CS 594 – Advanced Crypto (Spring 2026)

Alex Block

Lecture 16

March 11, 2026

Security of MPC

Real-Ideal Security Paradigm

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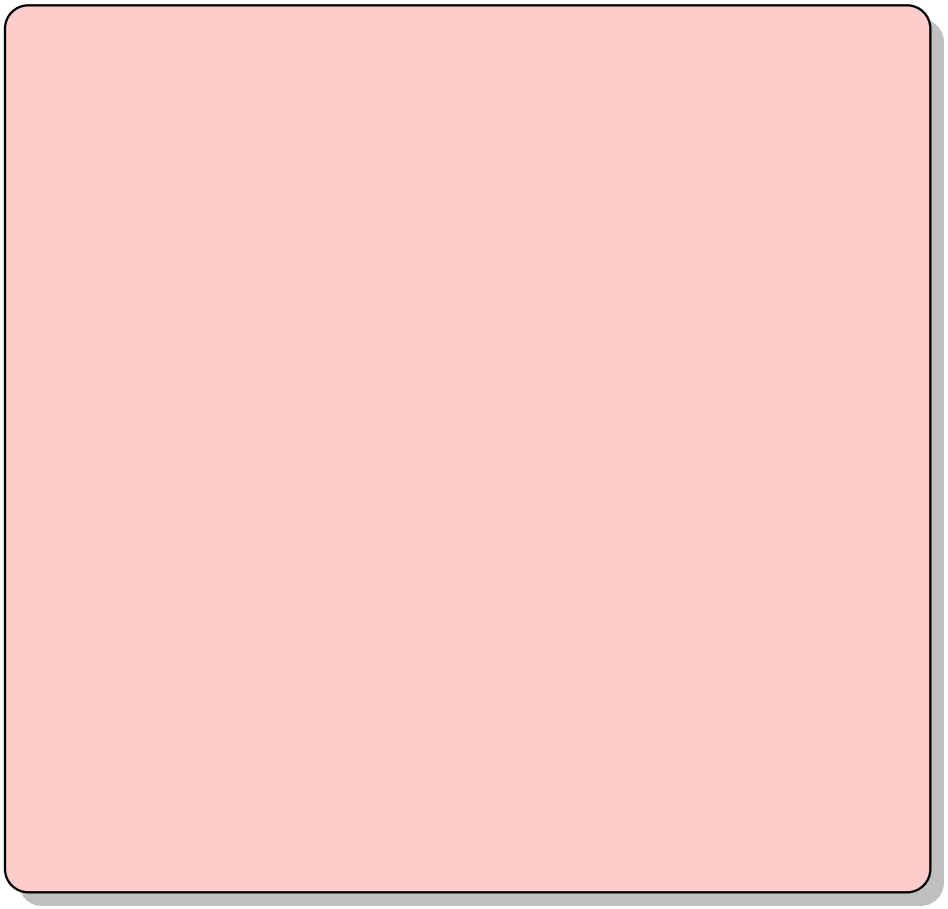

