{G}$ where $f(\mathbf{y}) = x$, where $\mathbf{y}$ is the binary representation of $x \in \mathbb{G}$.

Function Secret Sharing

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$f : \{0,1\}^{\ell} \to \mathbb{G}$

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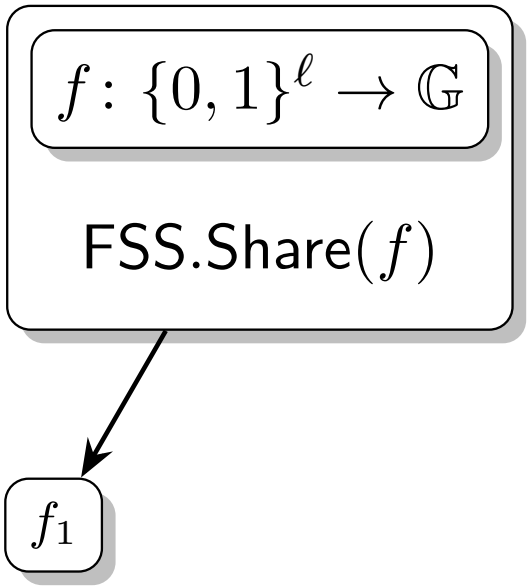
$\mathsf{FSS.Share}(f)$

Function Secret Sharing



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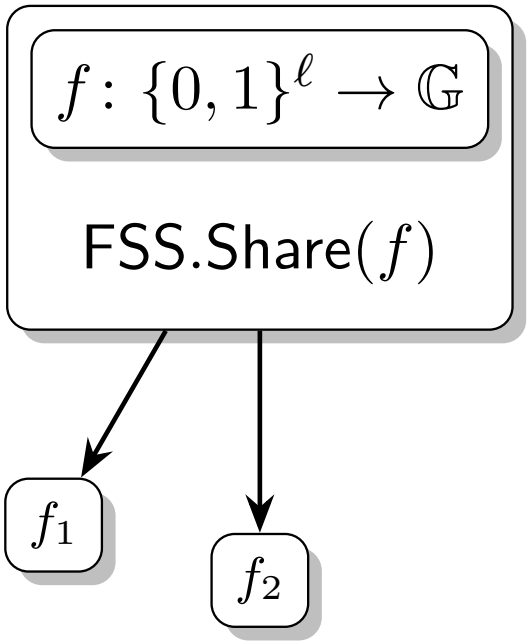

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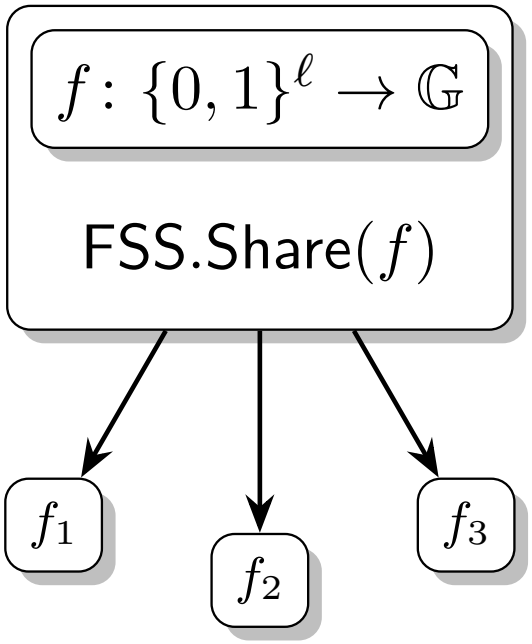

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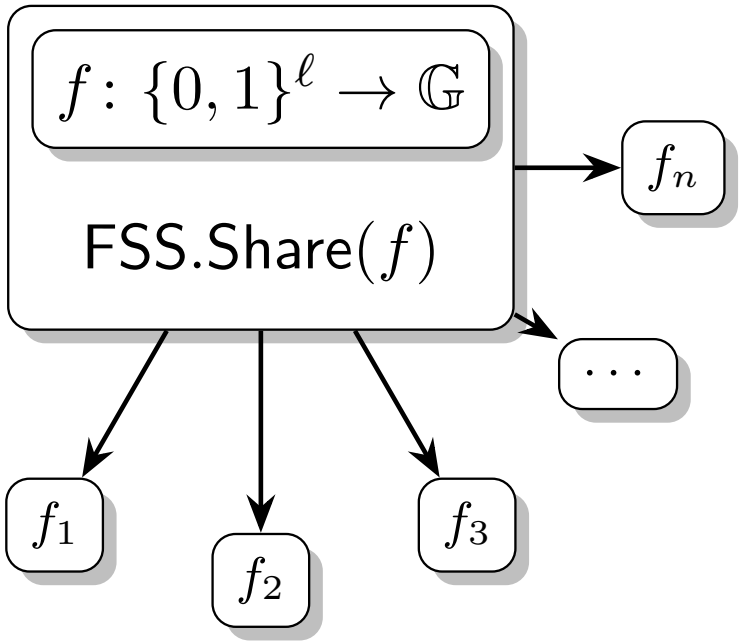

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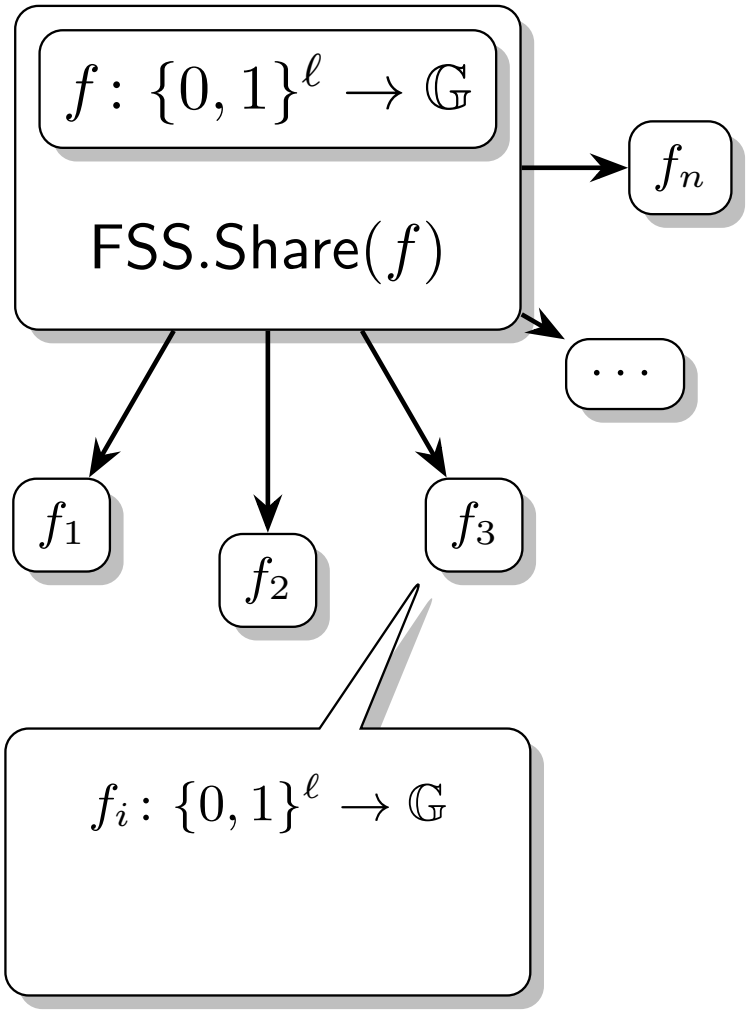

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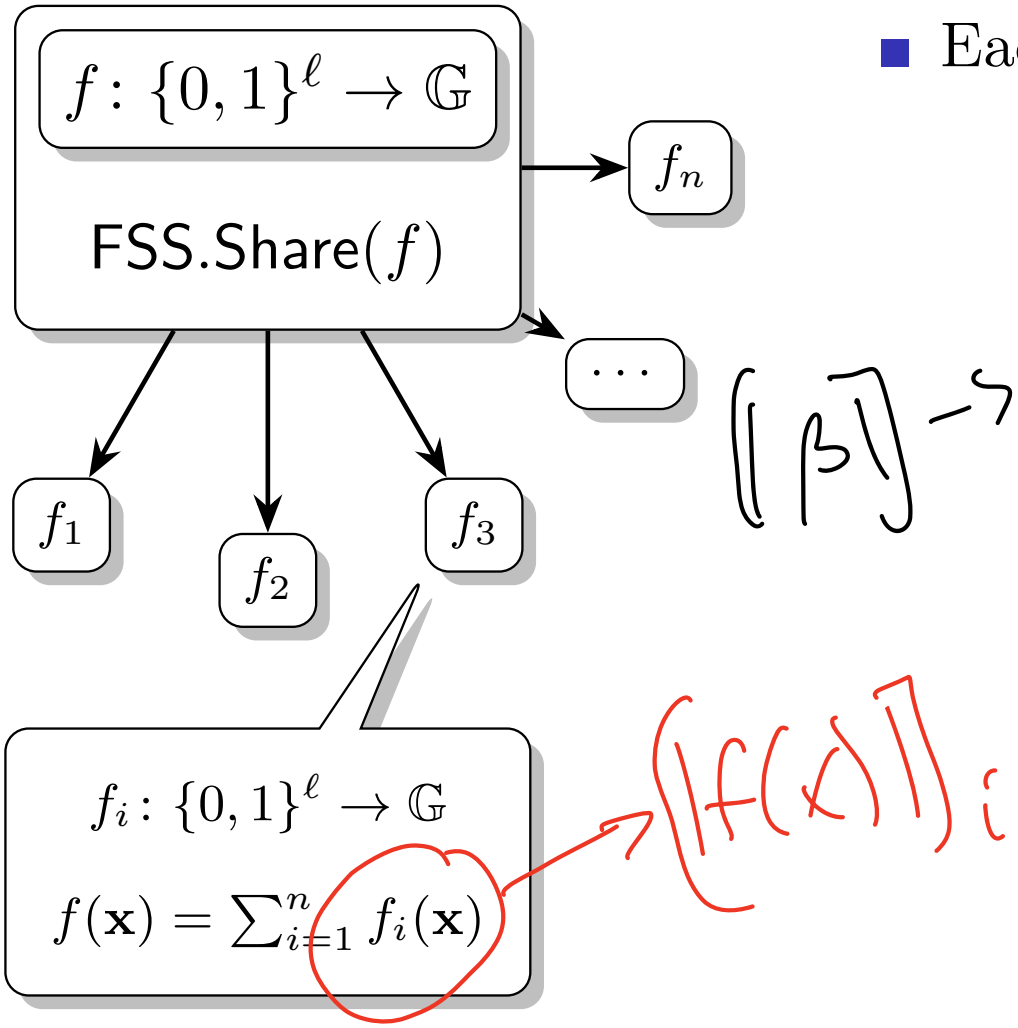

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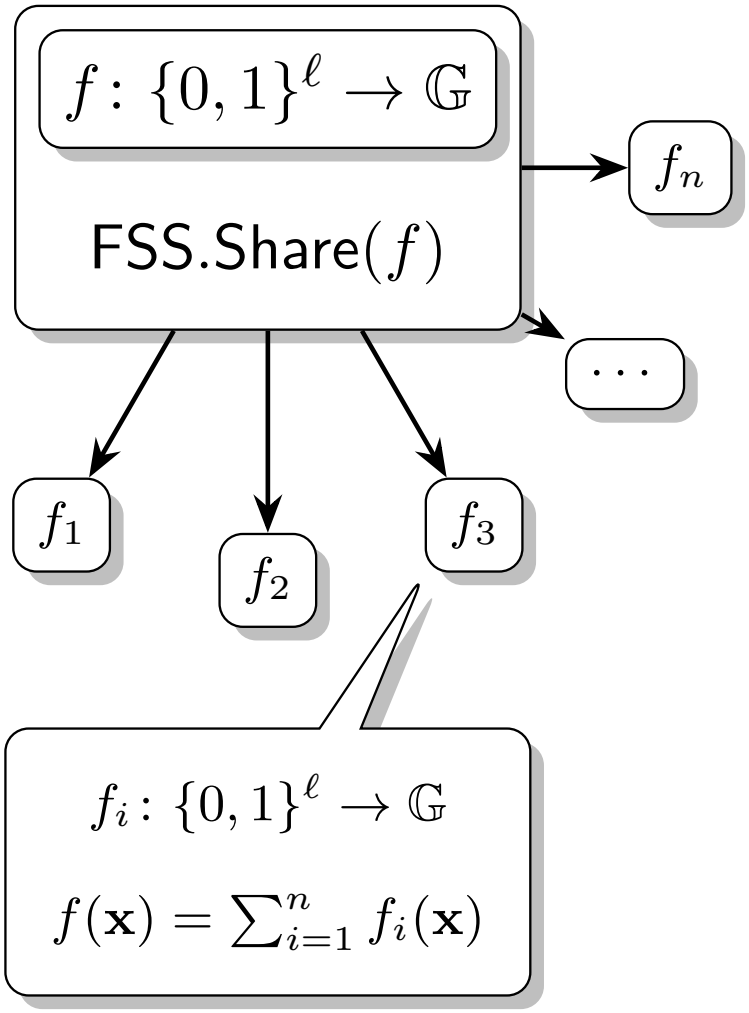

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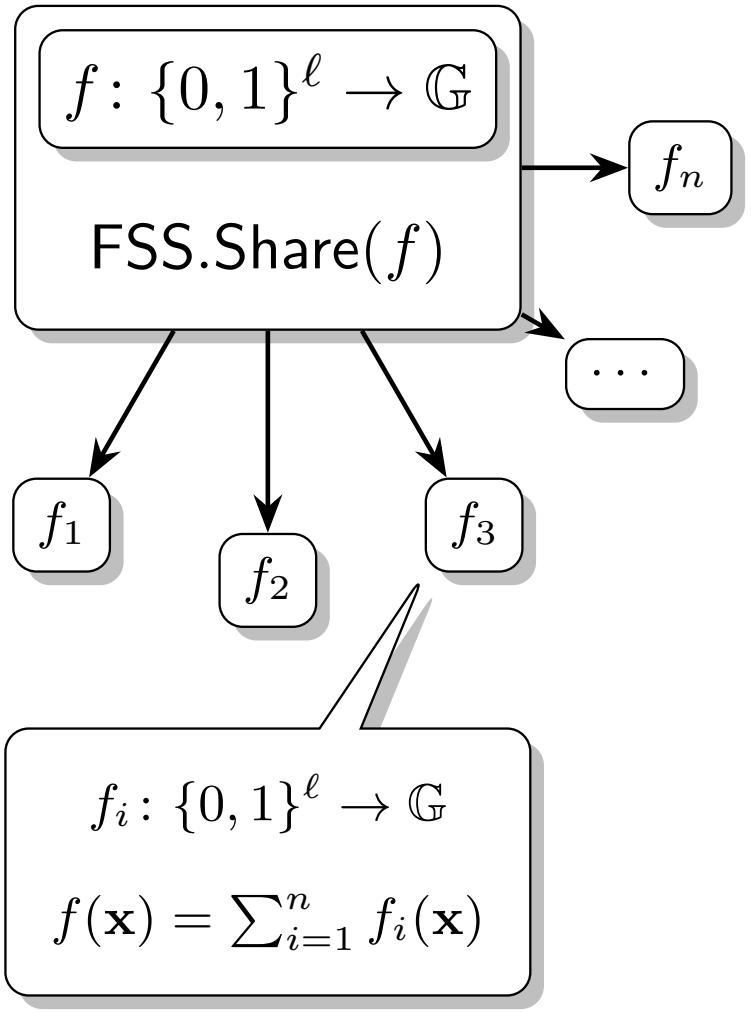

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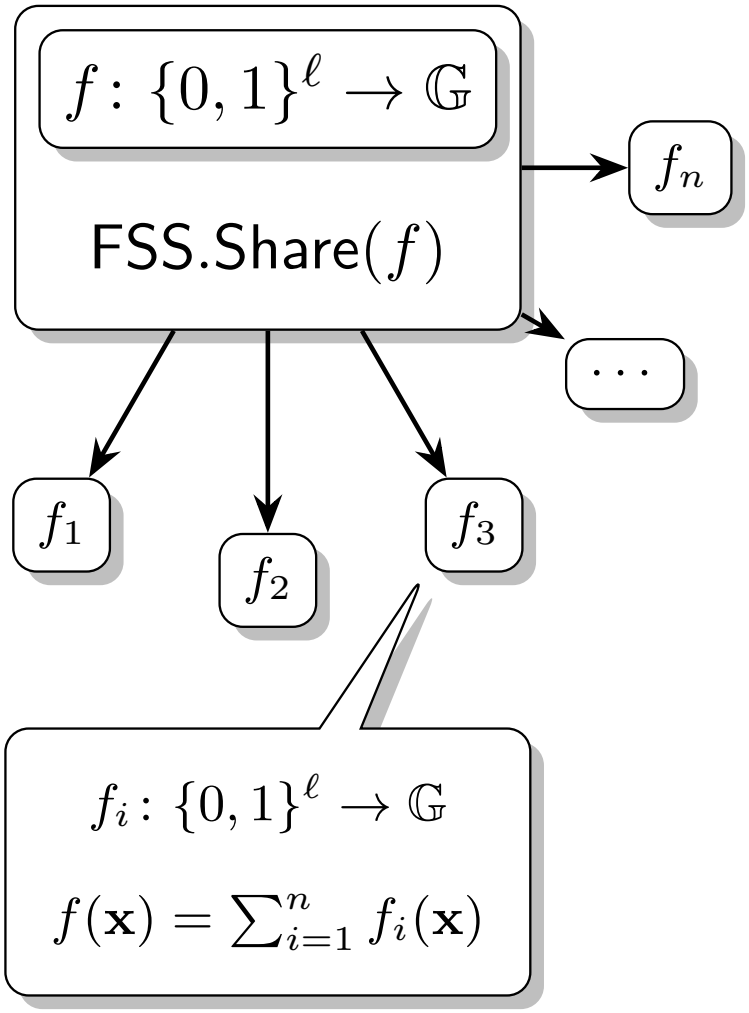

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11	g

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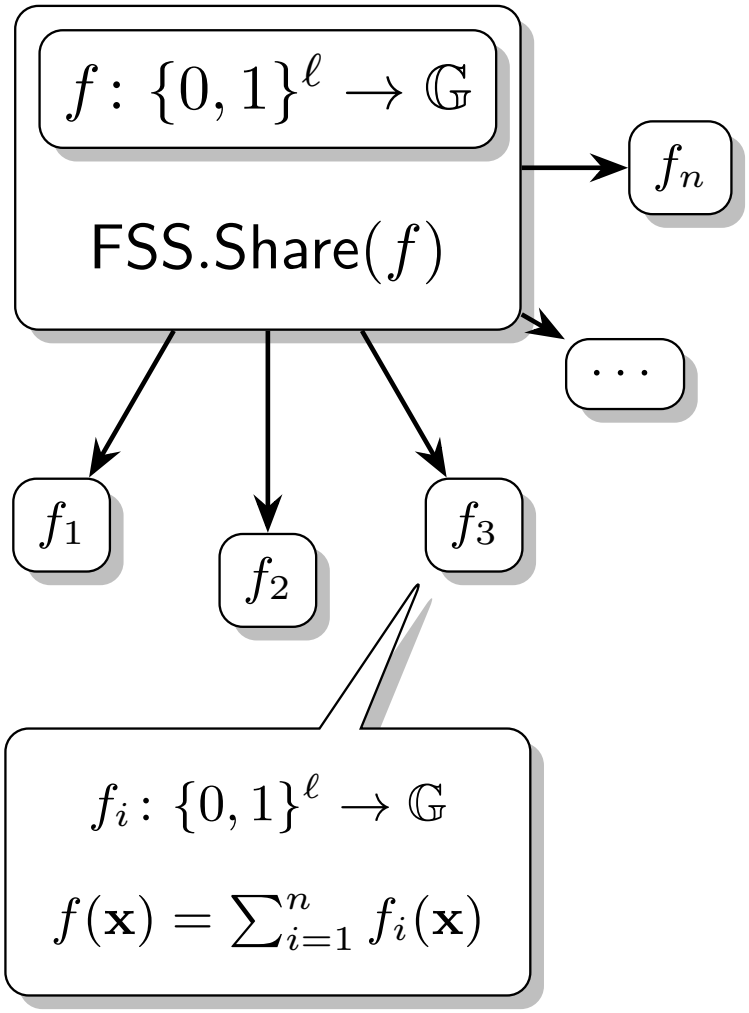

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(n, n) additive shares for each entry

↳ n shares, give each party 1 share per row

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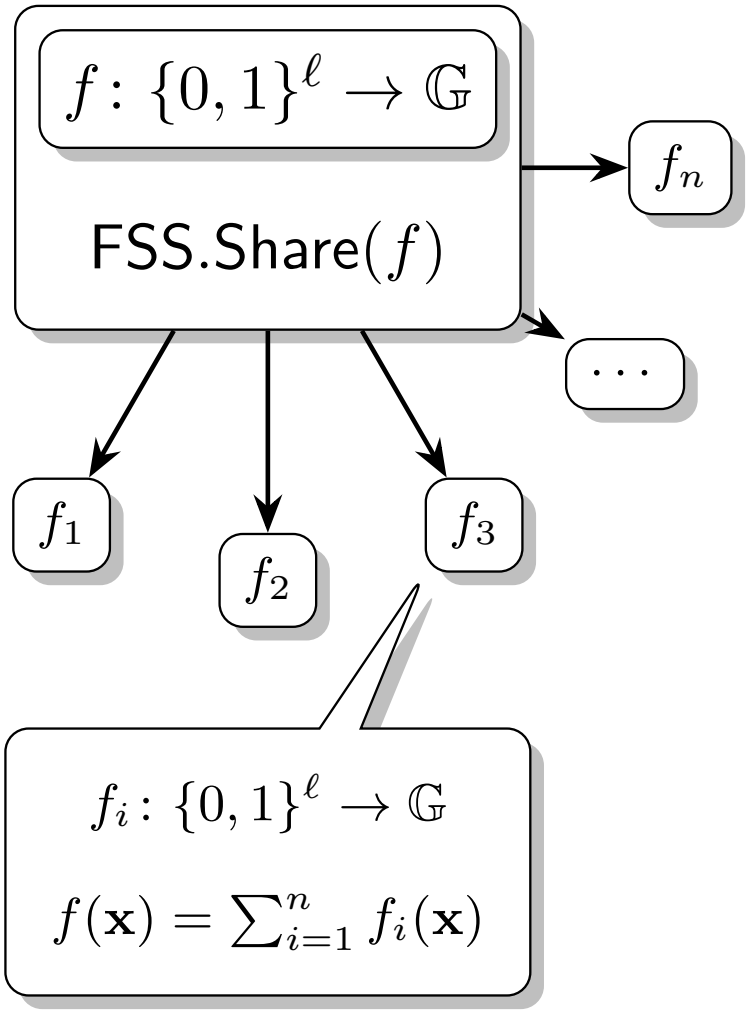

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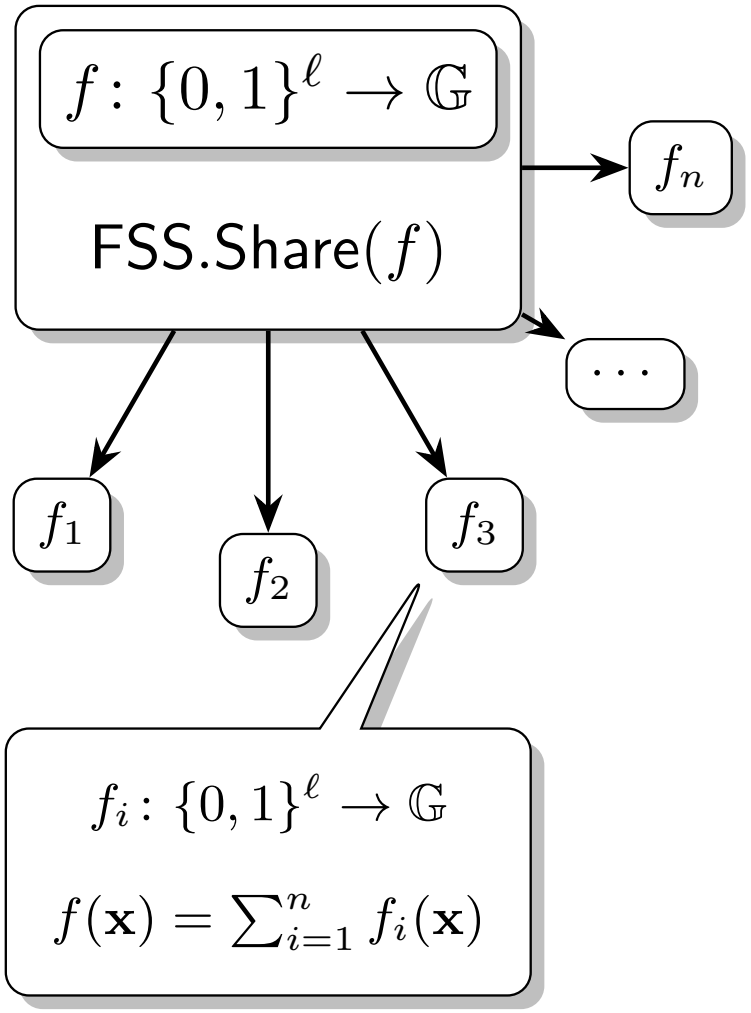
- Reconstruction: pick the right set of secret shares depending on $\mathbf{x}$.

Function Secret Sharing



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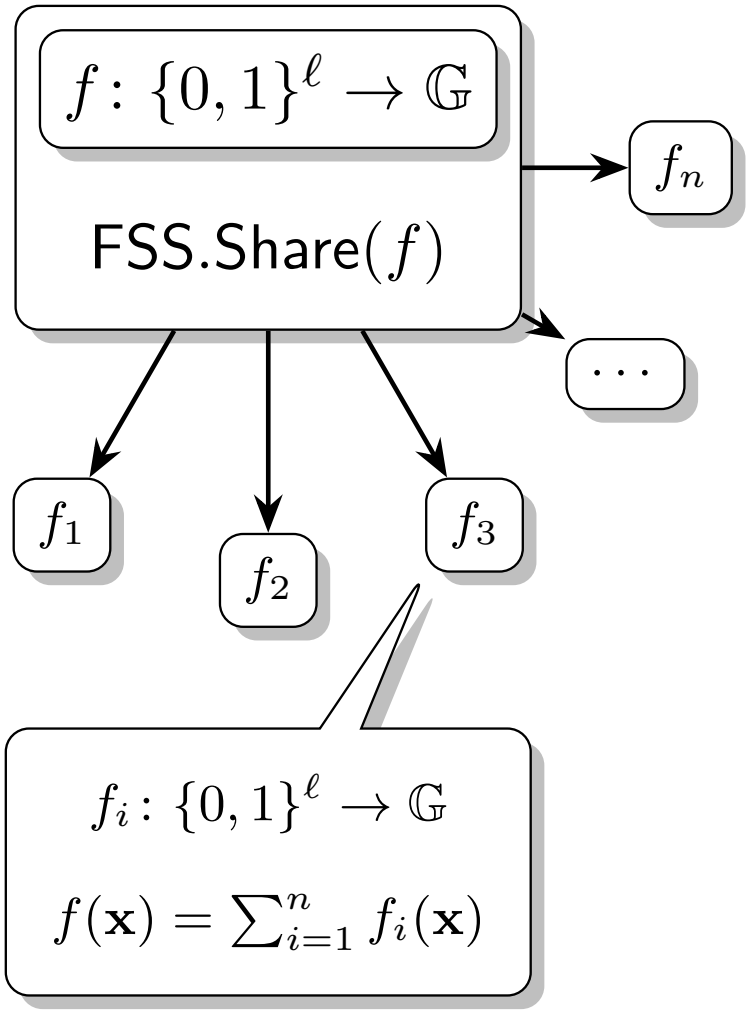

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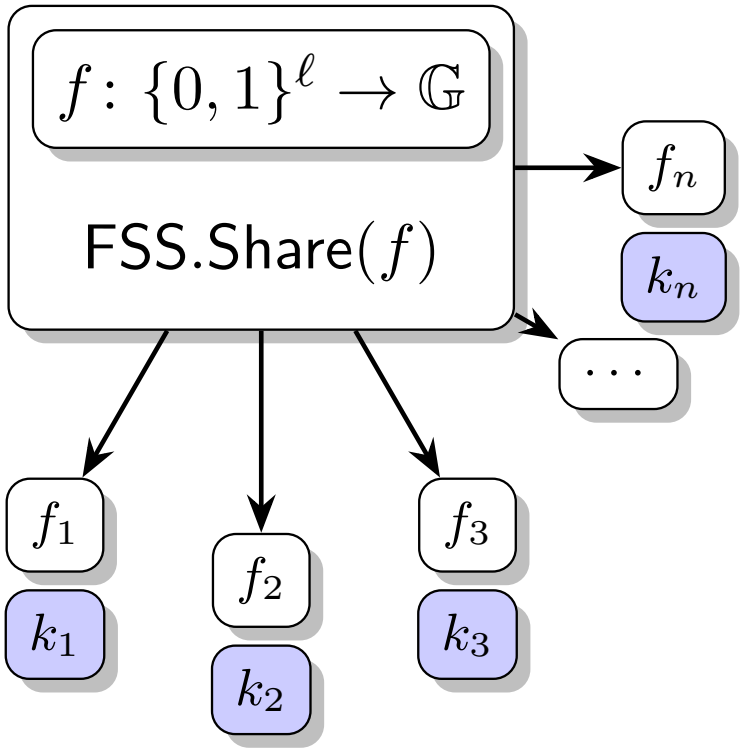

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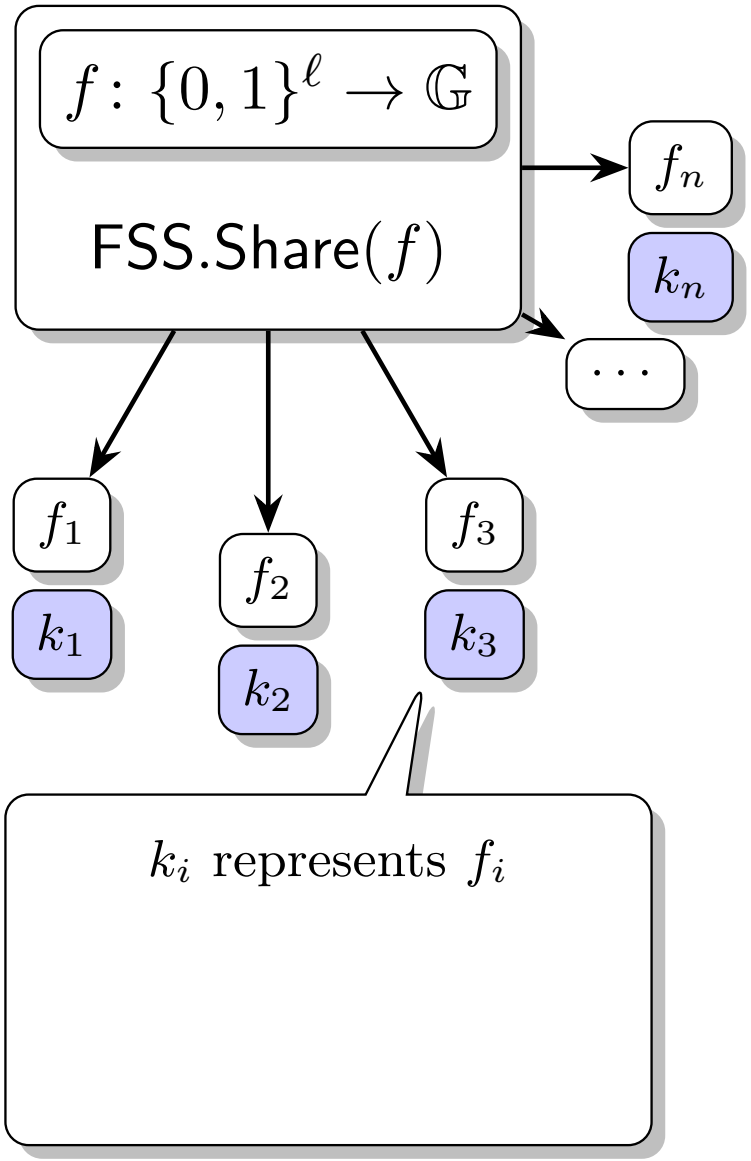

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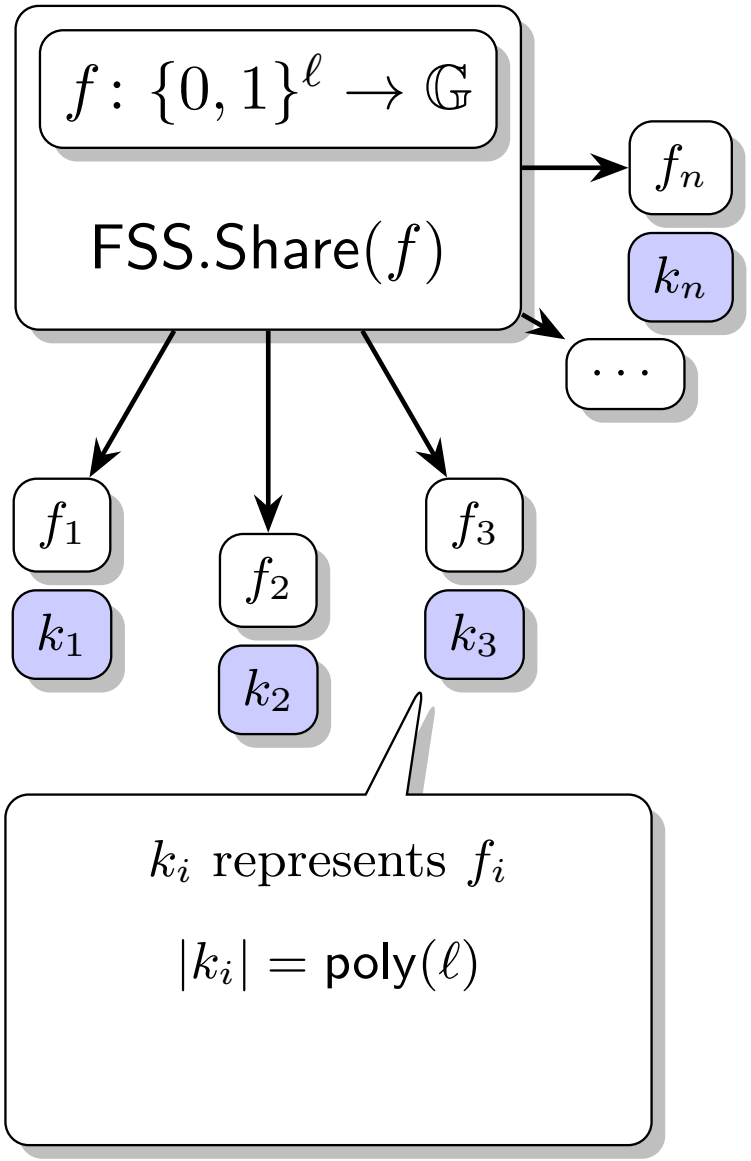

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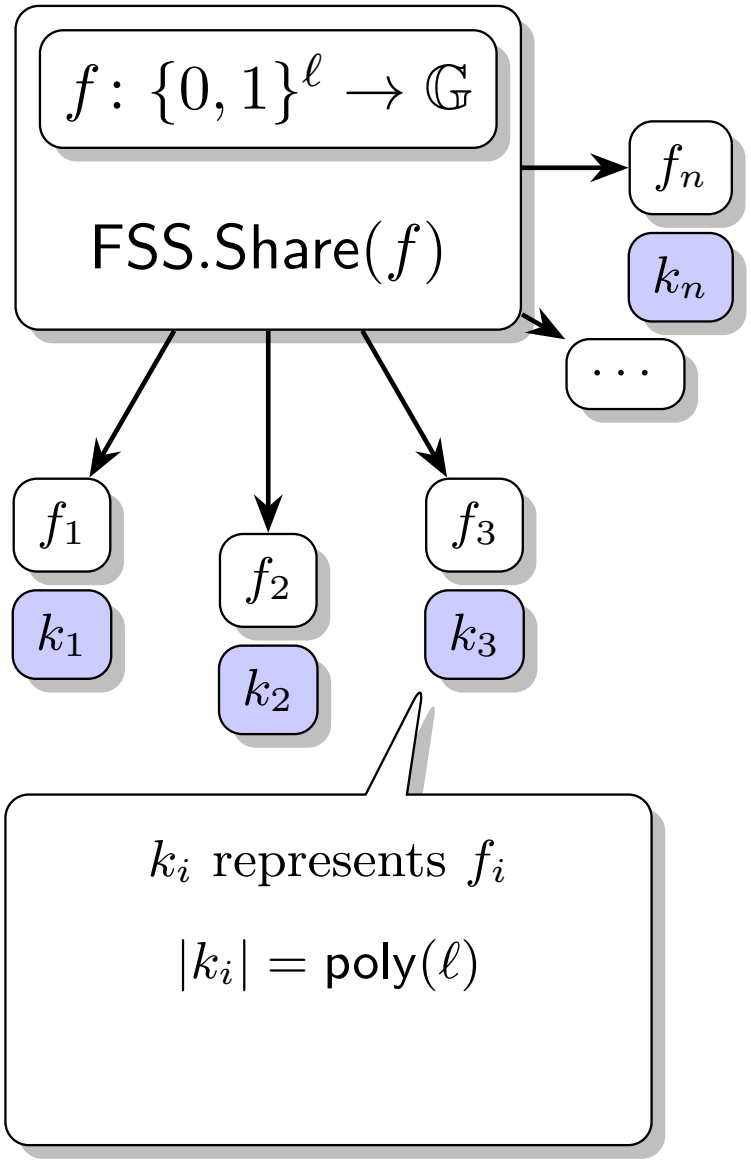

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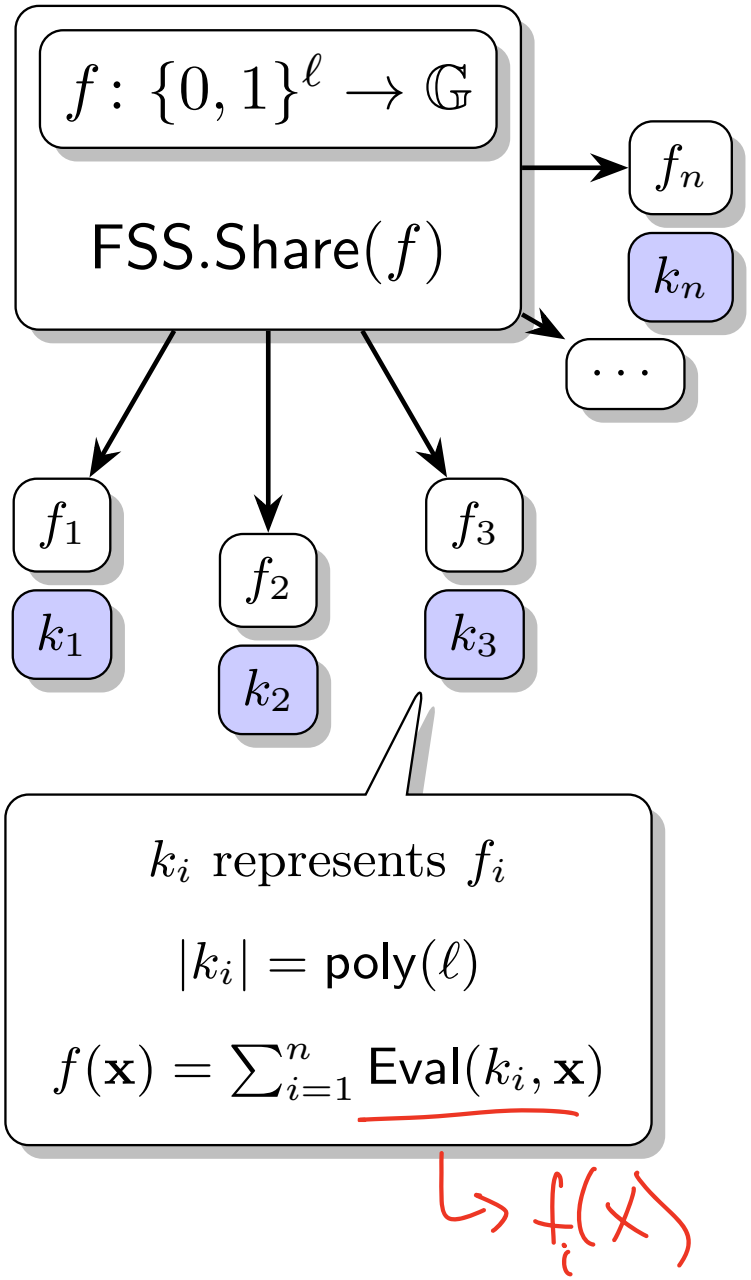

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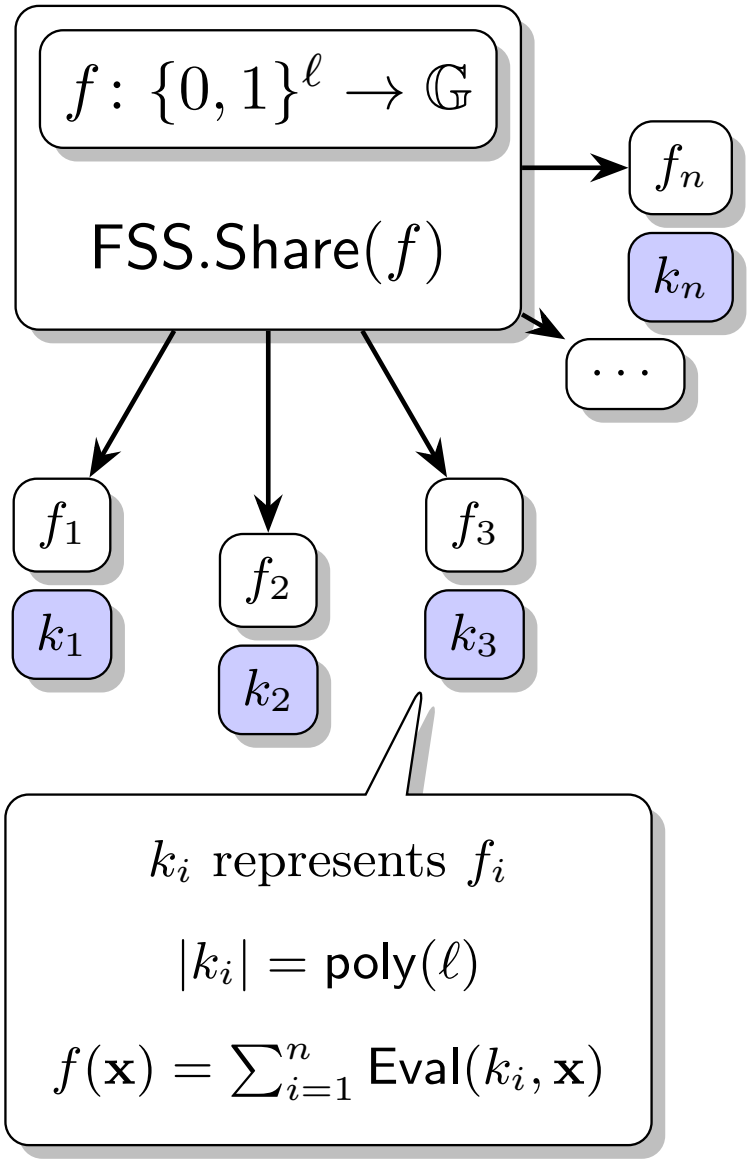

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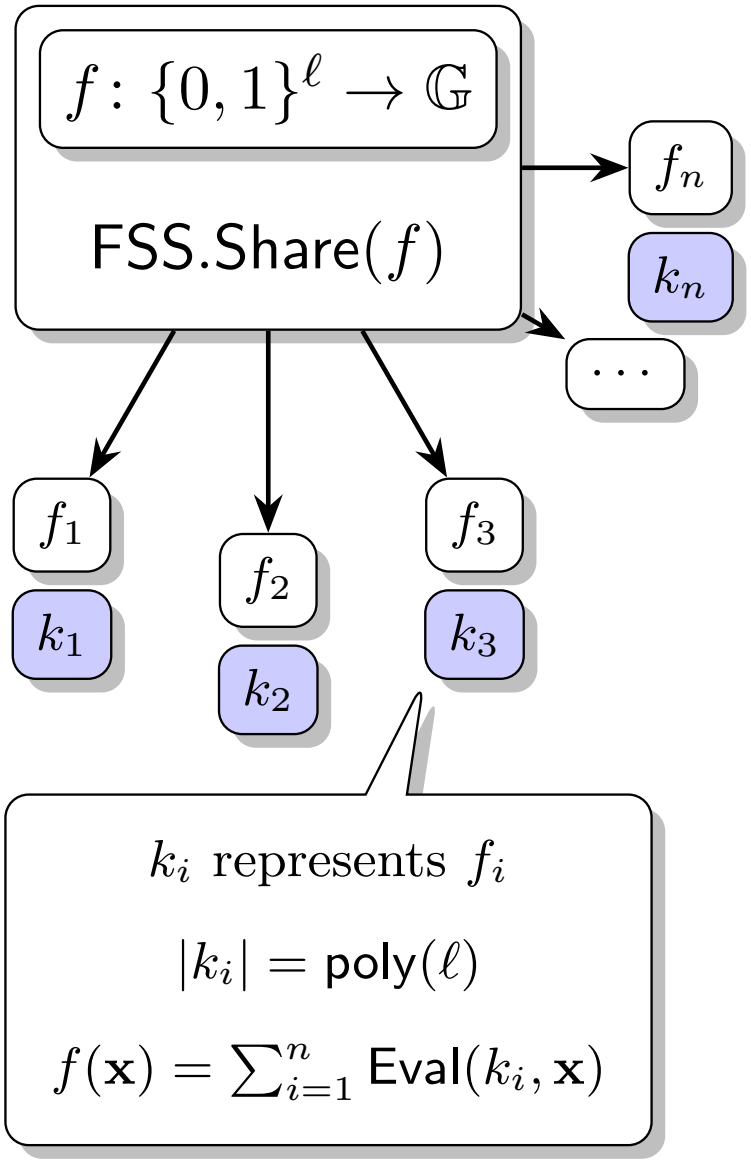

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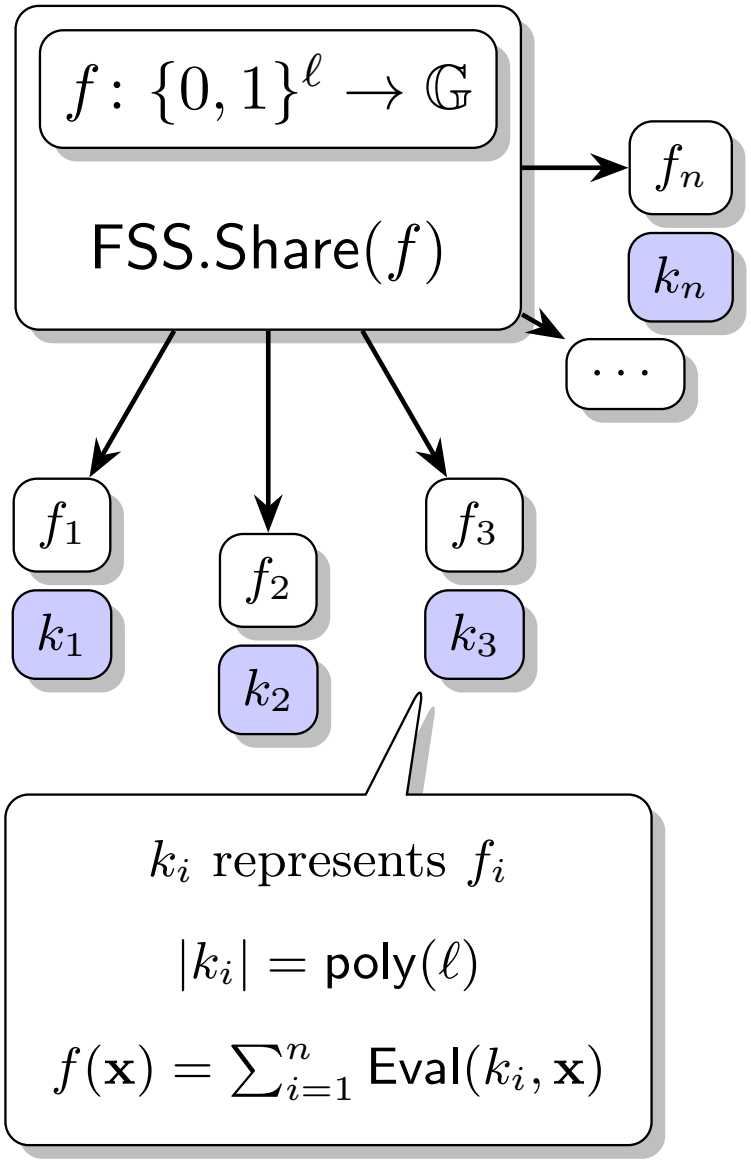

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Moreover, $(\mathsf{Gen}, \mathsf{Eval})$ also satisfy the following correctness and security requirements (given in the next two definitions).