Real World

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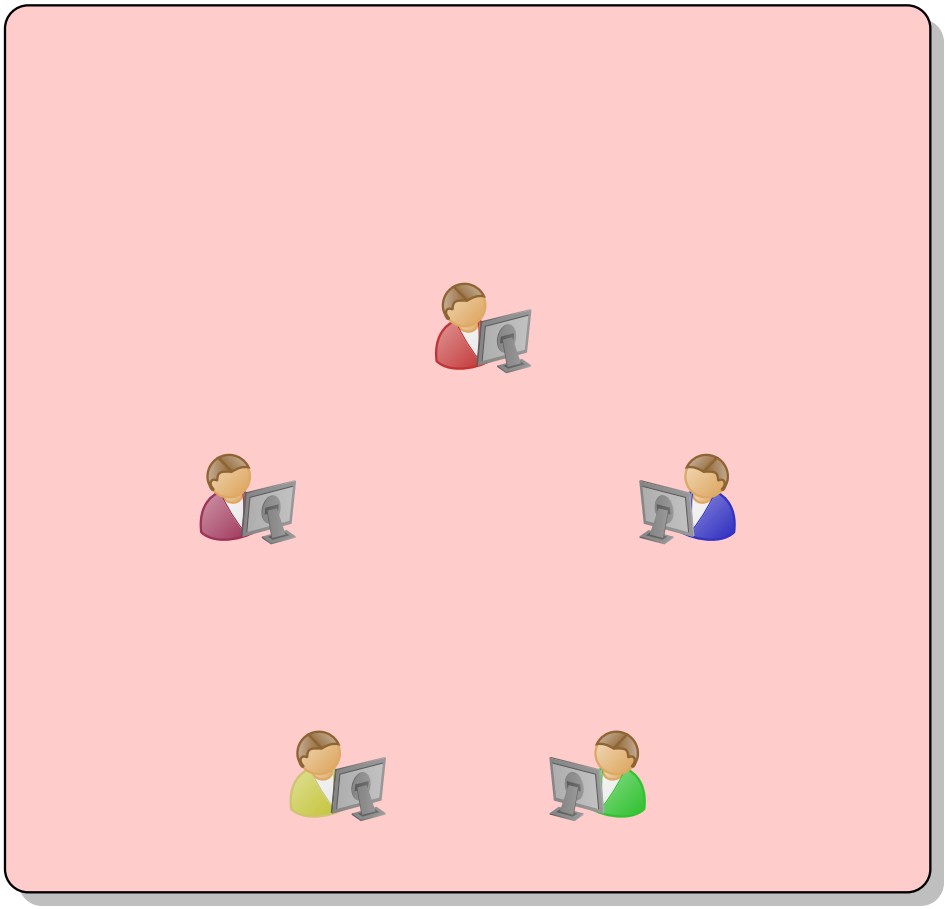

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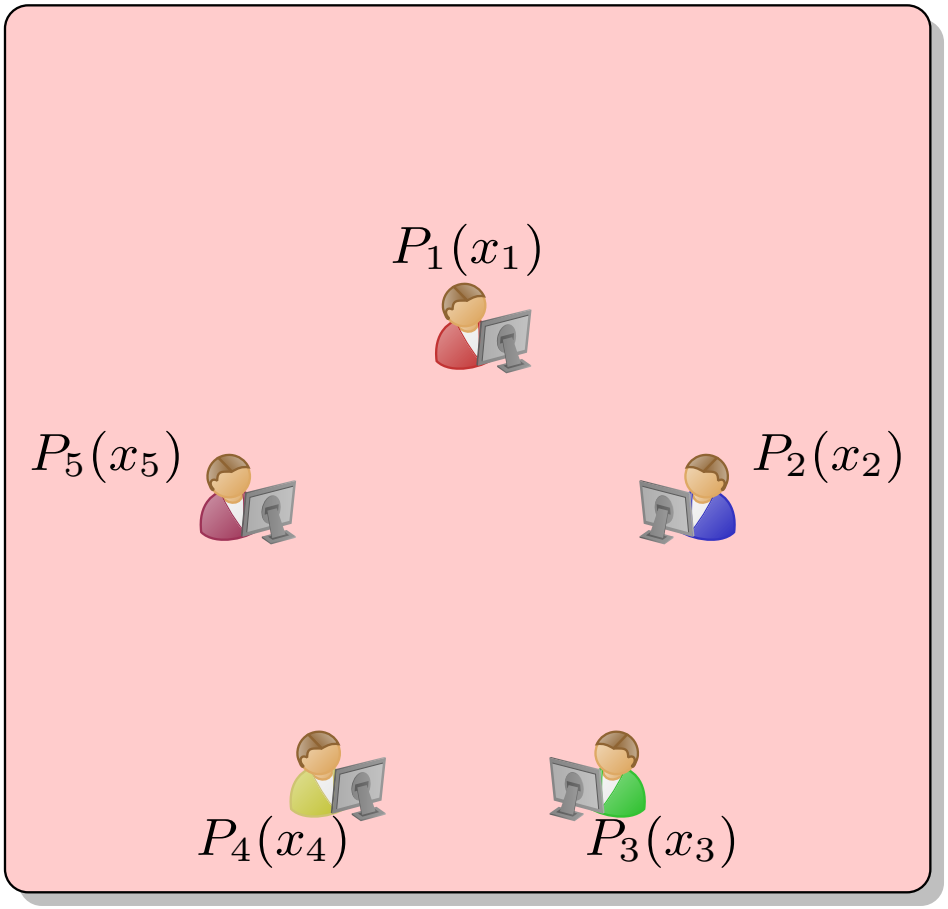

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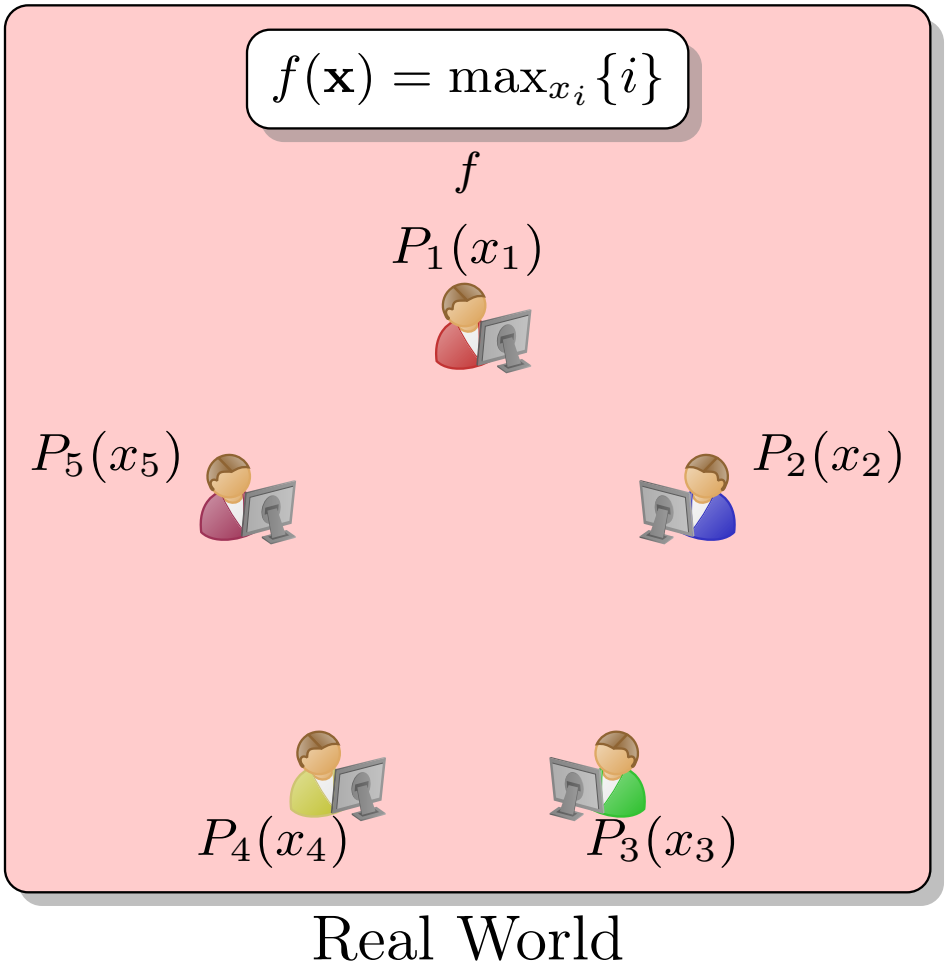