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$$\Pr\left[(k_1, \ldots, k_n) \leftarrow \mathsf{Gen}(1^{\lambda}, f) \ : \ f(\mathbf{x}) = \sum_{i=1}^{n} \mathsf{Eval}(i, k_i, \mathbf{x})\right] = 1.$$

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Let $\mathsf{Adv}(1^\lambda, \mathcal{A}) := |\Pr[b = b'] - 1/2|$ be the advantage of the adversary in the above game. We say that $(\mathsf{Gen}, \mathsf{Eval})$ is *t-secure* if

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Definition 5 (Distributed Point Function)

A *distributed point function* (DPF) is a $(2,2)$-FSS for the family of all point functions $\mathcal{F} = \{f_{\alpha,\beta} \colon \alpha \in \{0,1\}^\ell, \beta \in \mathbb{G}\}_{\ell \in \mathbb{N}}$.

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- To build a (single-bit) DPF, we will need two primitives: *puncturable PRF* and *PRGs*.

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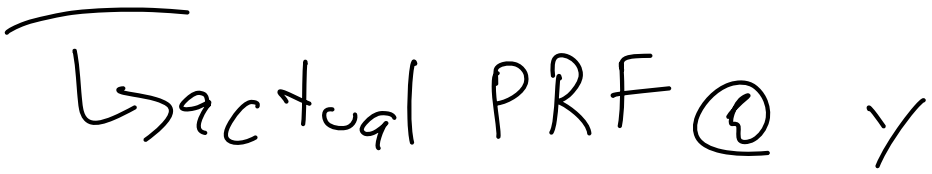

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  - When $s = \lambda$, we call G a *length-doubling PRG*.

PRFs from PRGs via GGM Construction

- We can construct PRFs from PRGs using the Goldreich-Goldwasser-Micali Construction.

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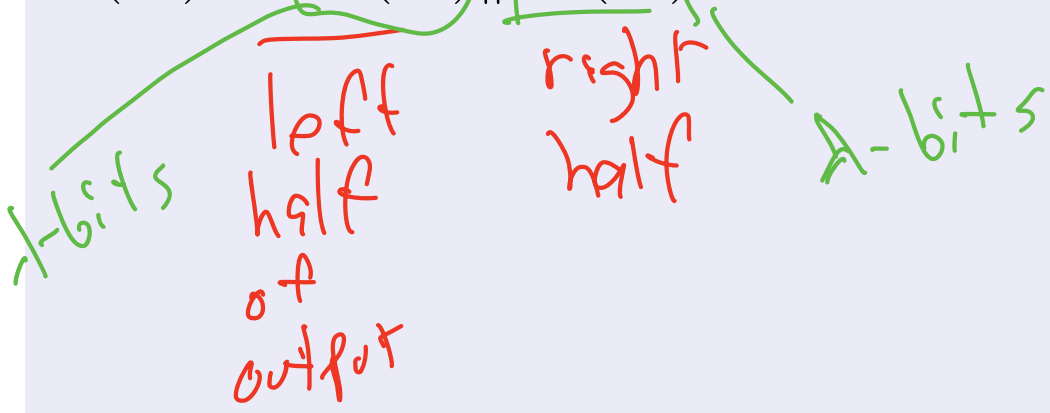

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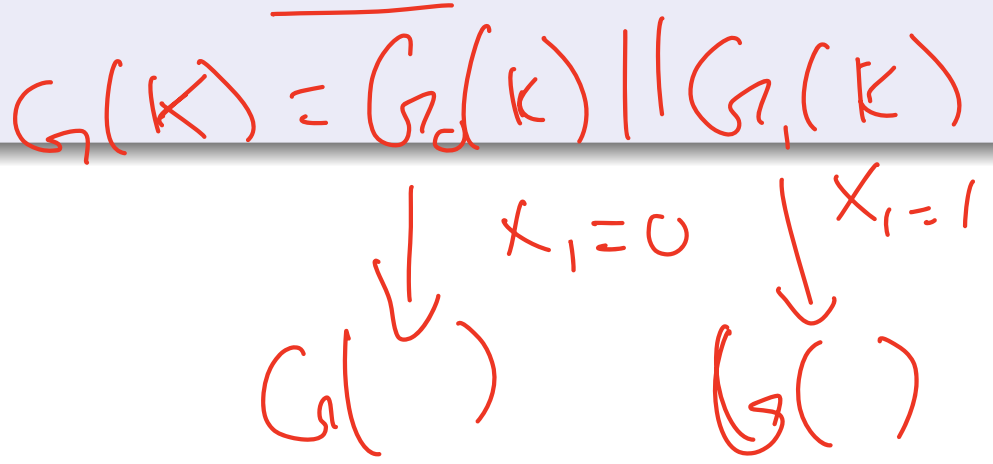

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where $x = (x_1, \ldots, x_n)$ *is the input to the function.*

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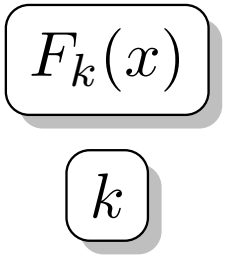

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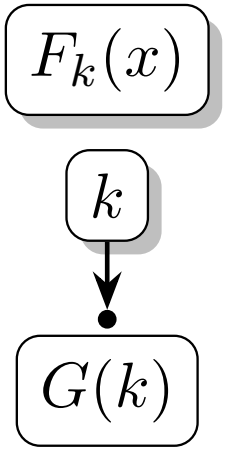

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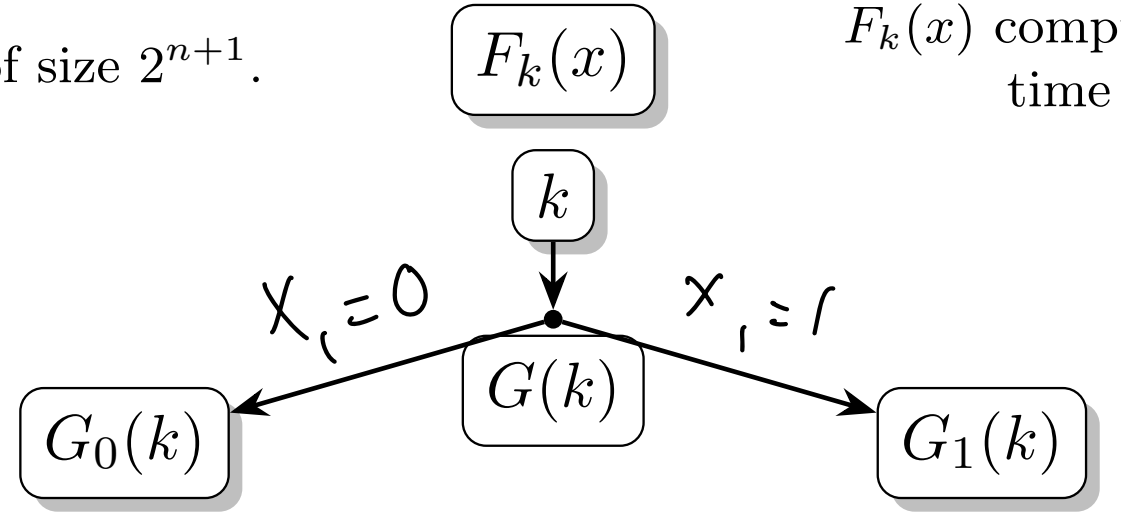

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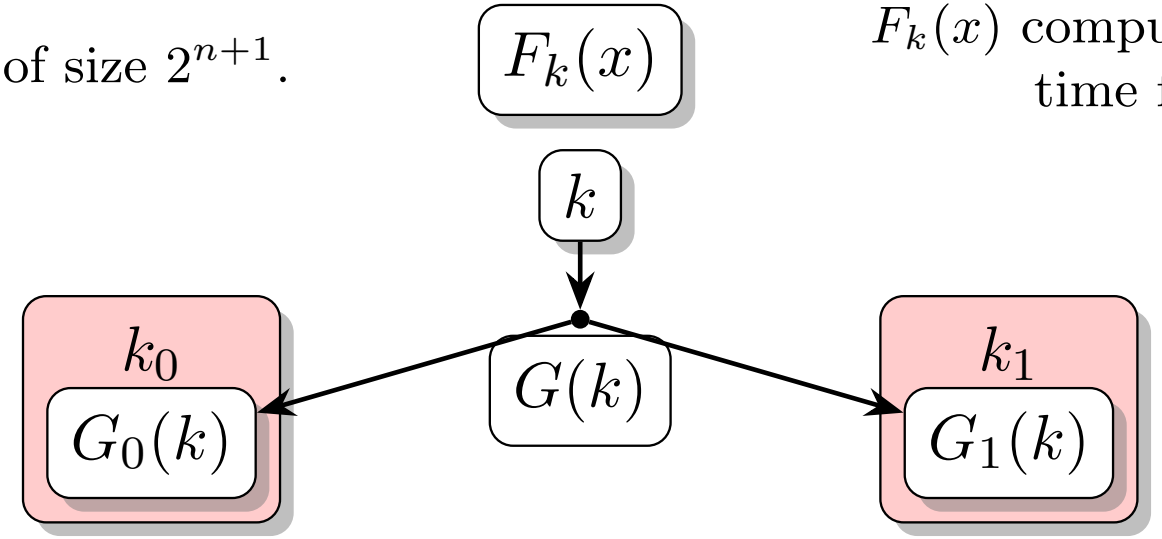

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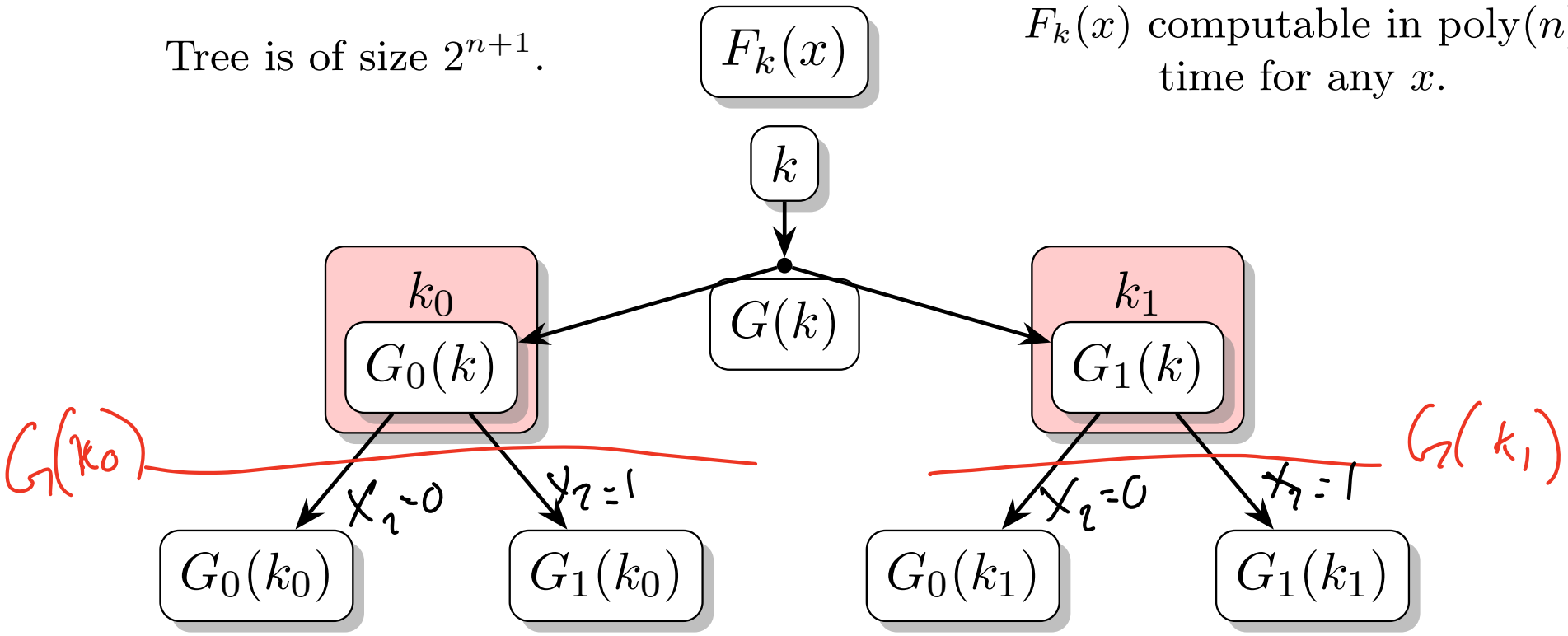

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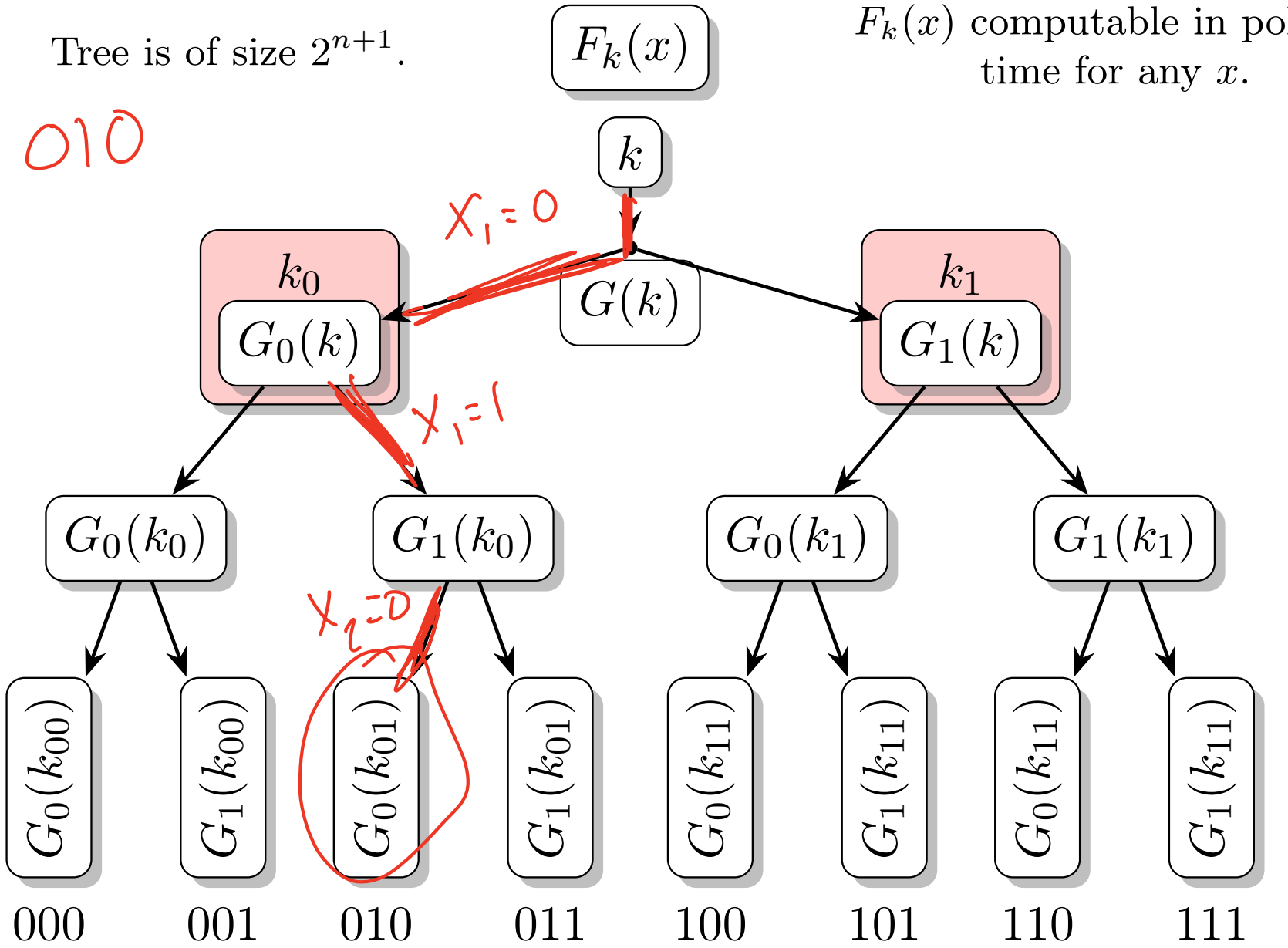

$X = 010$

$X_1 = 0$

$X_1 = 1$

$X_2 = 0$

$F_k(010) = G_0(k_{01}) = k_{010} = G_0(G_1(G_0(k)))$

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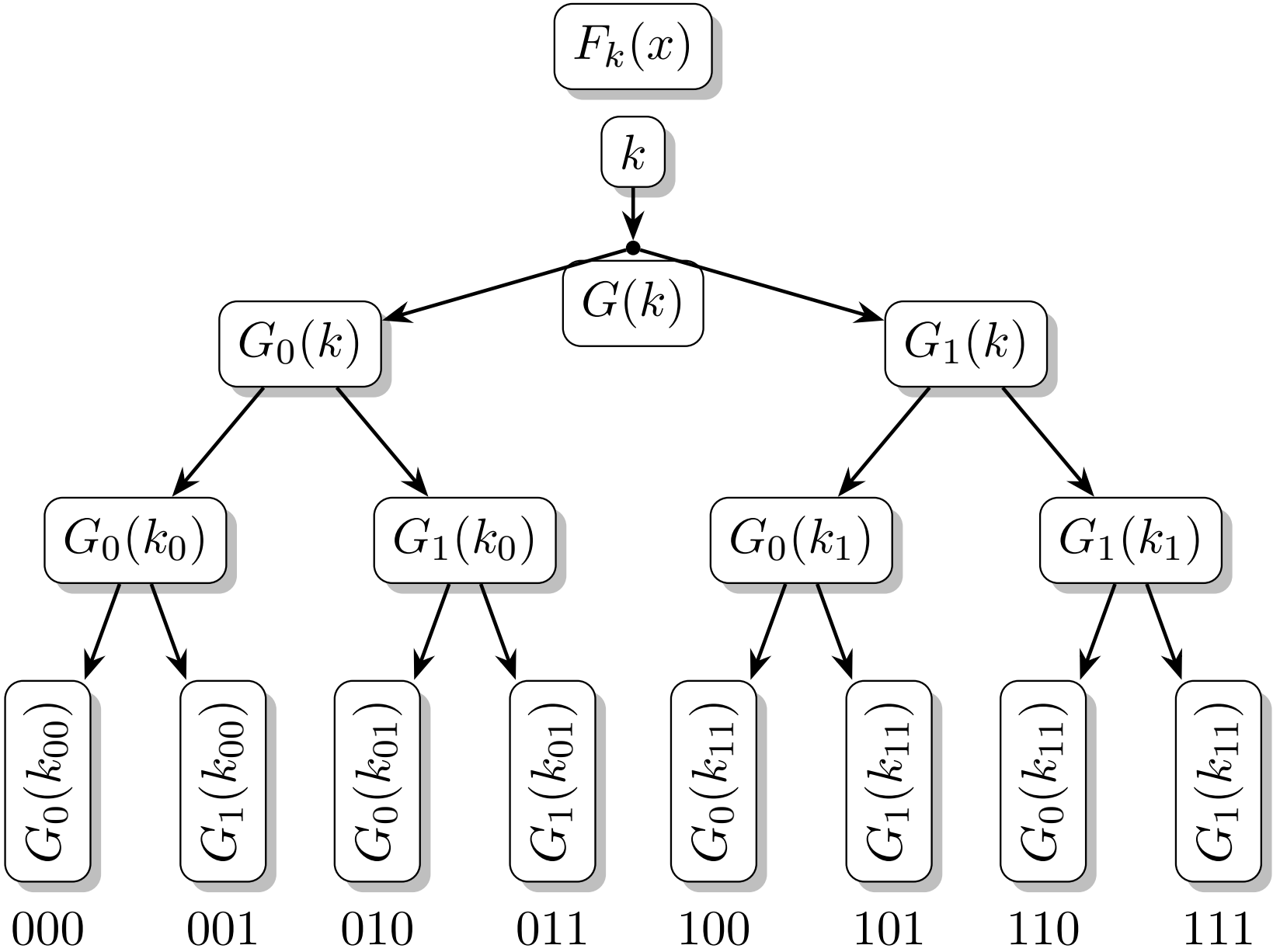
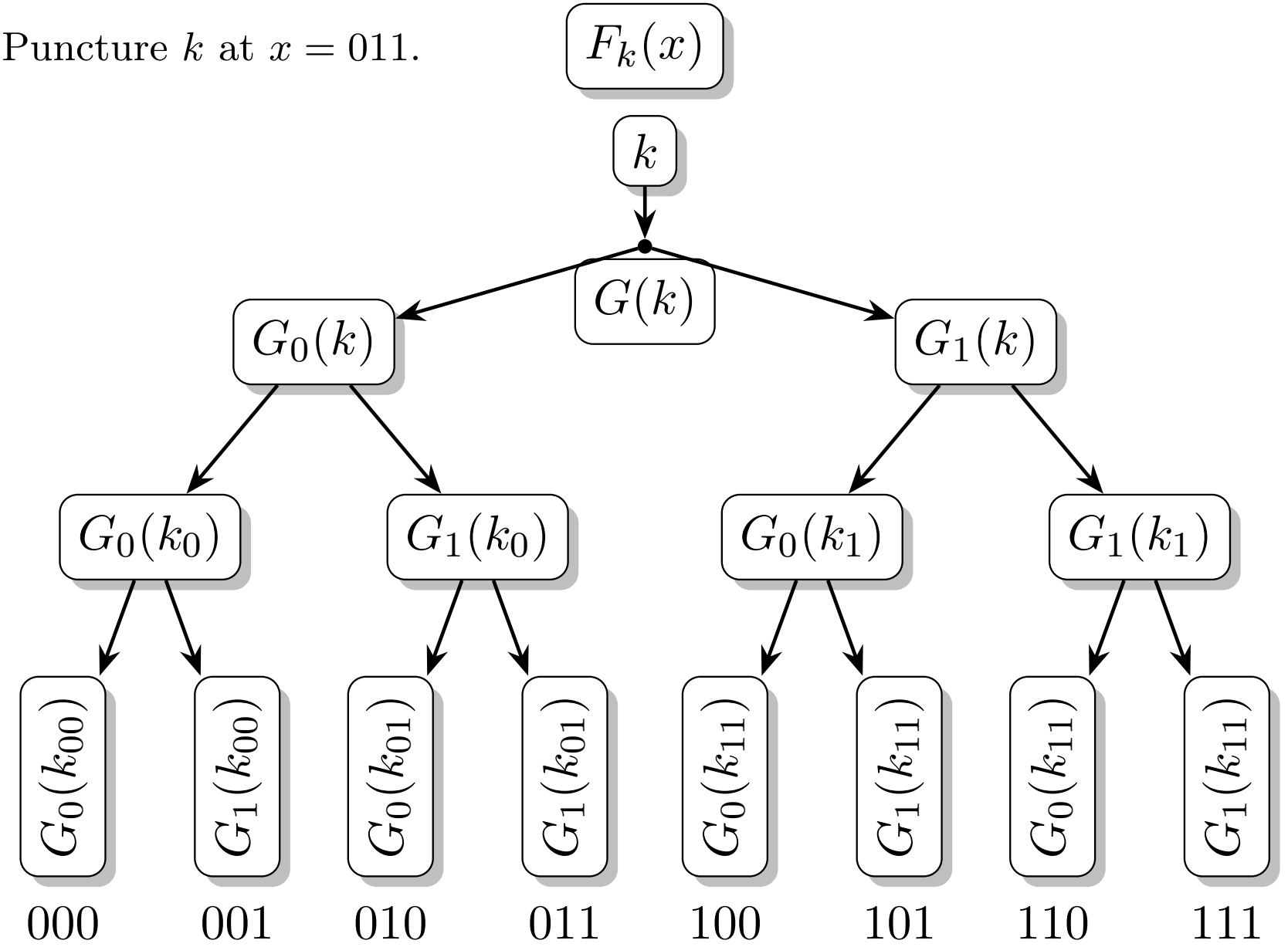


PPRFs from PRGs via GGM Construction

Puncture k at $x = 011$.



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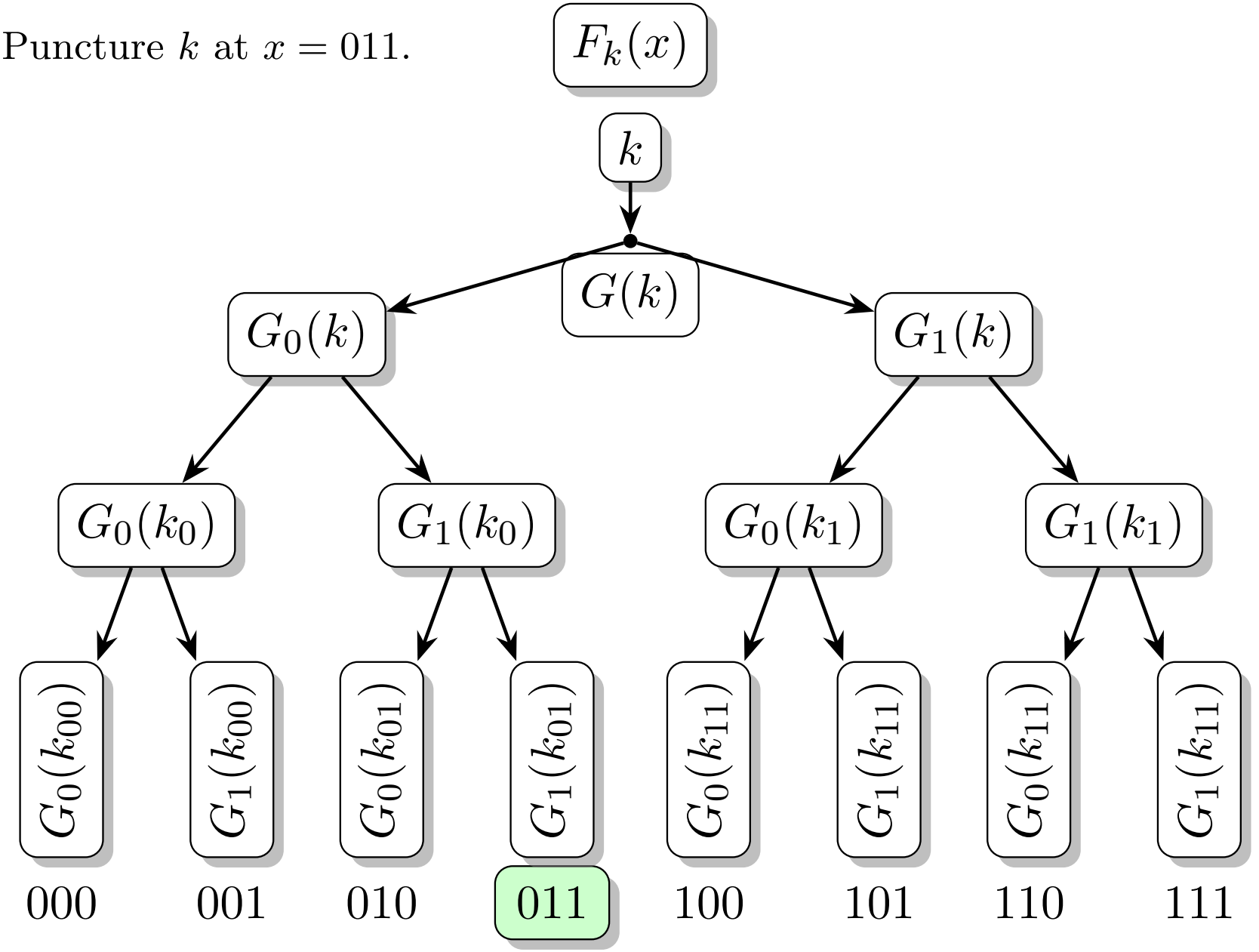

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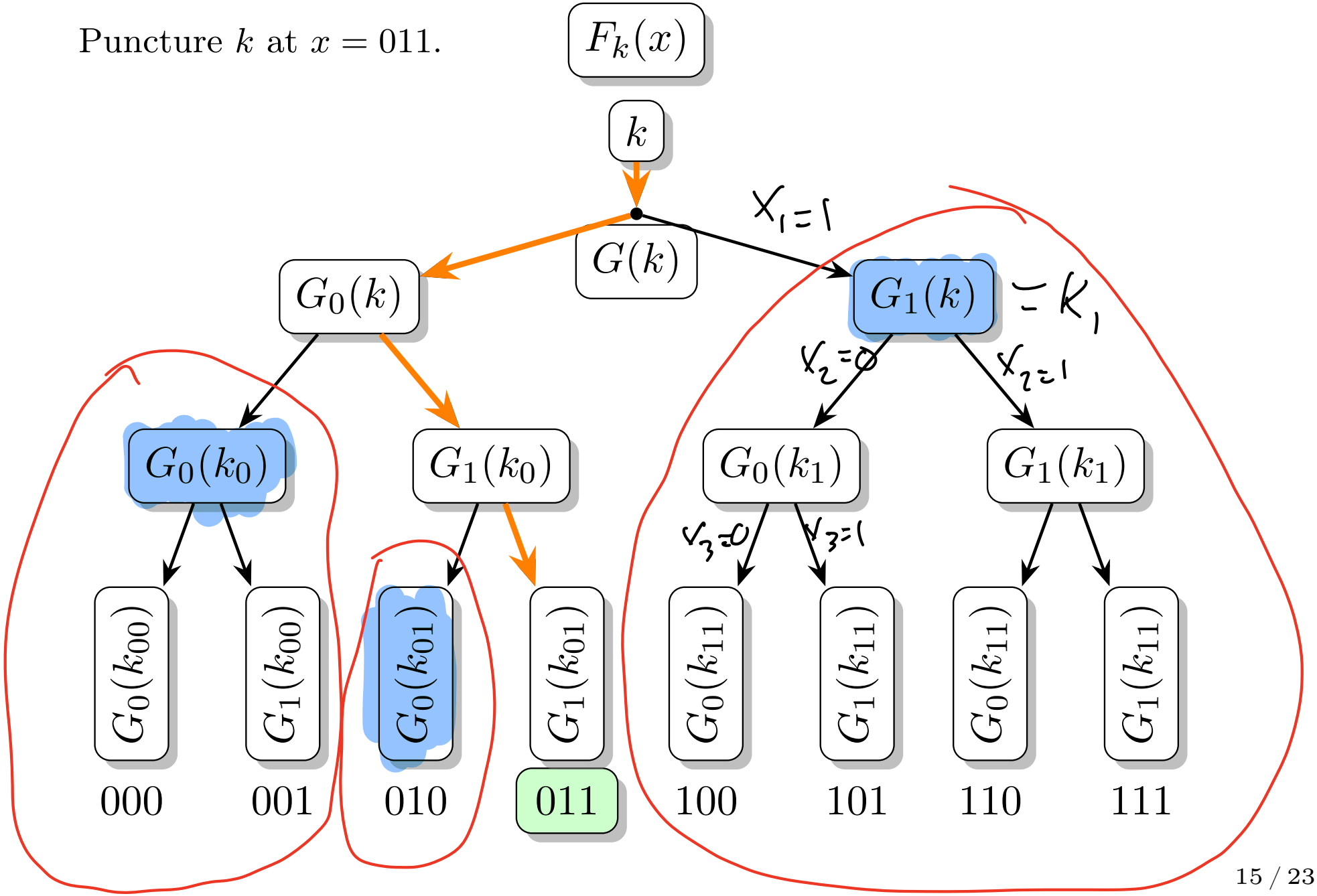

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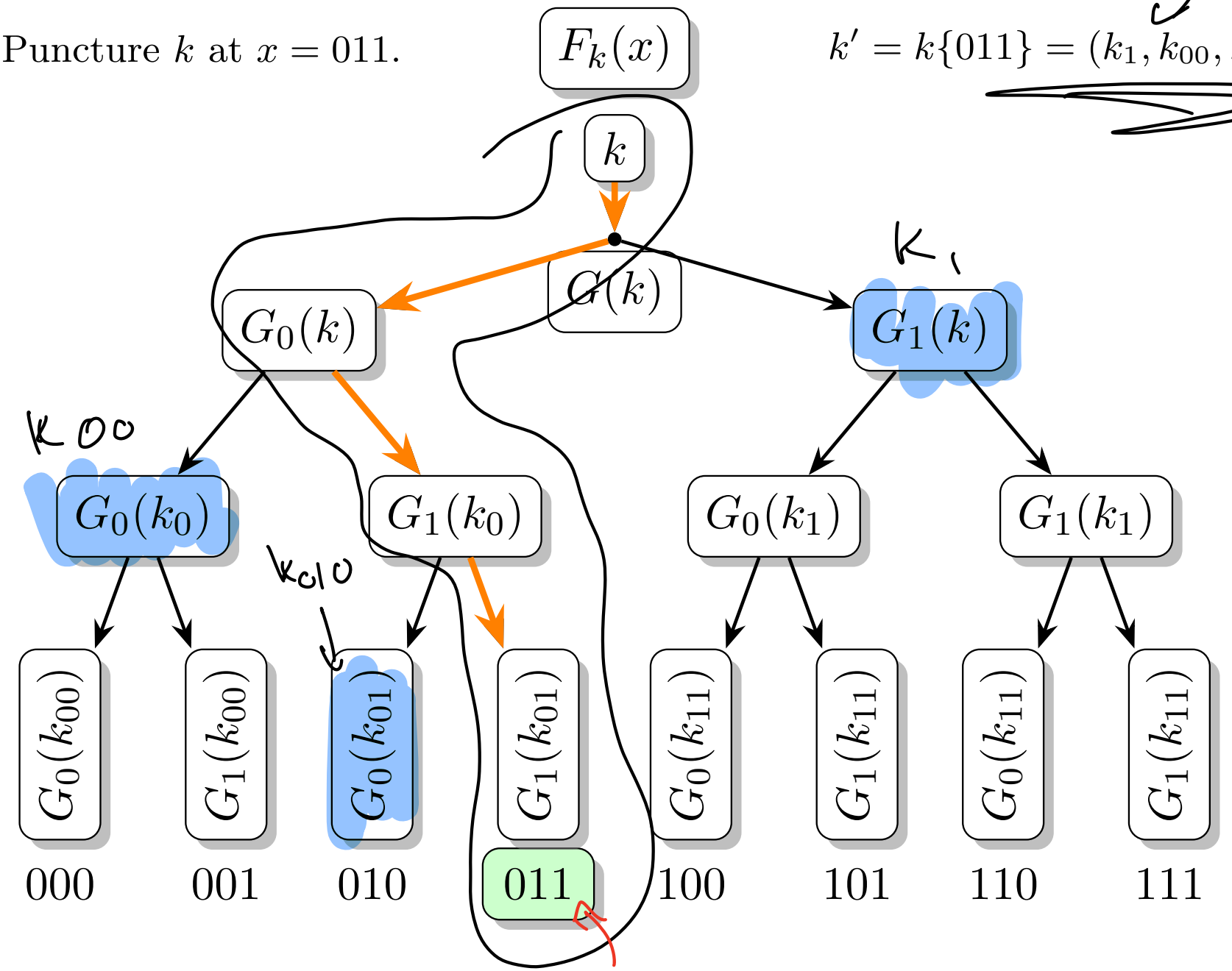
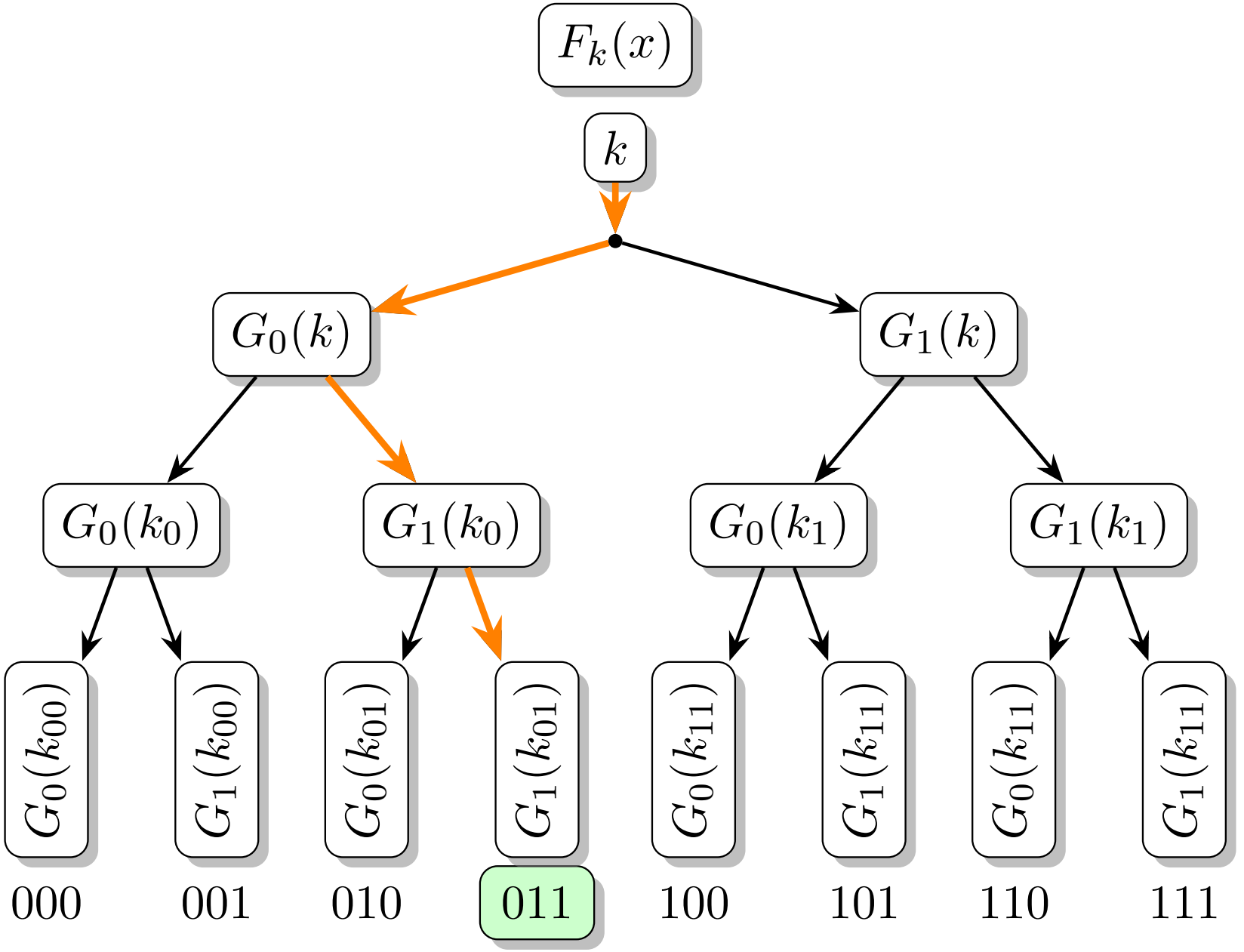
$k' = k\{011\} = (k_1, k_{00}, k_{010})$



DPFs from PRGs via GGM-like Construction



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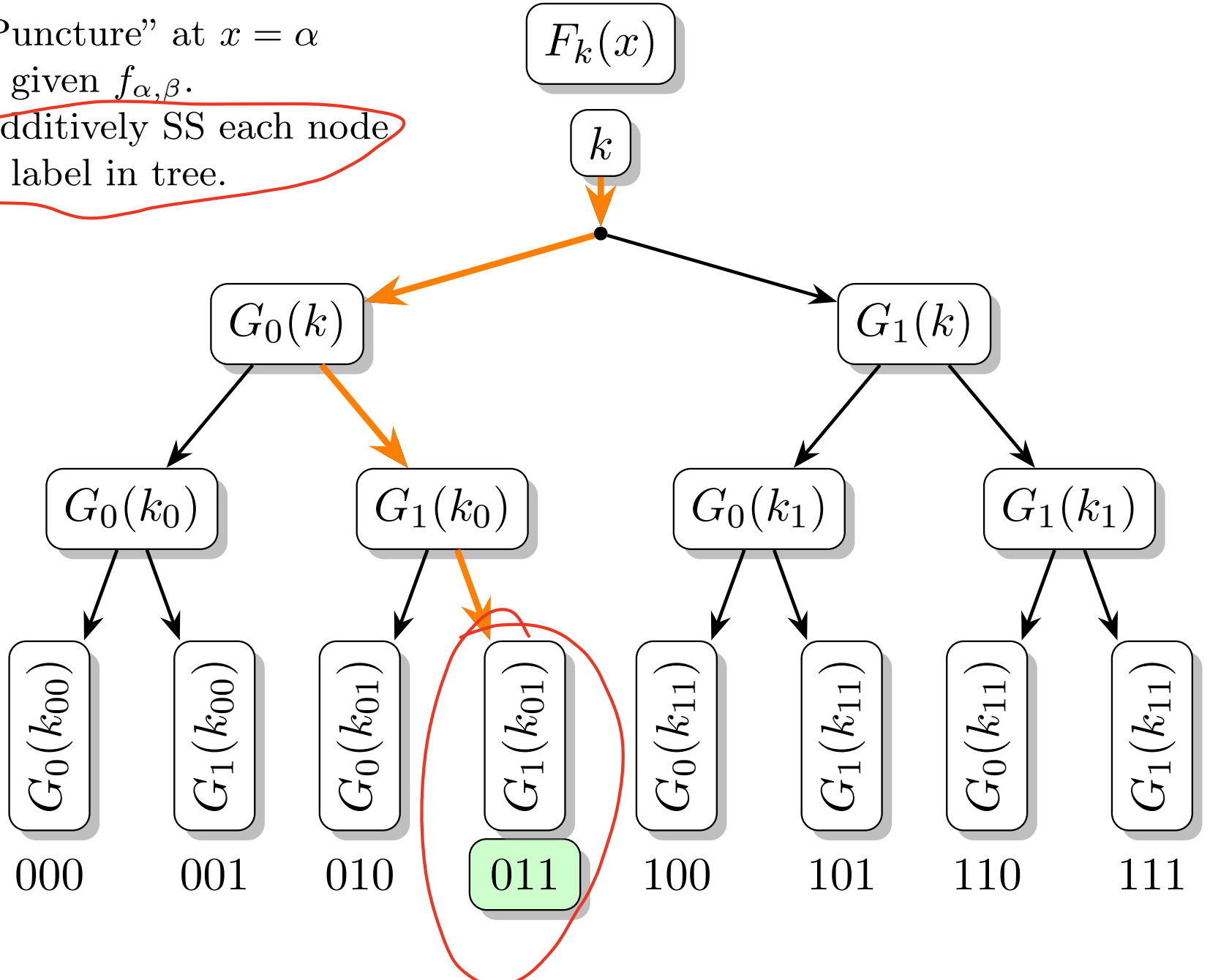
"Puncture" at $x = \alpha$ given $f_{\alpha,\beta}$.
Additively SS each node label in tree.