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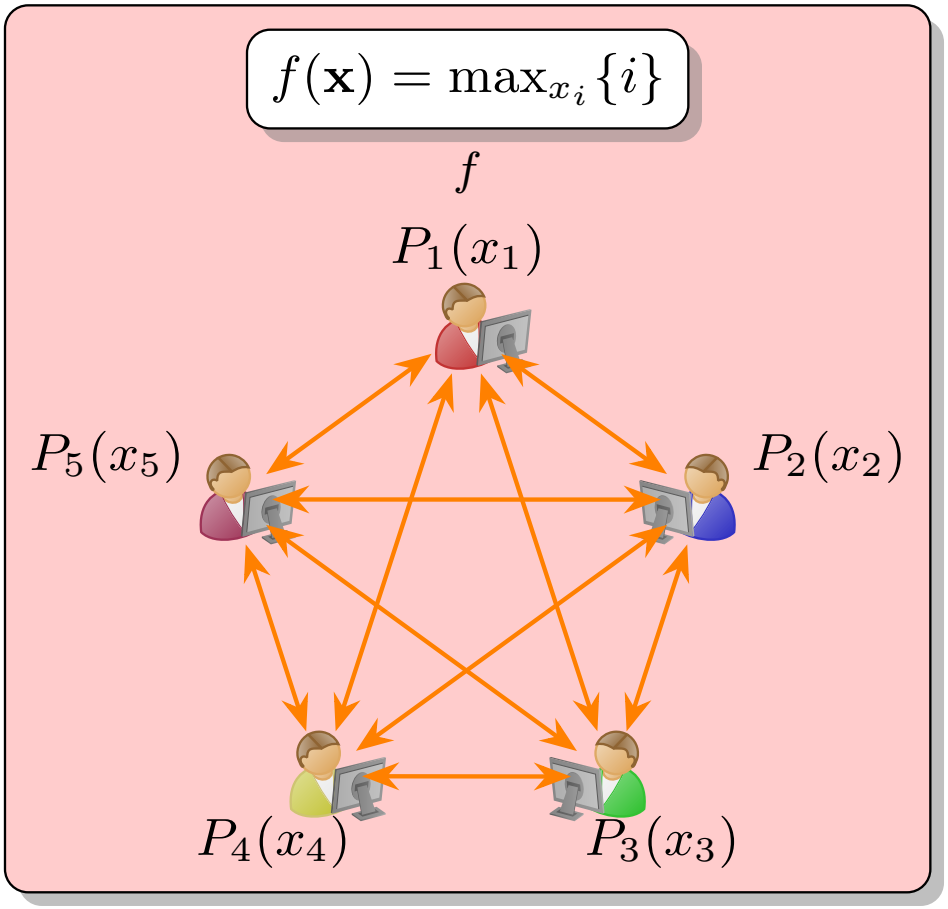

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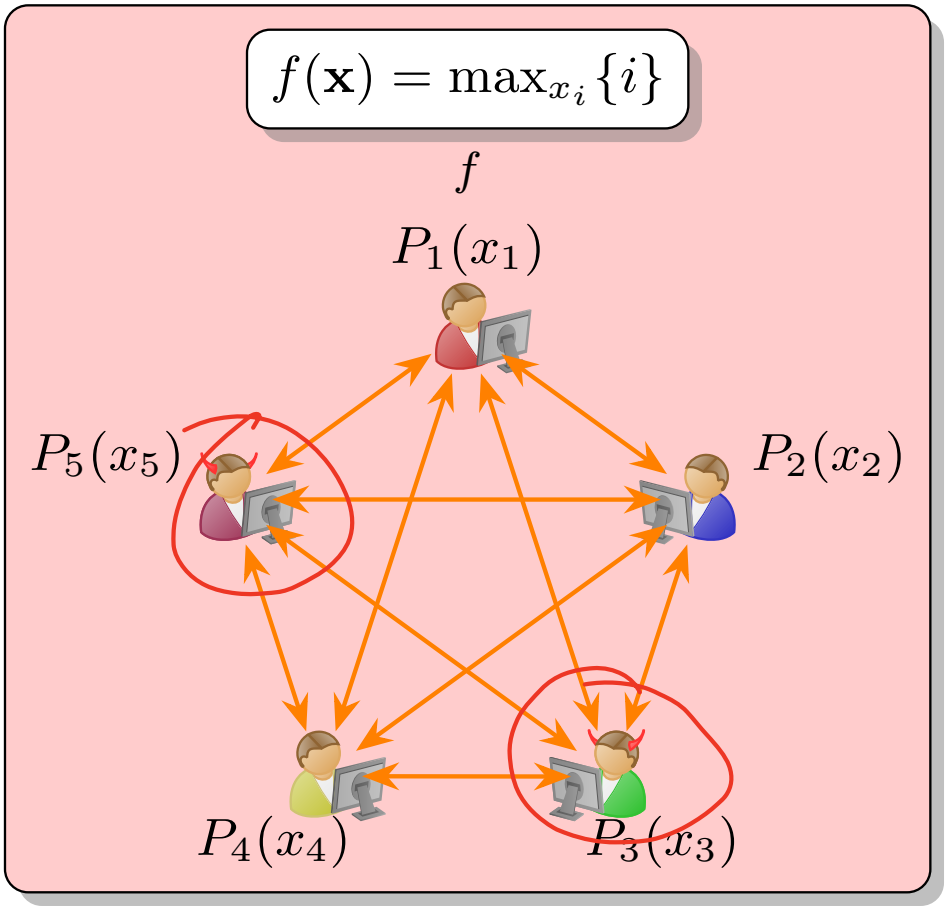

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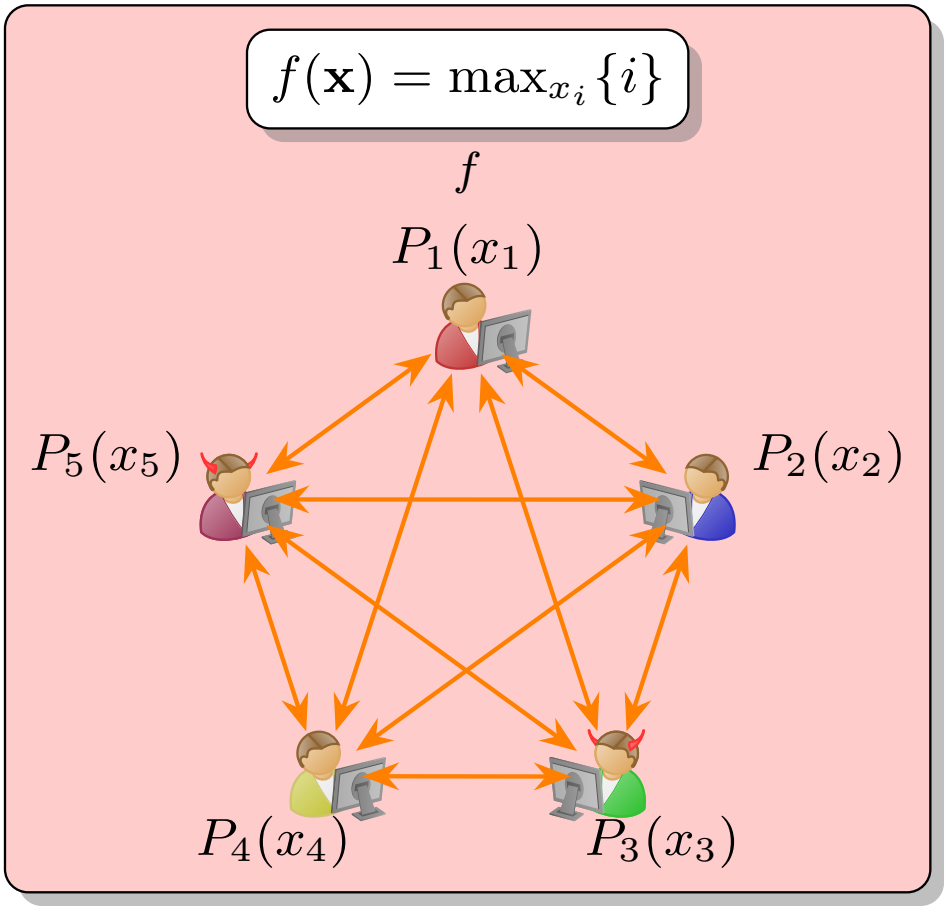