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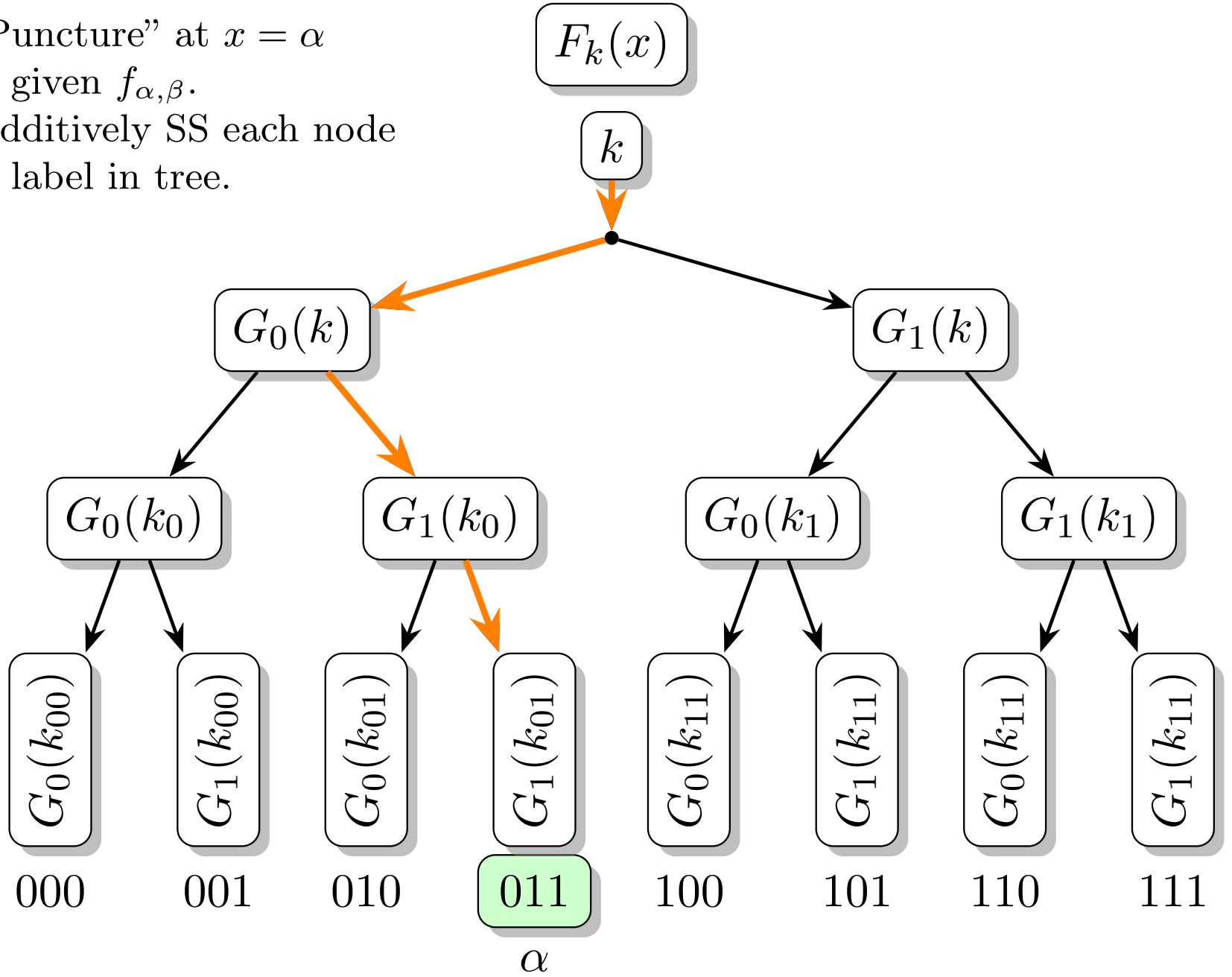

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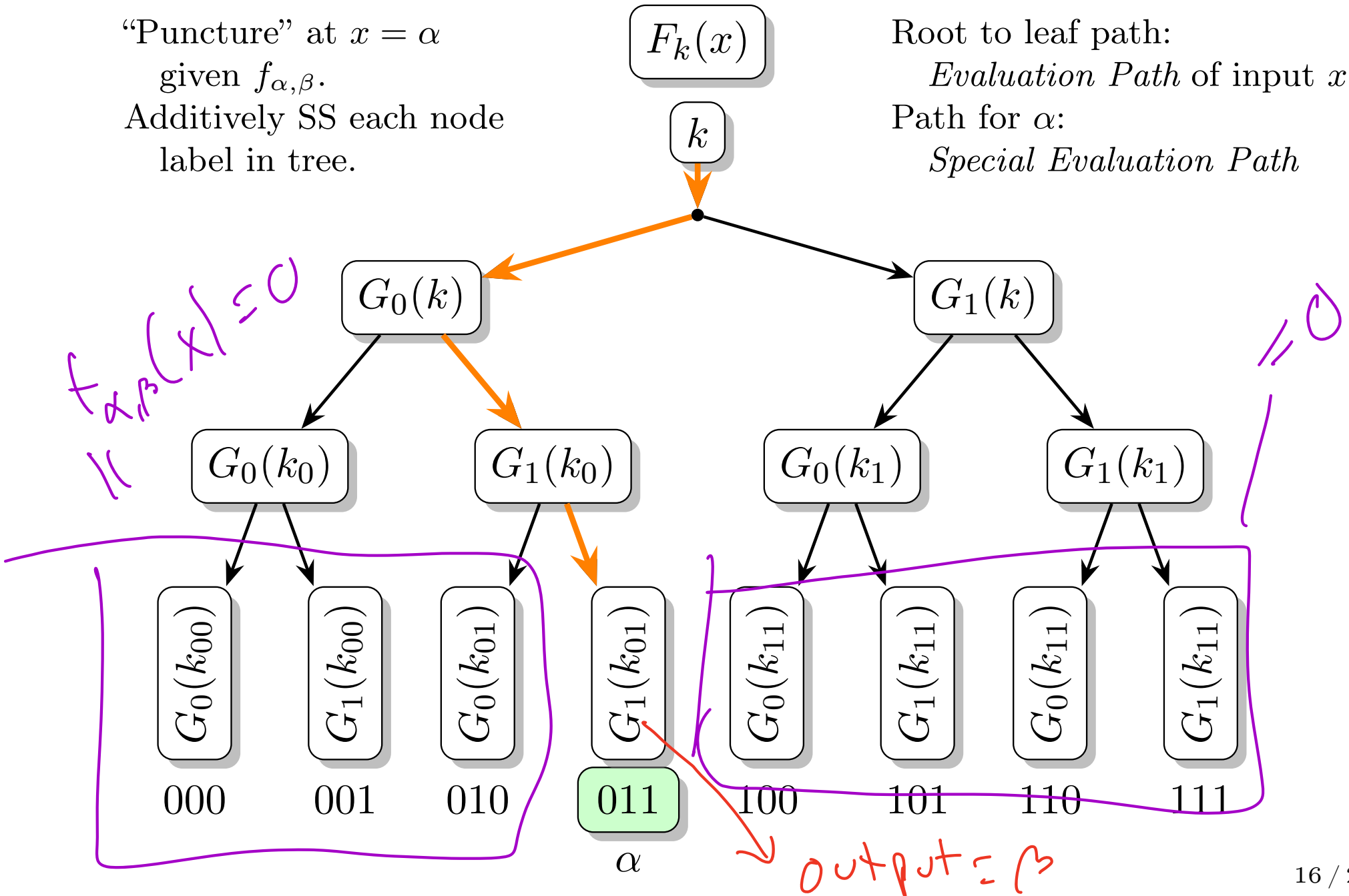
Root to leaf path:
Evaluation Path of input x
Path for α:
Special Evaluation Path



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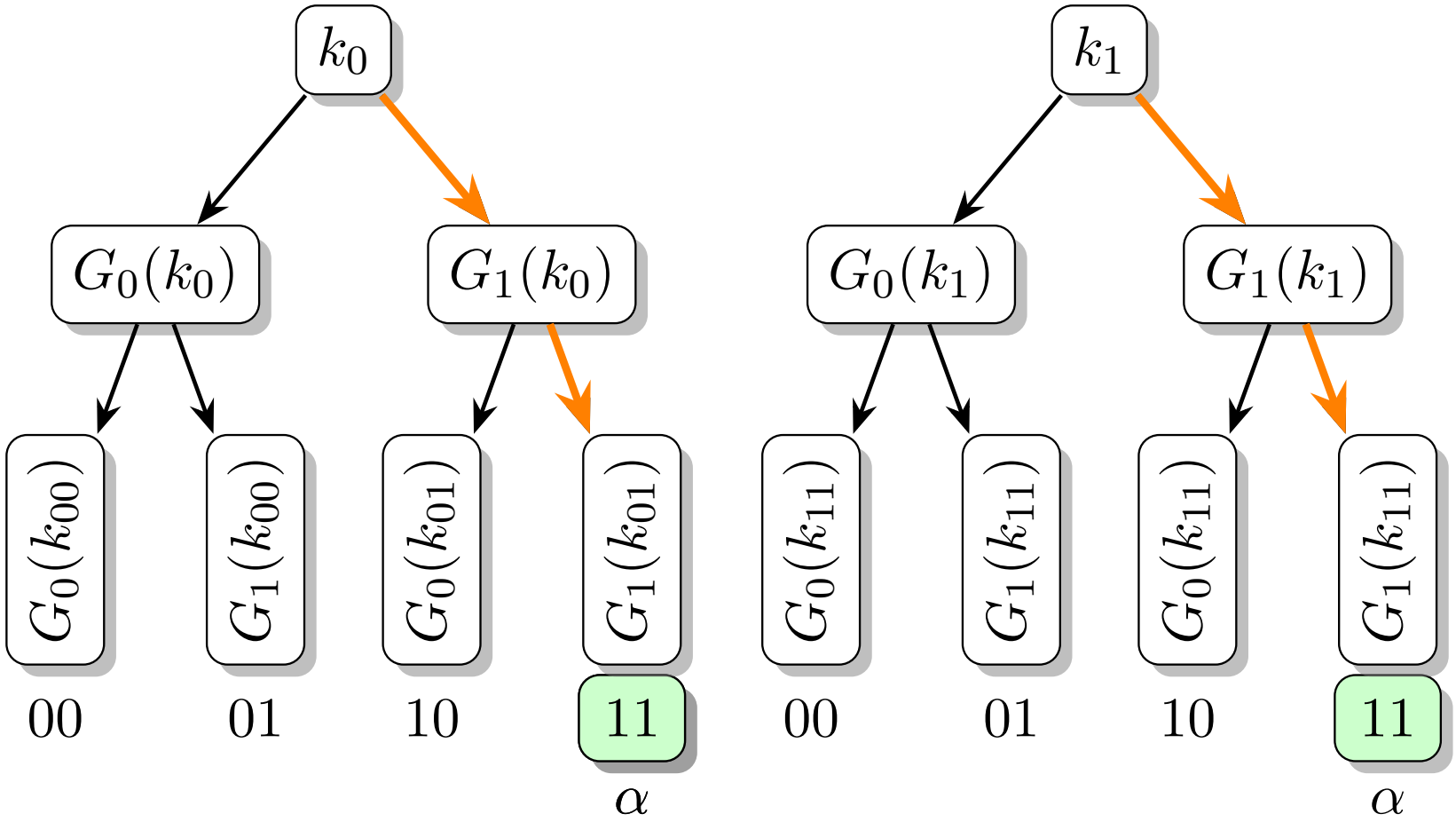

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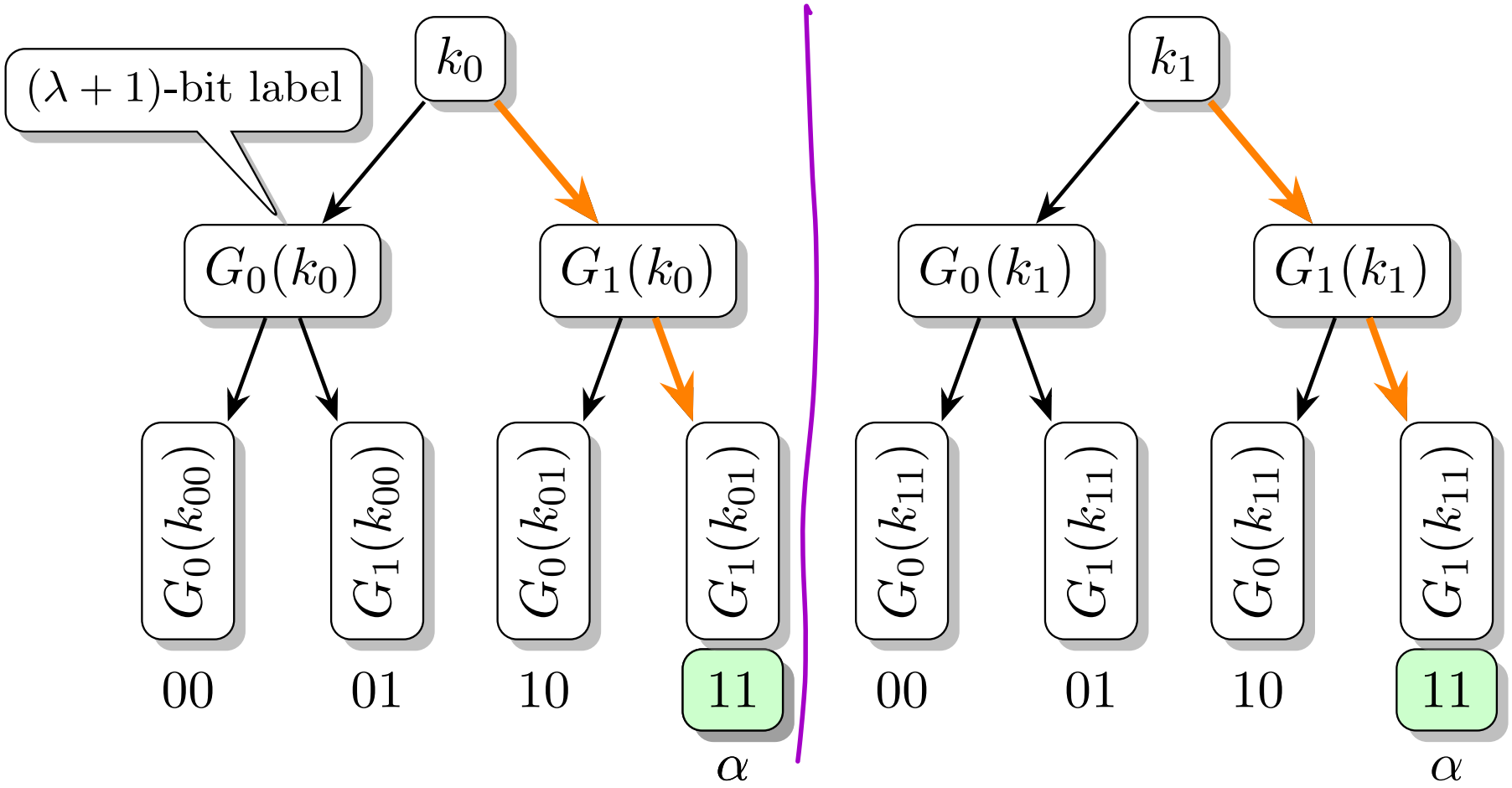

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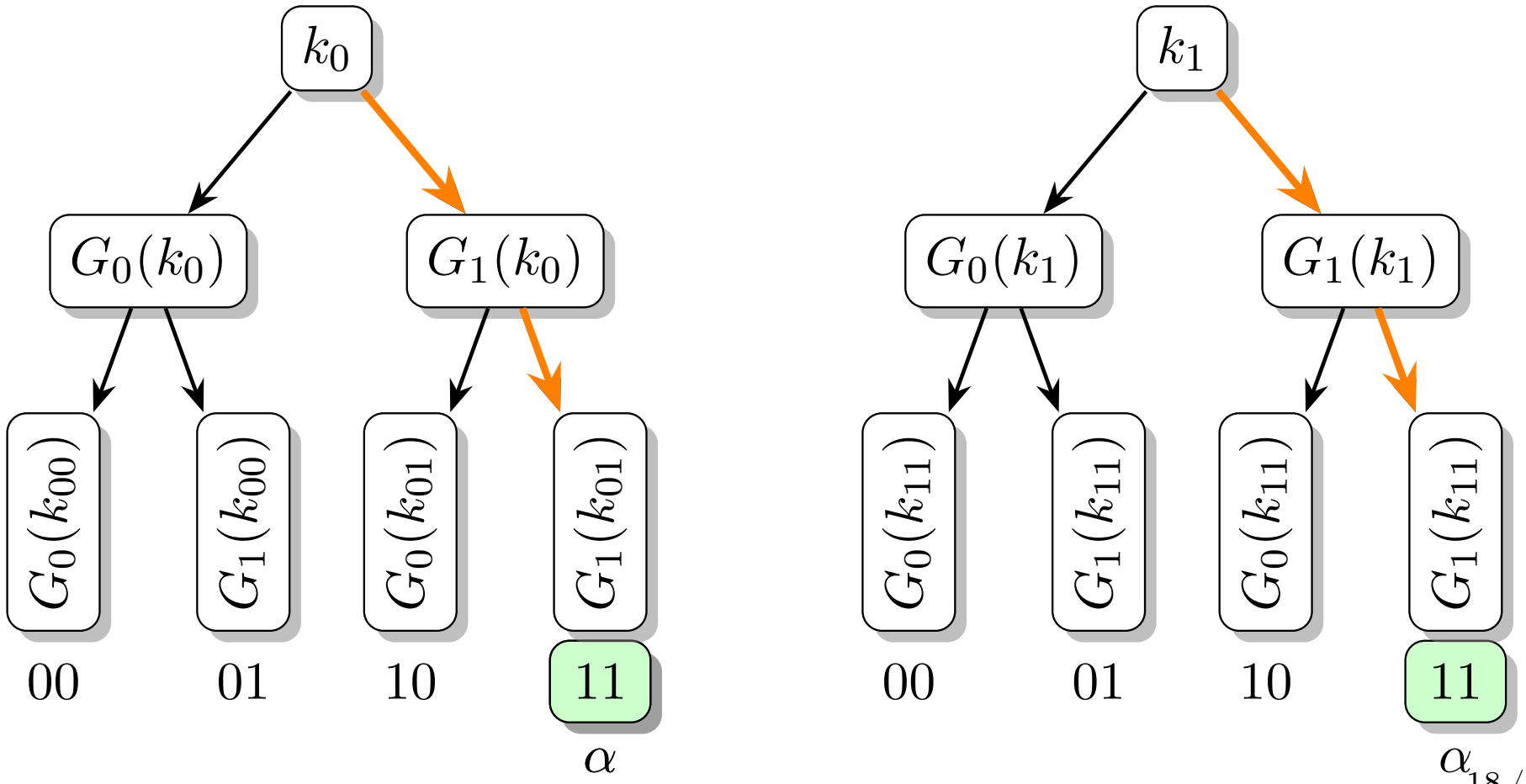

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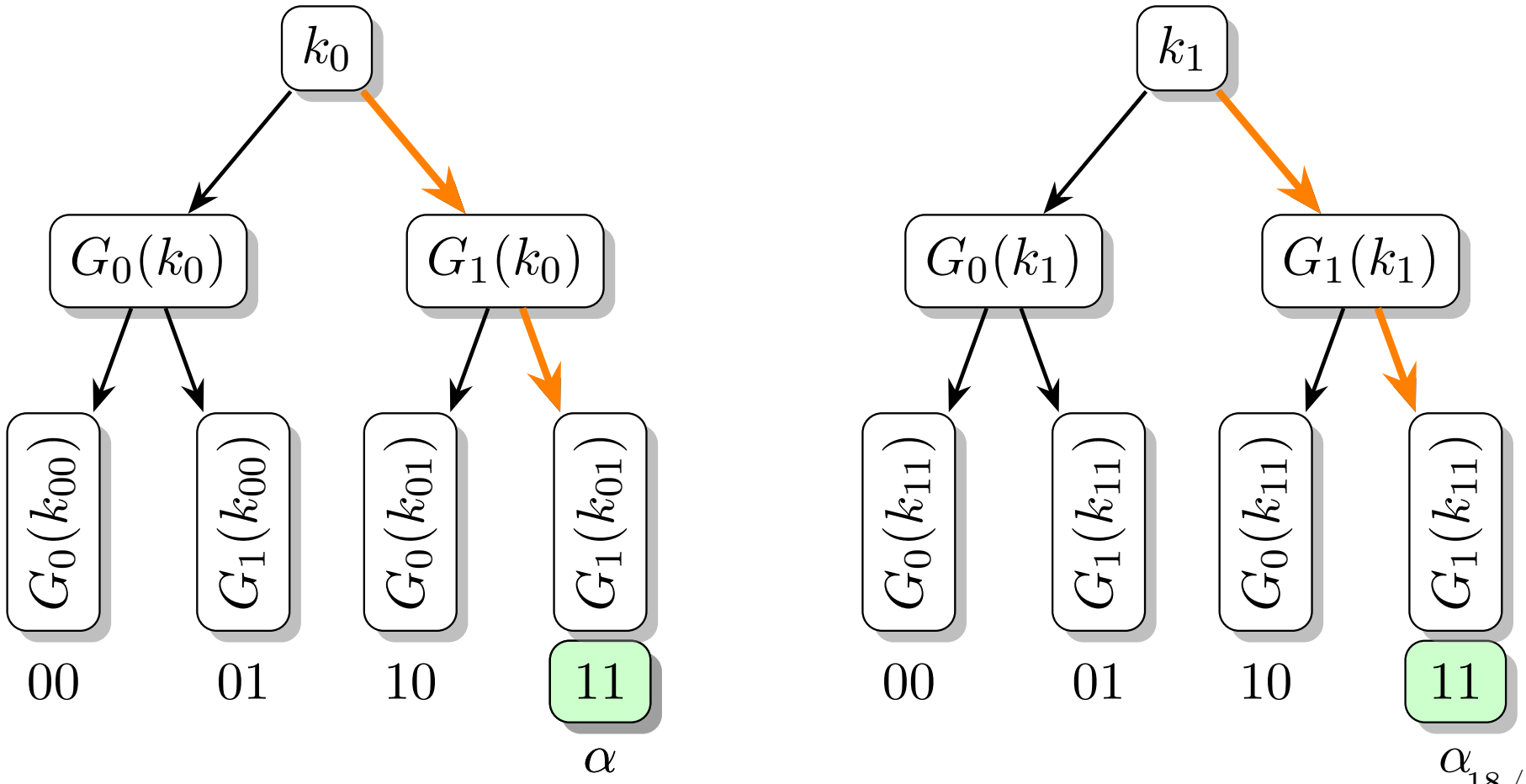

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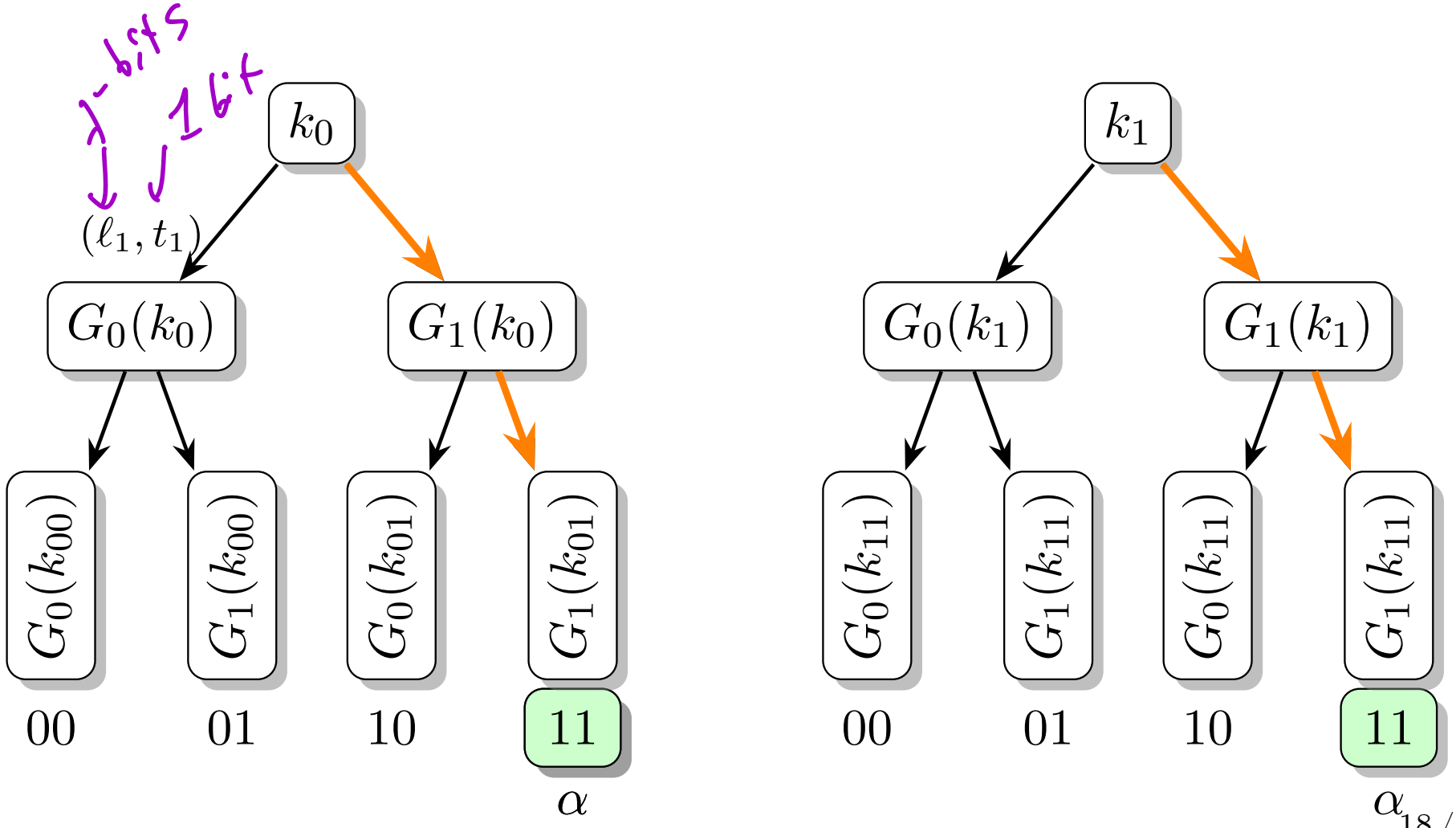

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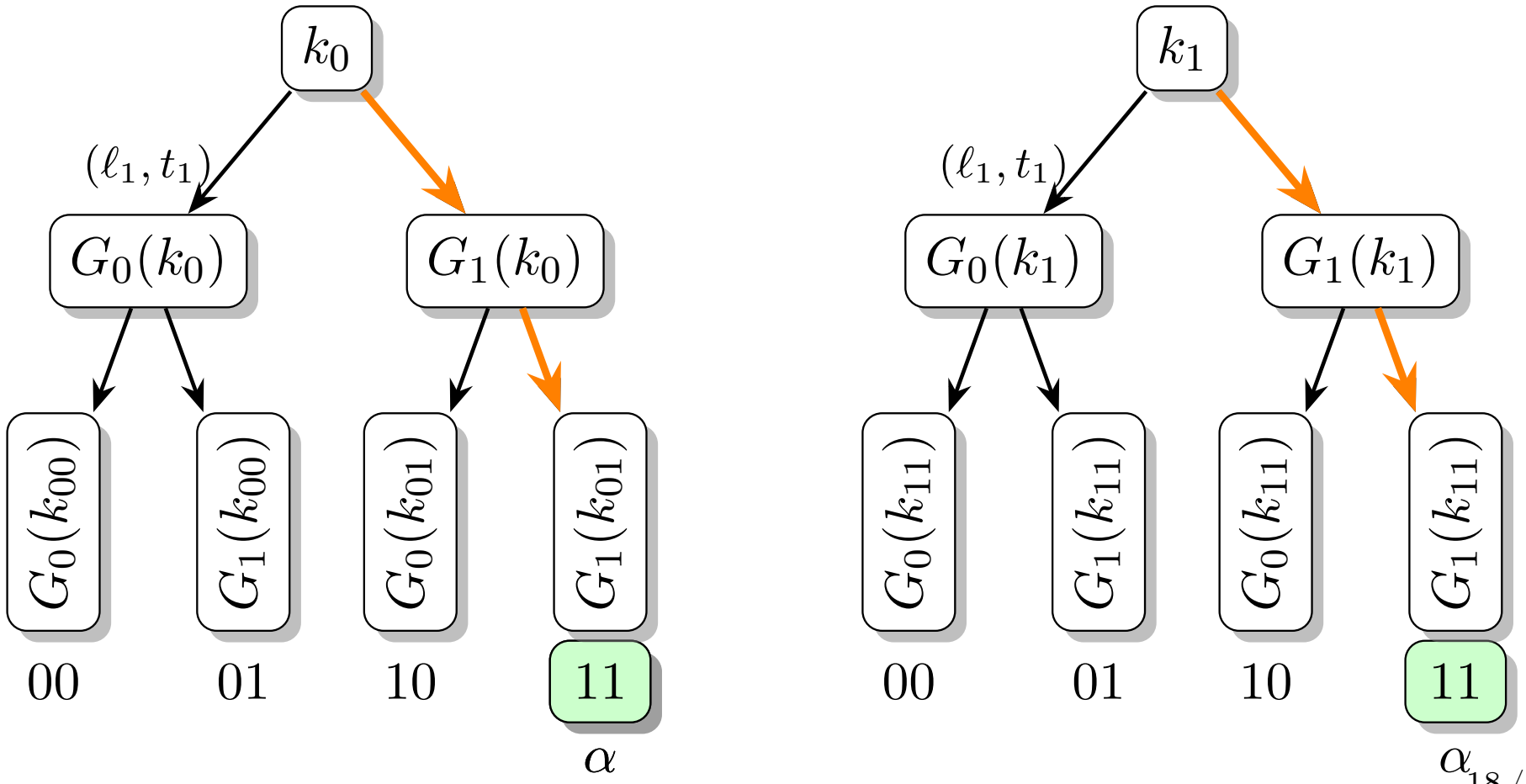

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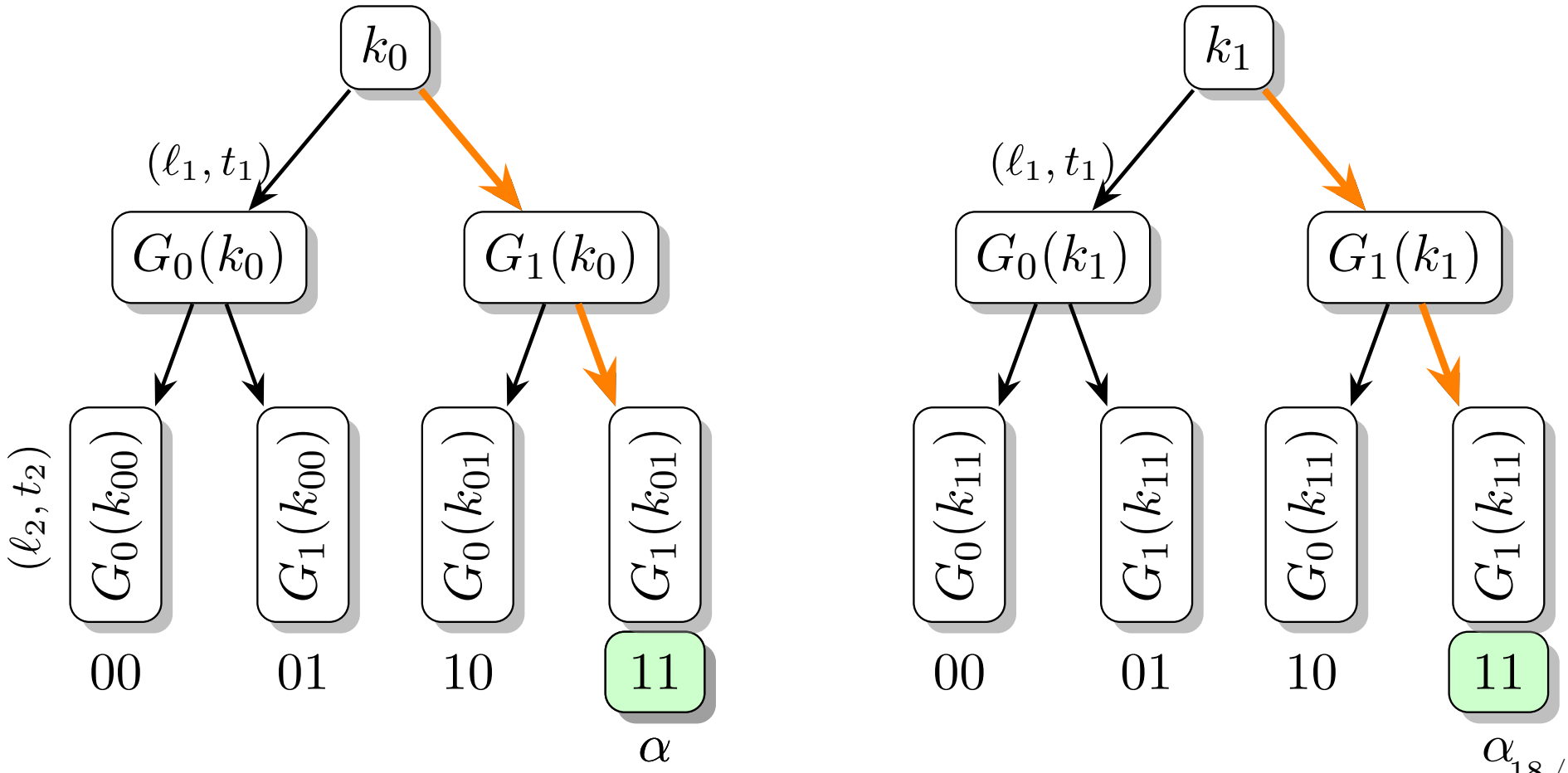

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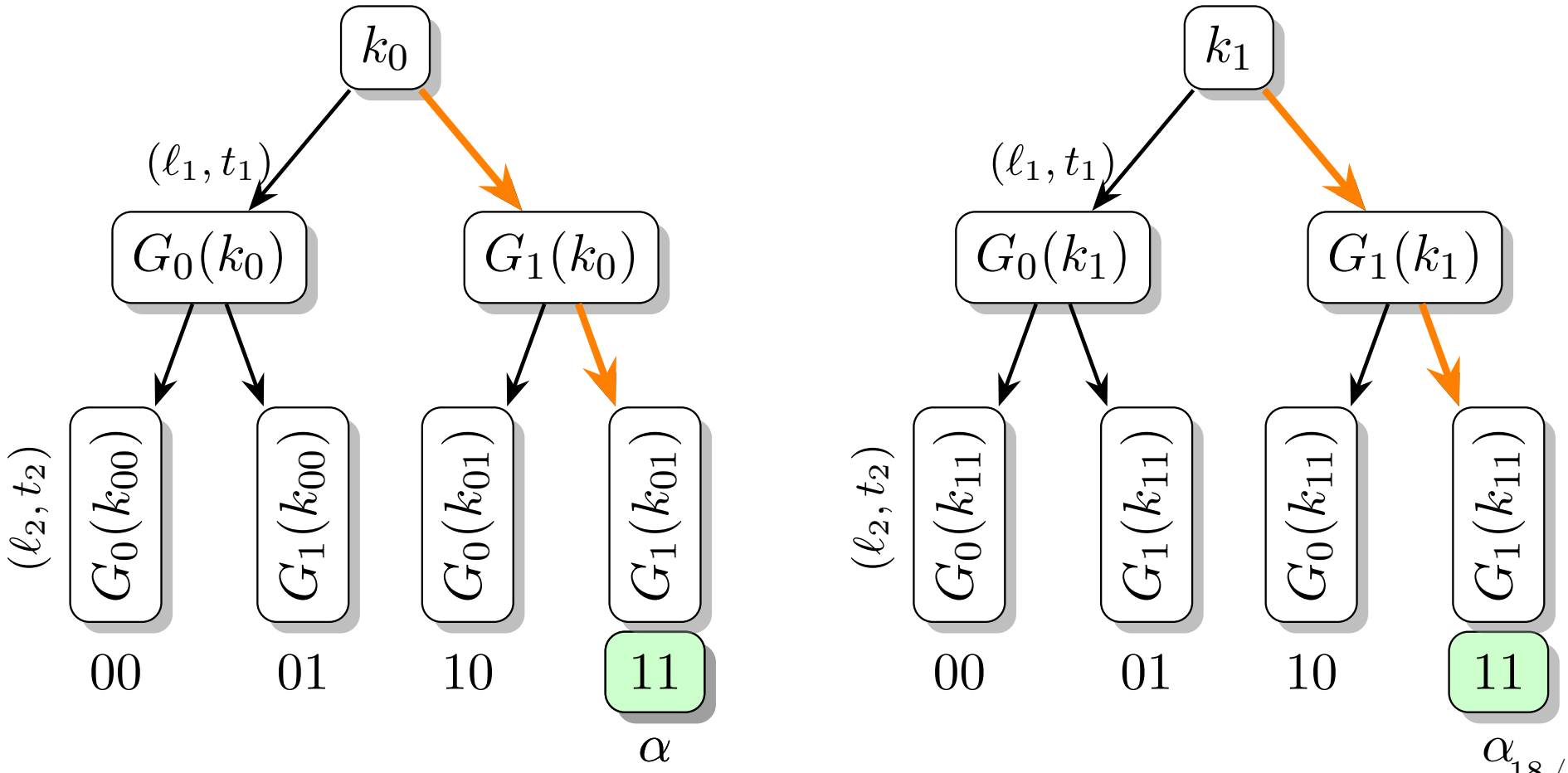

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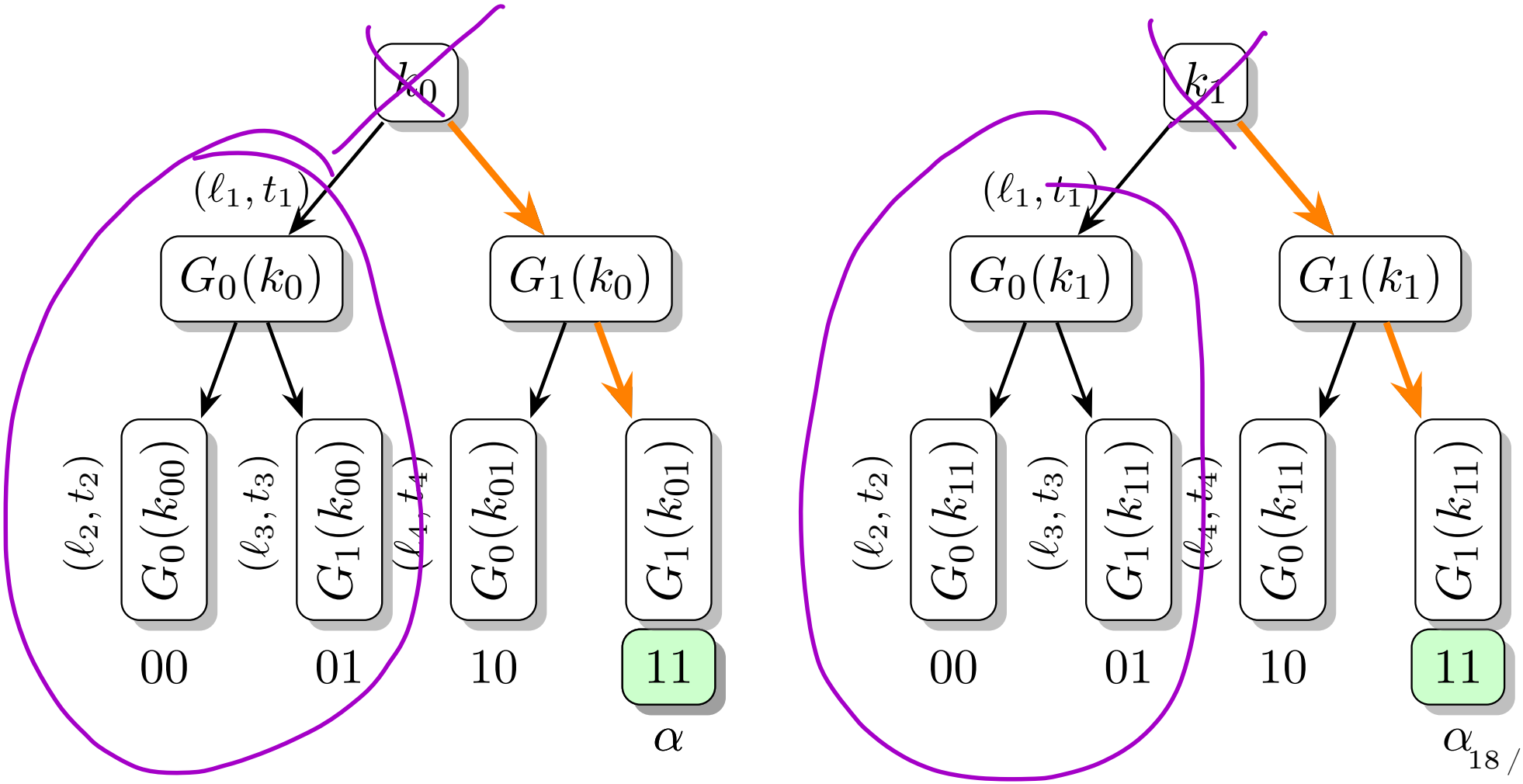

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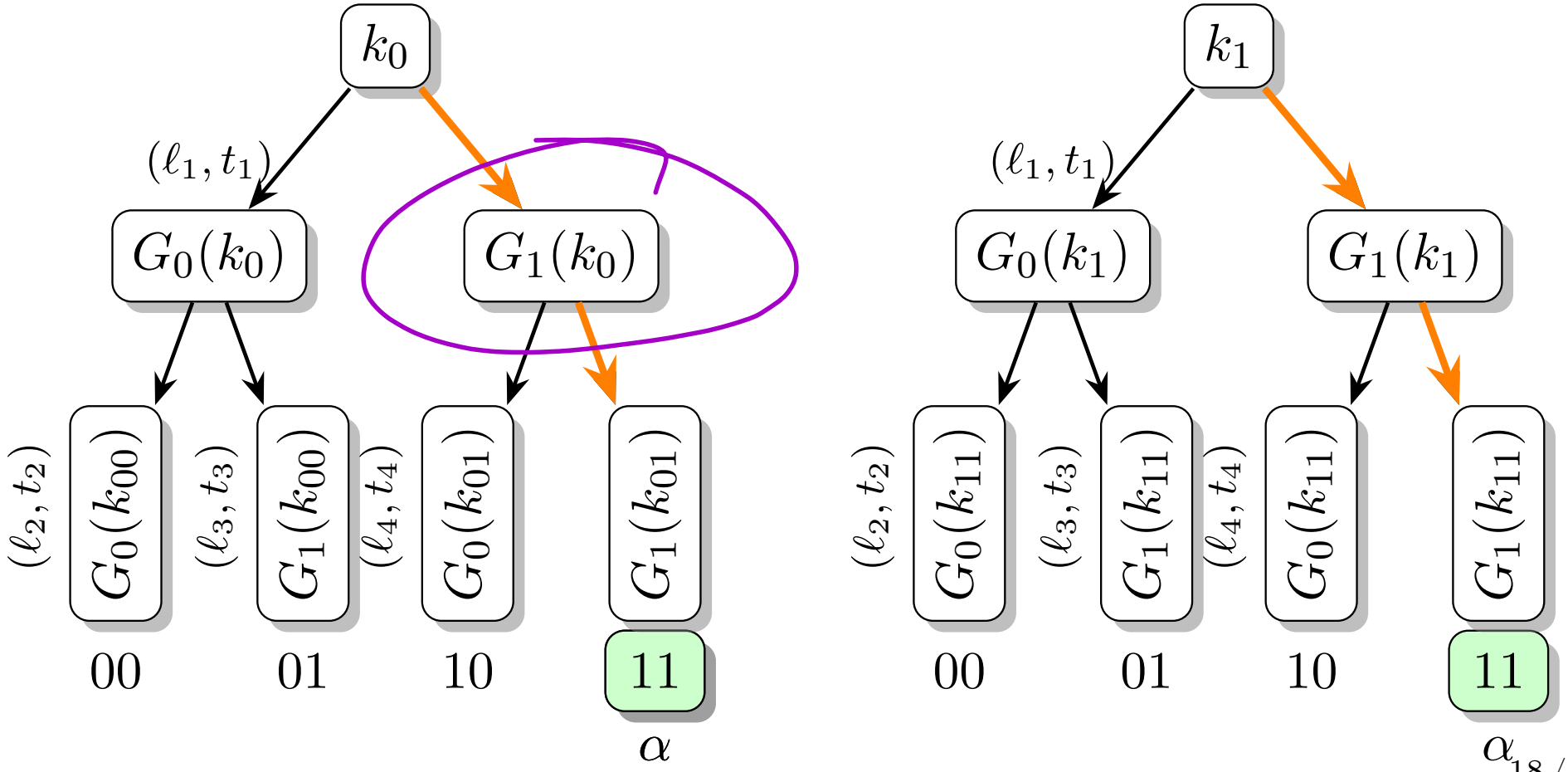

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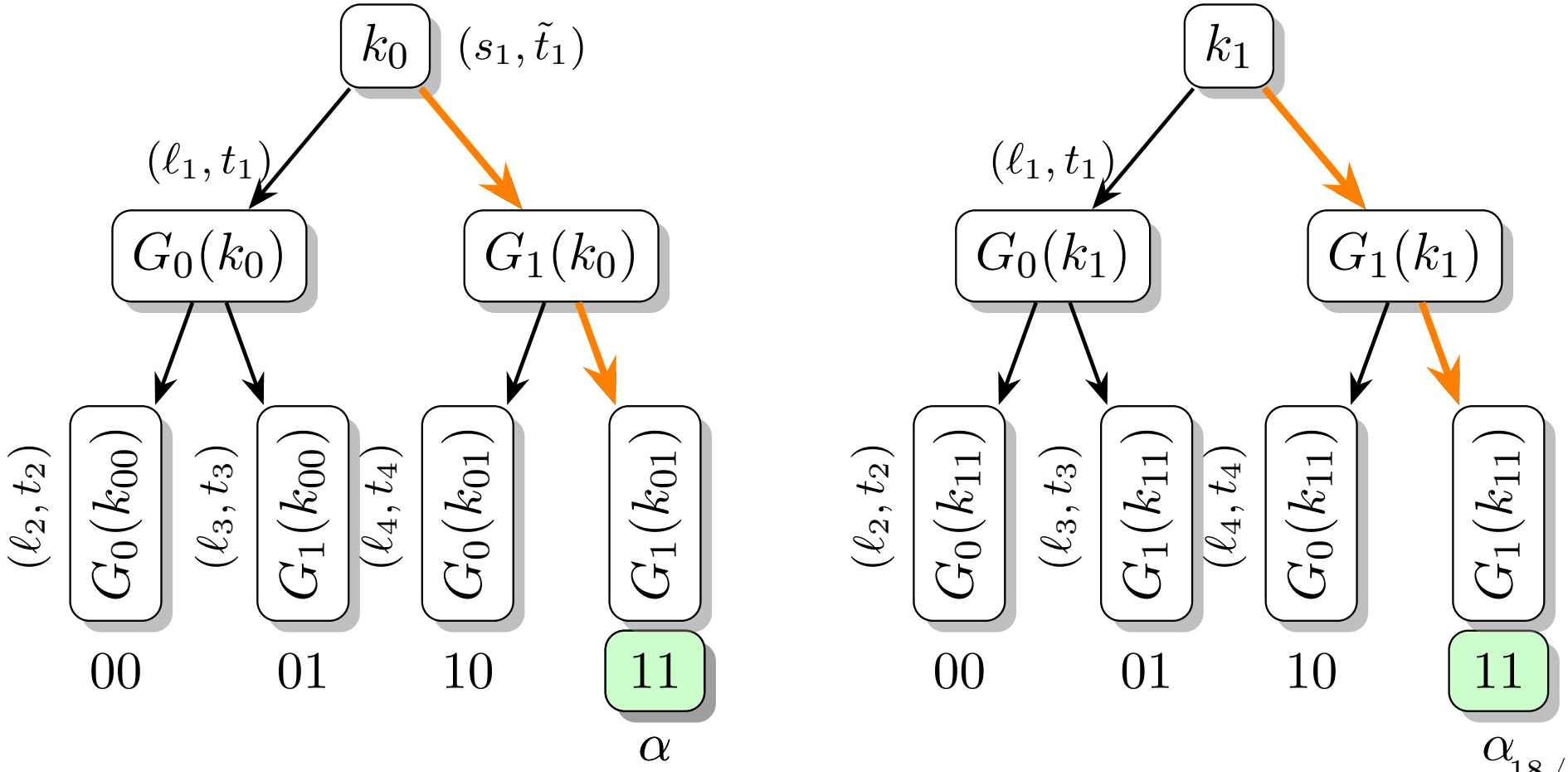

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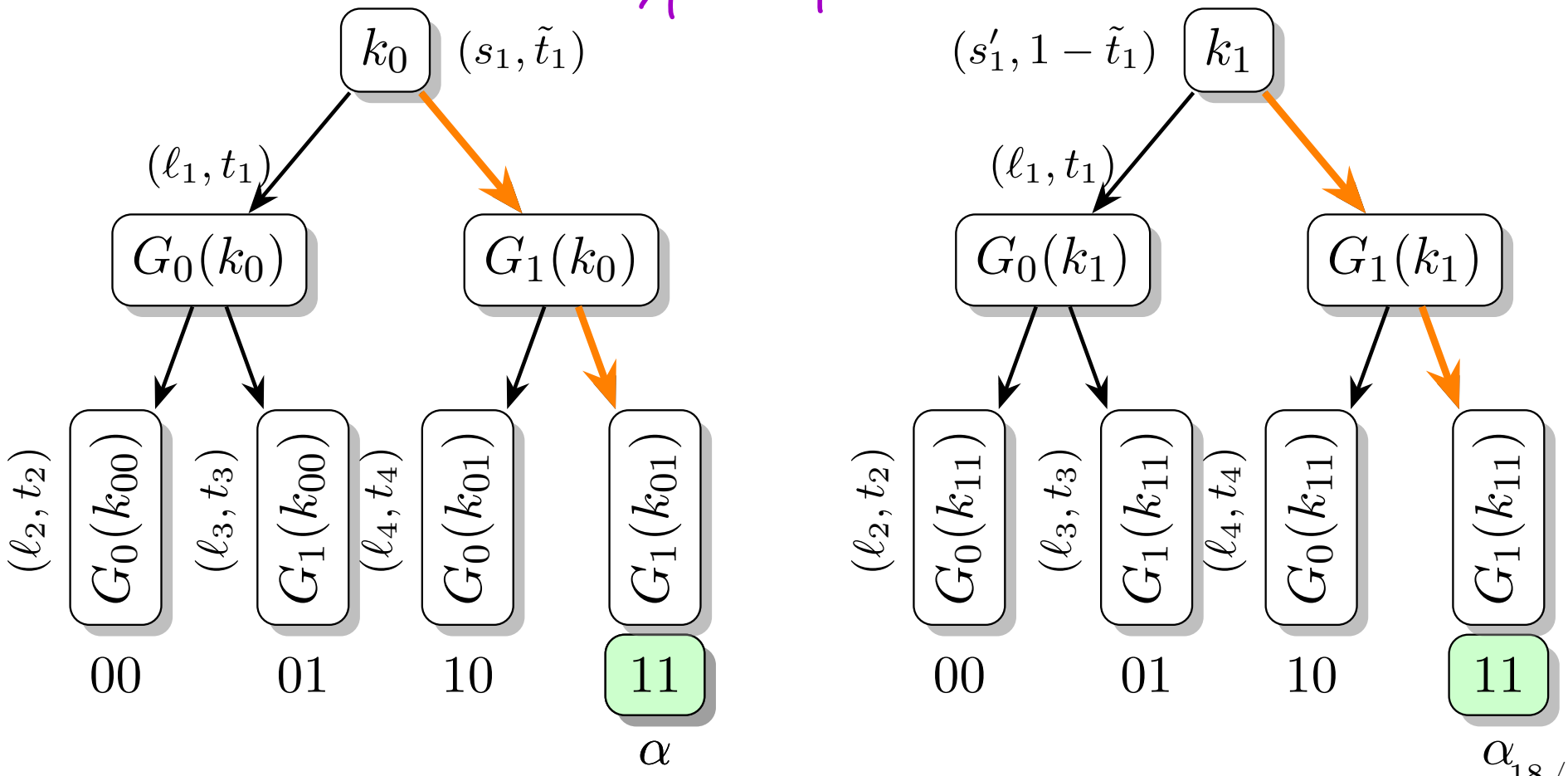

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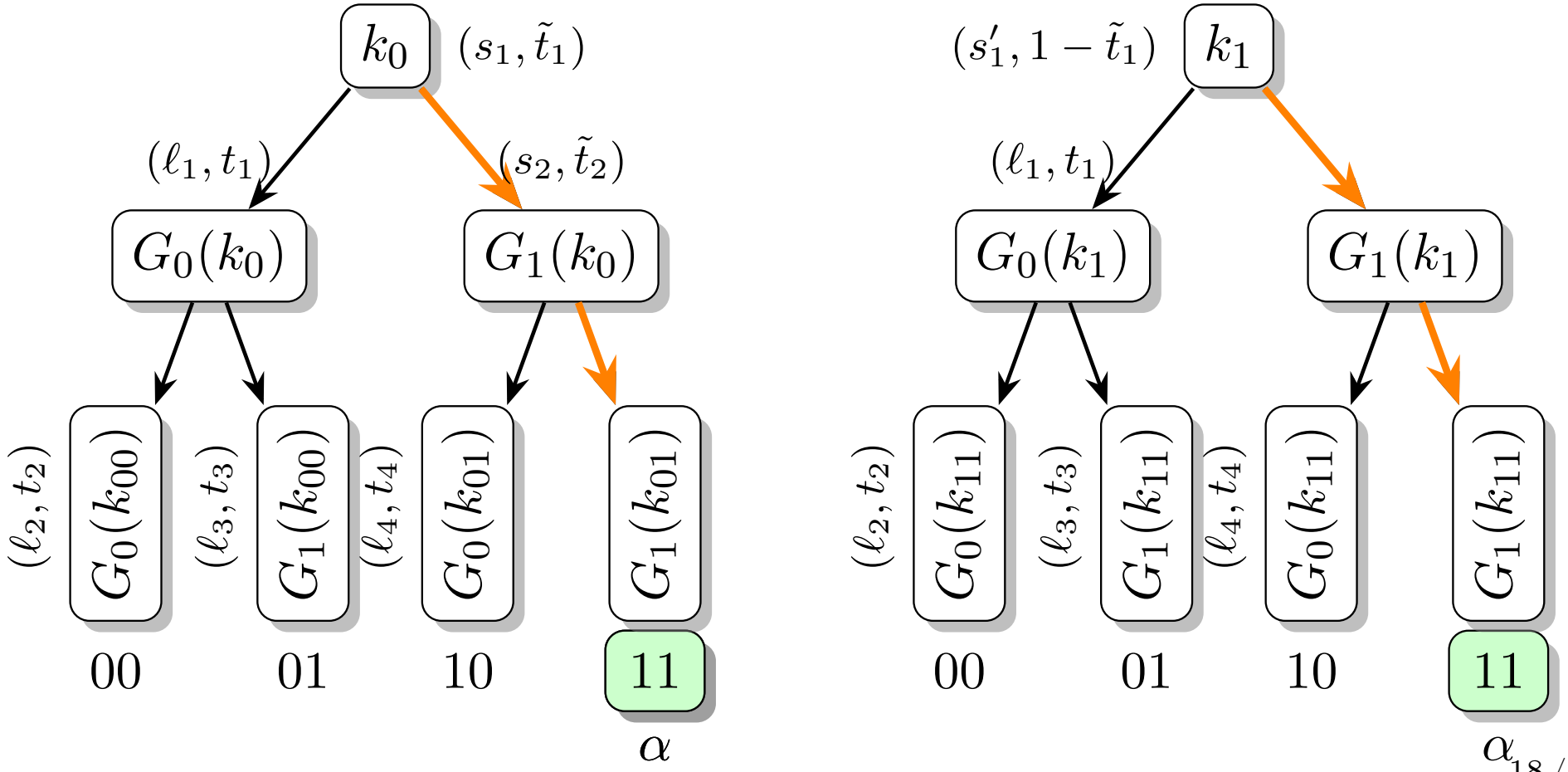

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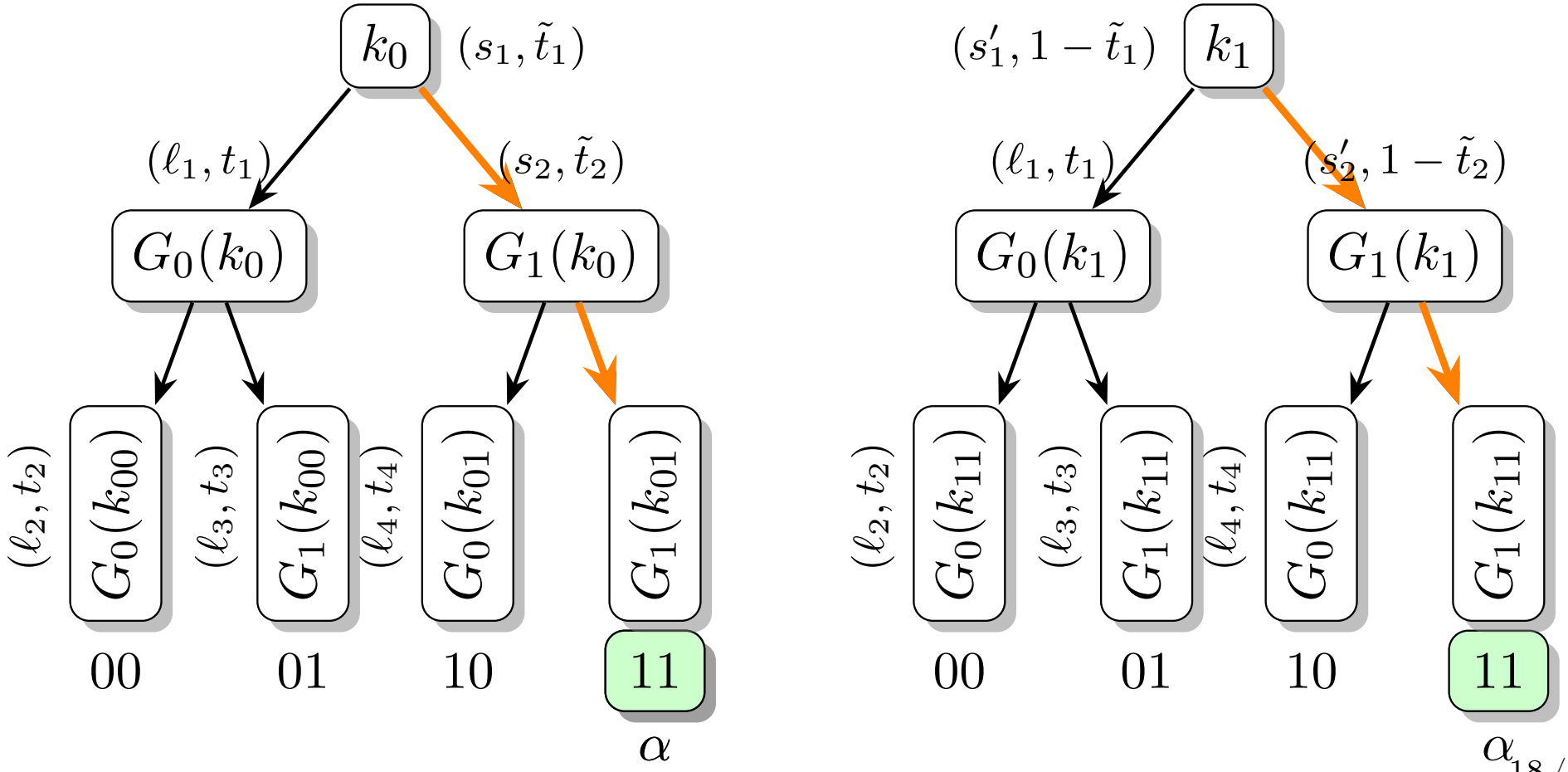

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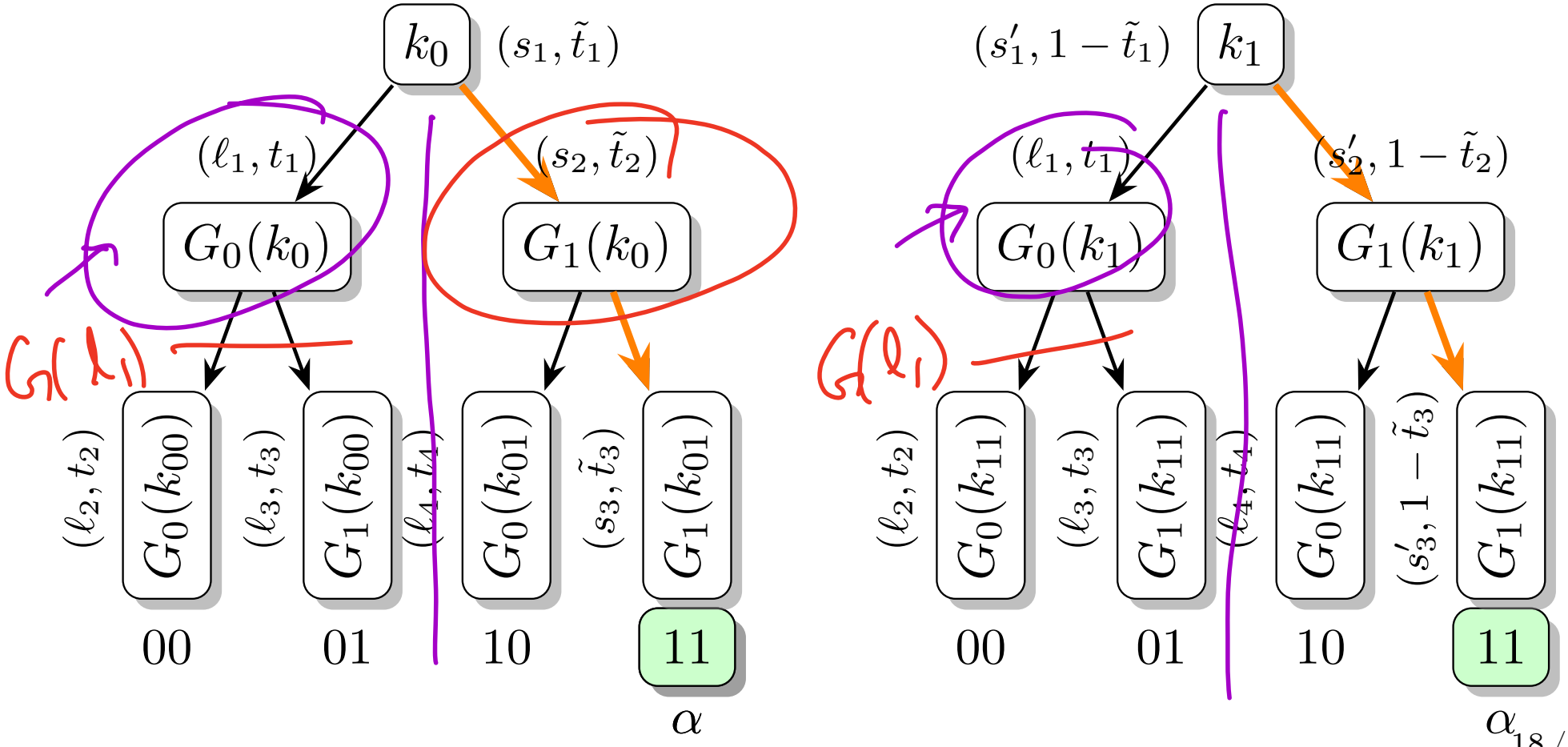

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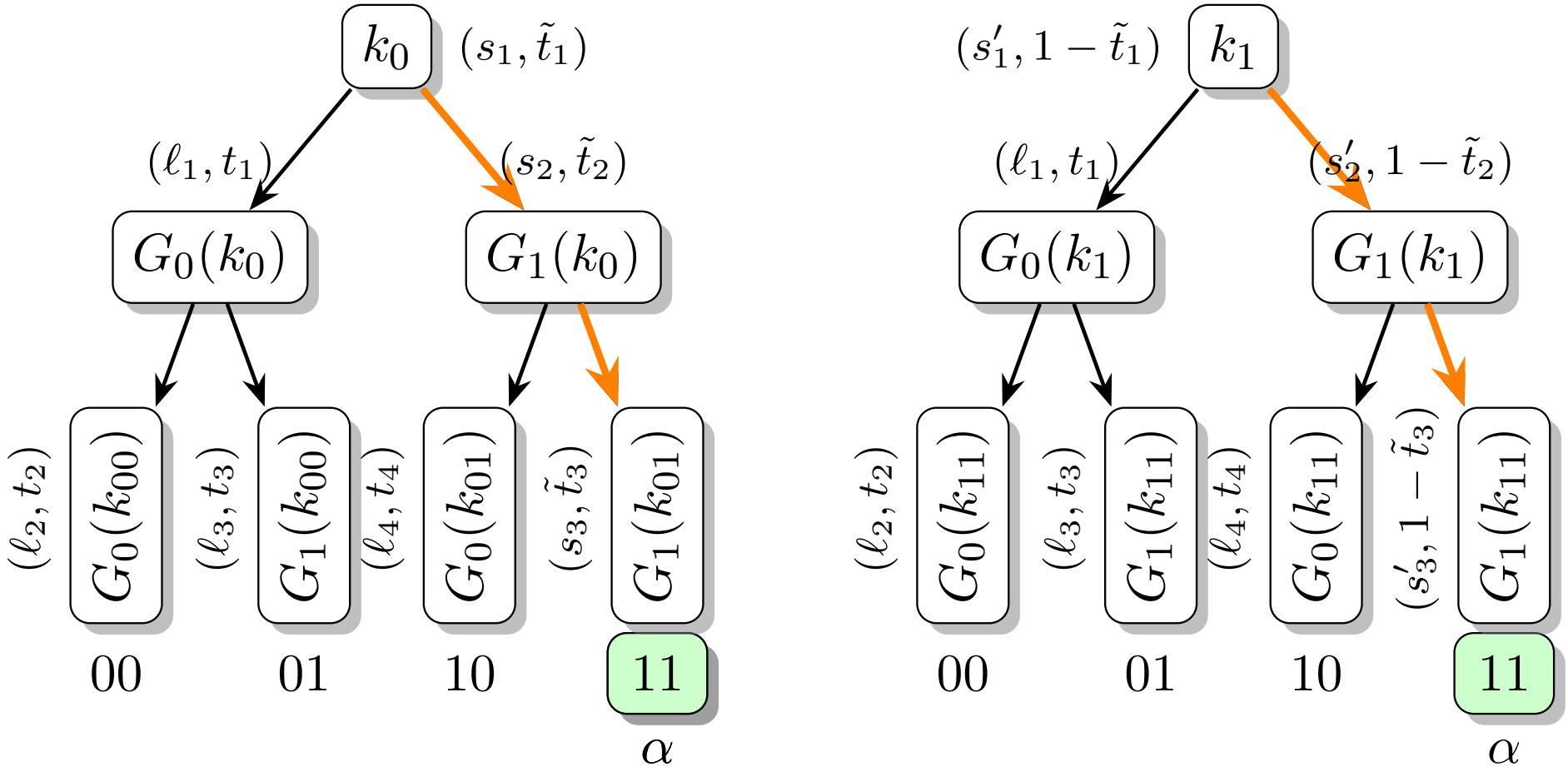

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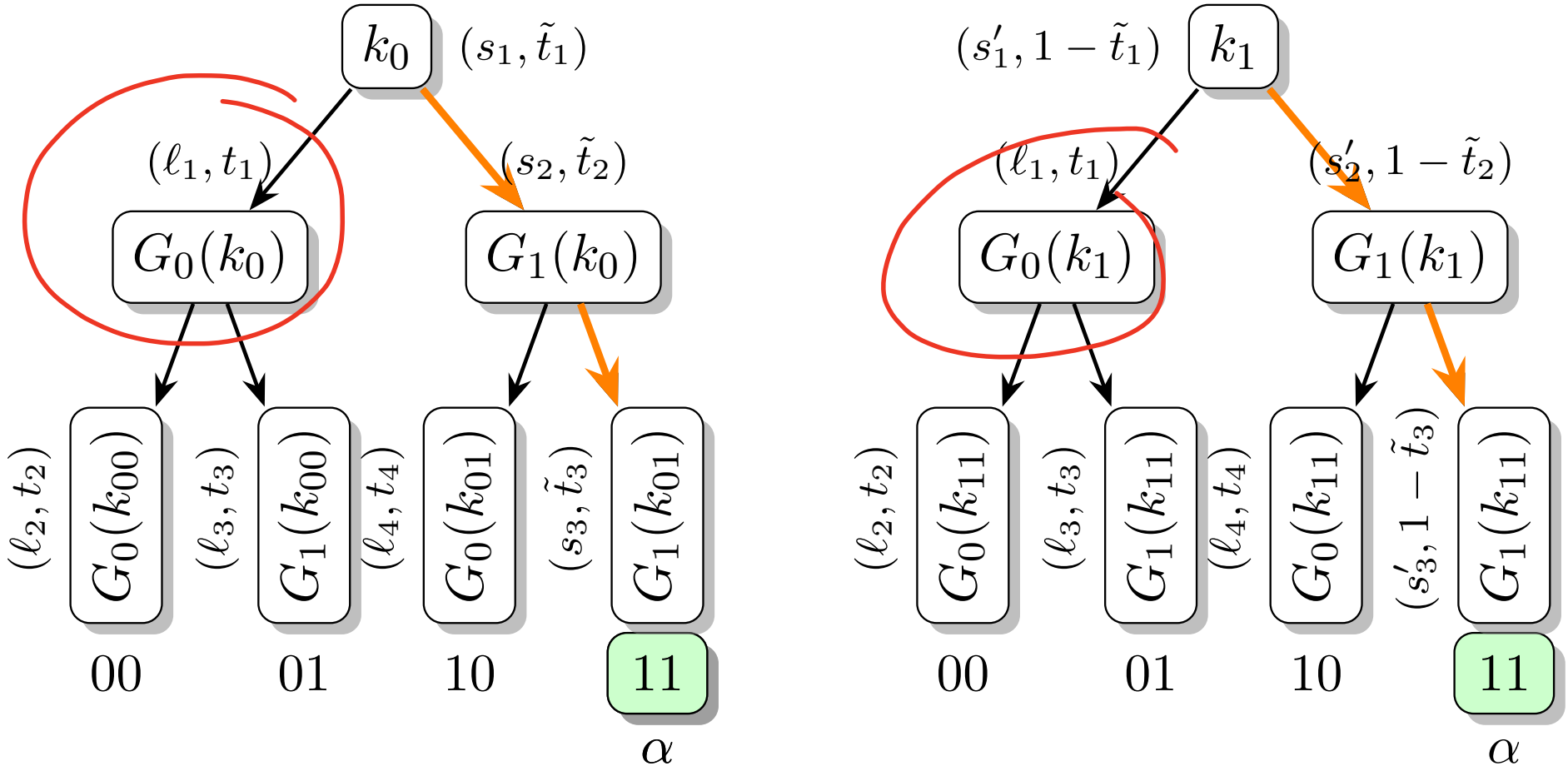

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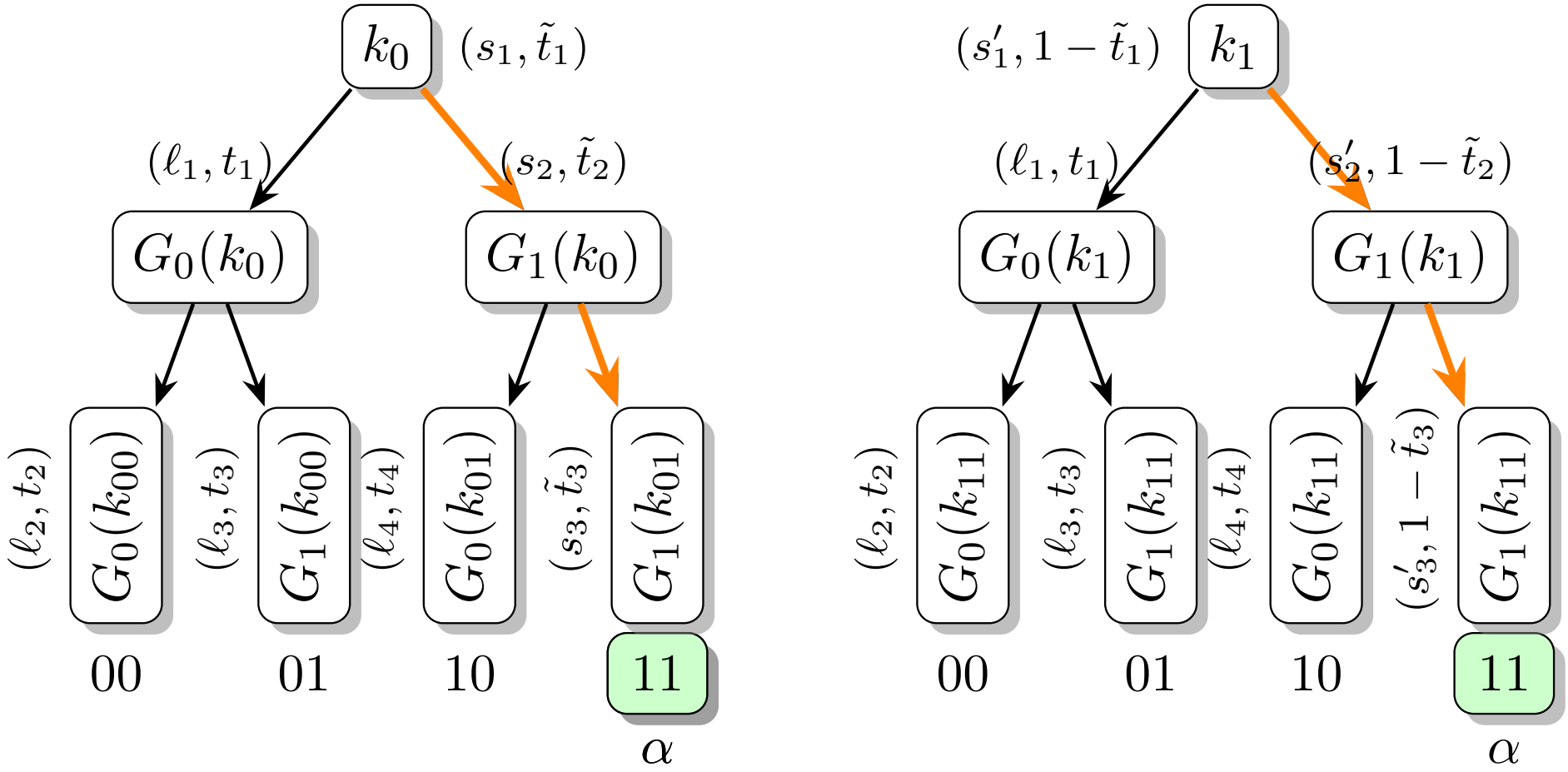

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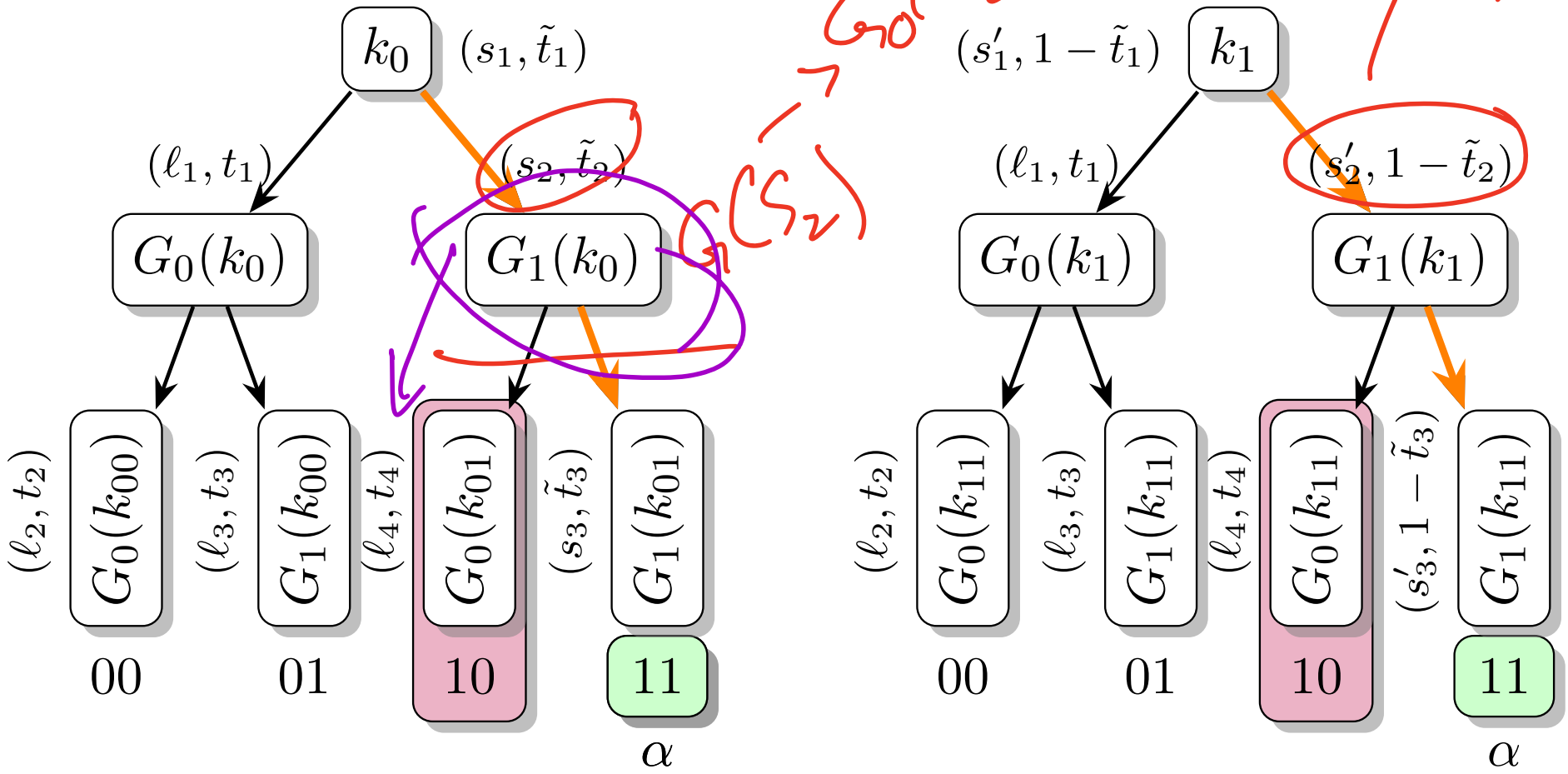

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$t \in \{0,1\}$ $CW \cdot t = \begin{cases} CW & t=1 \\ 0 & t=0 \end{cases}$

$(s_0 \oplus (t_0 \cdot CW)) \oplus (s_1 \oplus (t_1 \cdot CW)) = s \oplus \underbrace{(t_0 \oplus t_1)}_{t} \cdot CW$

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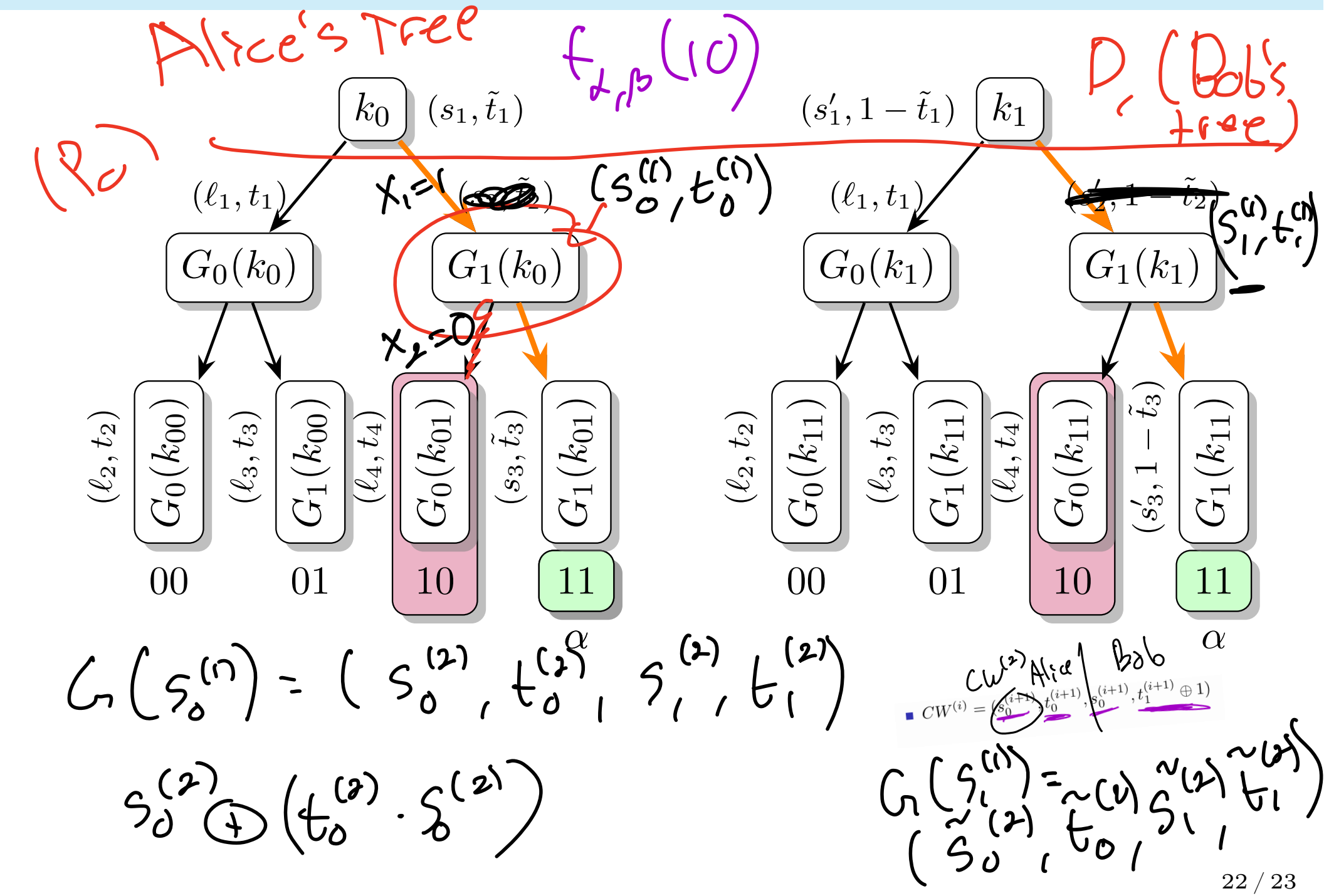

$x_2 = 0$

$G(s_0^{(1)}) = (s_0^{(2)}, t_0^{(2)}, s_1^{(2)}, t_1^{(2)})$

$s_0^{(2)} \oplus (t_0^{(2)} \cdot s_0^{(2)})$

$\rightarrow$

$t_0^{(2)} = 0$

$\rightarrow s_0^{(2)}$

Alice

$G(\tilde{s}_1^{(1)}) = (\tilde{s}_0^{(2)}, \tilde{t}_0^{(2)}, \tilde{s}_1^{(2)}, \tilde{t}_1^{(2)})$

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$CW^{(2)}$

Alice | Bob

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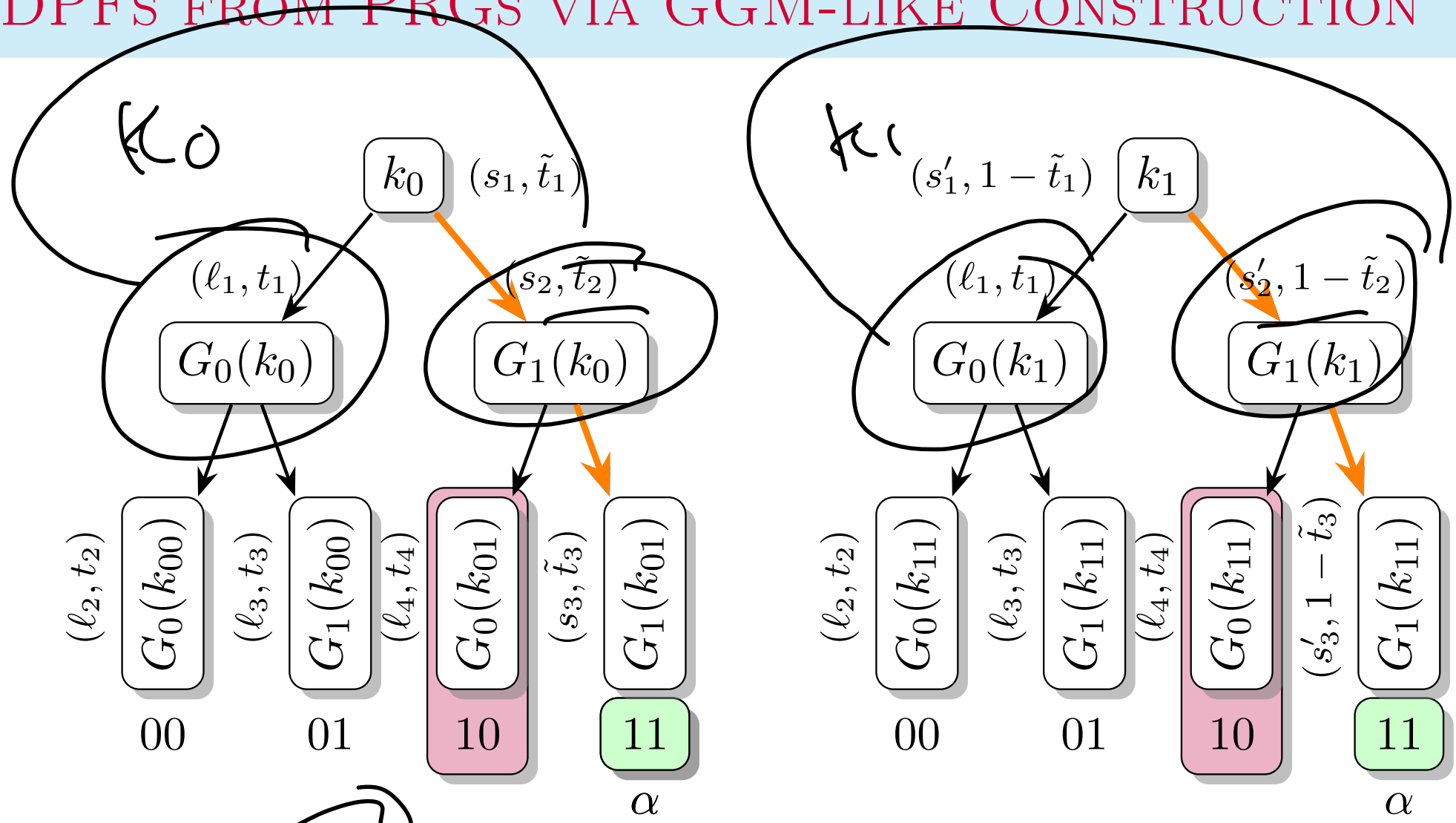

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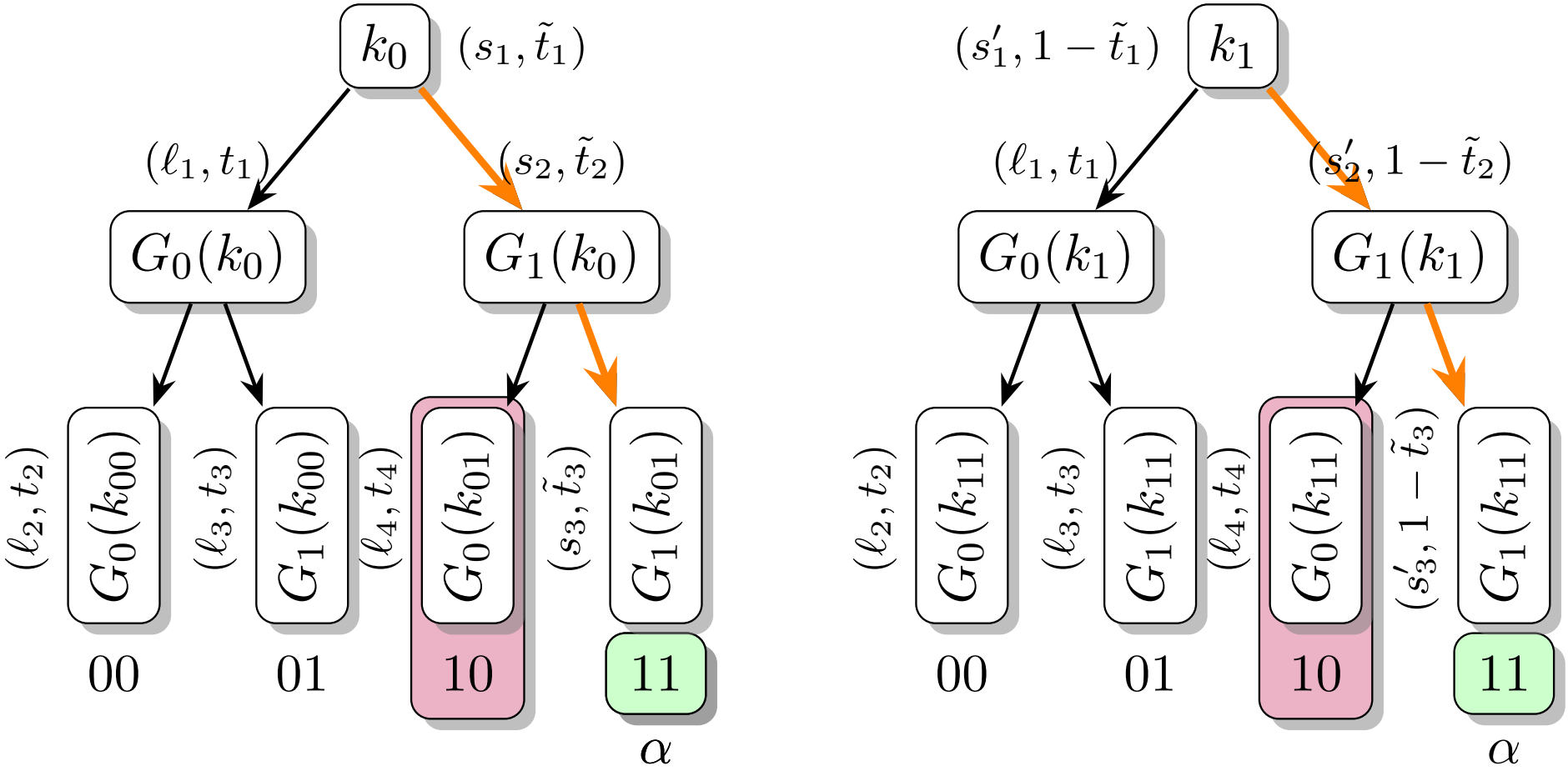

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- Reconstruction: use $CW^{(i)}$ to compute the label of the leaf corresponding to α, output the (sum) of shares of t for this node.

Next Time: Zero-knowledge Proofs