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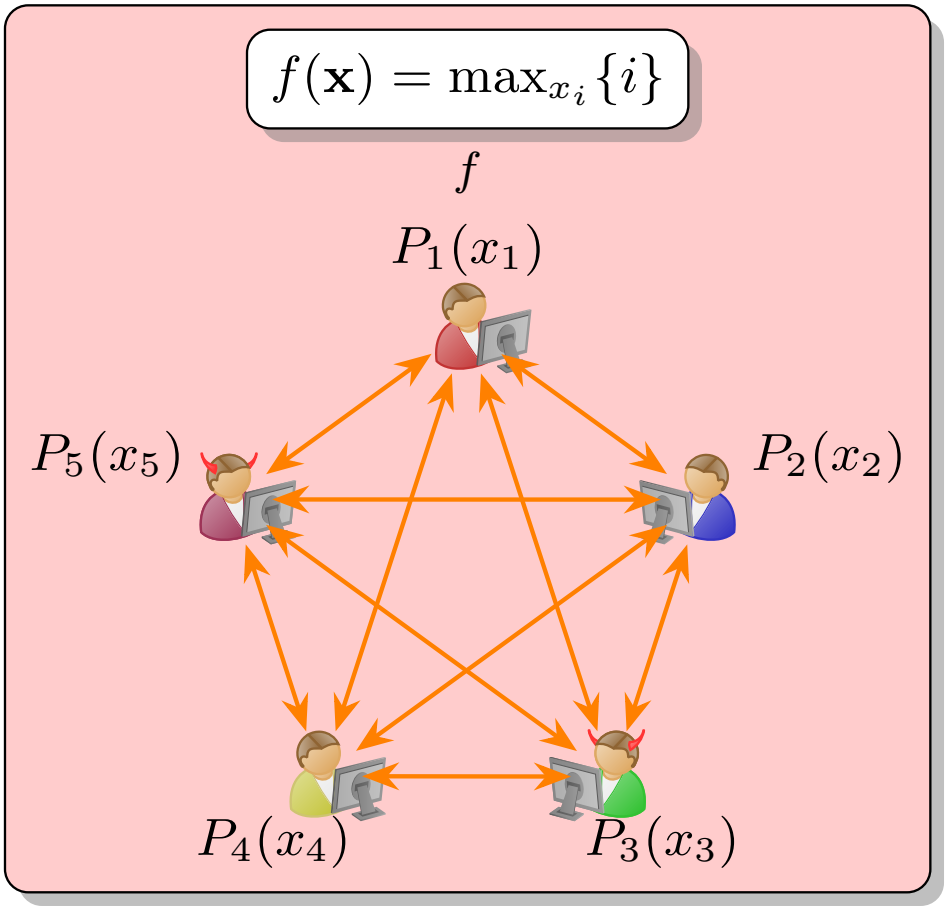

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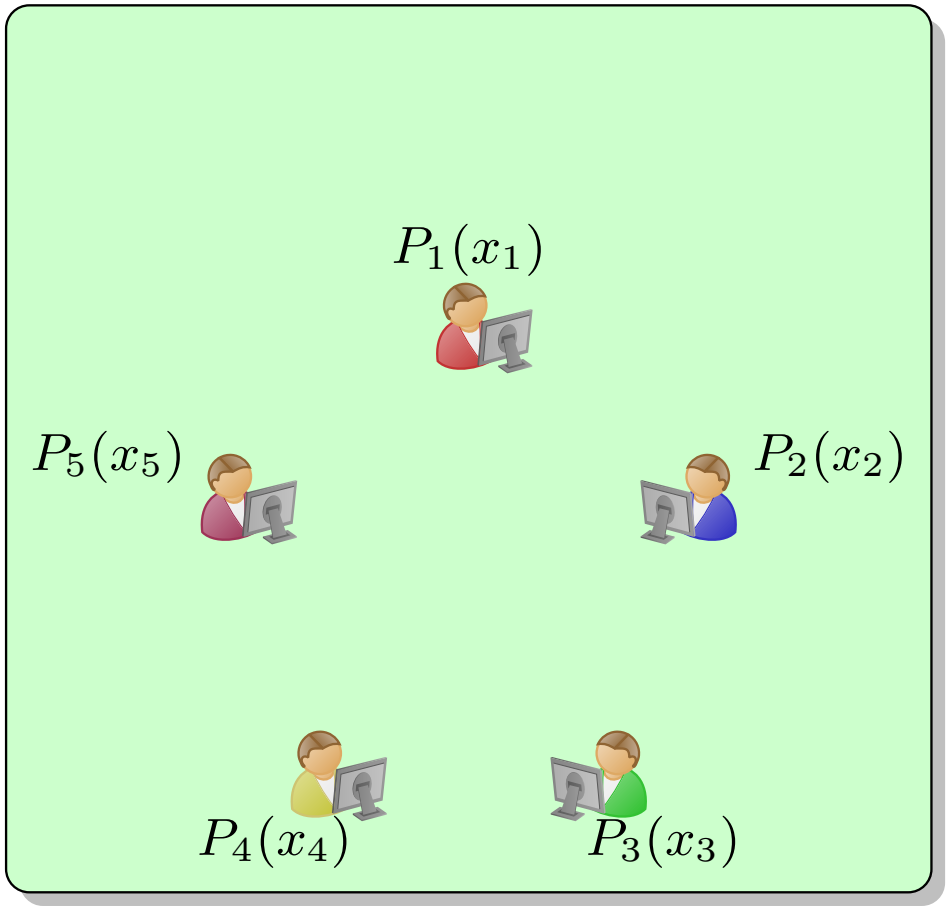

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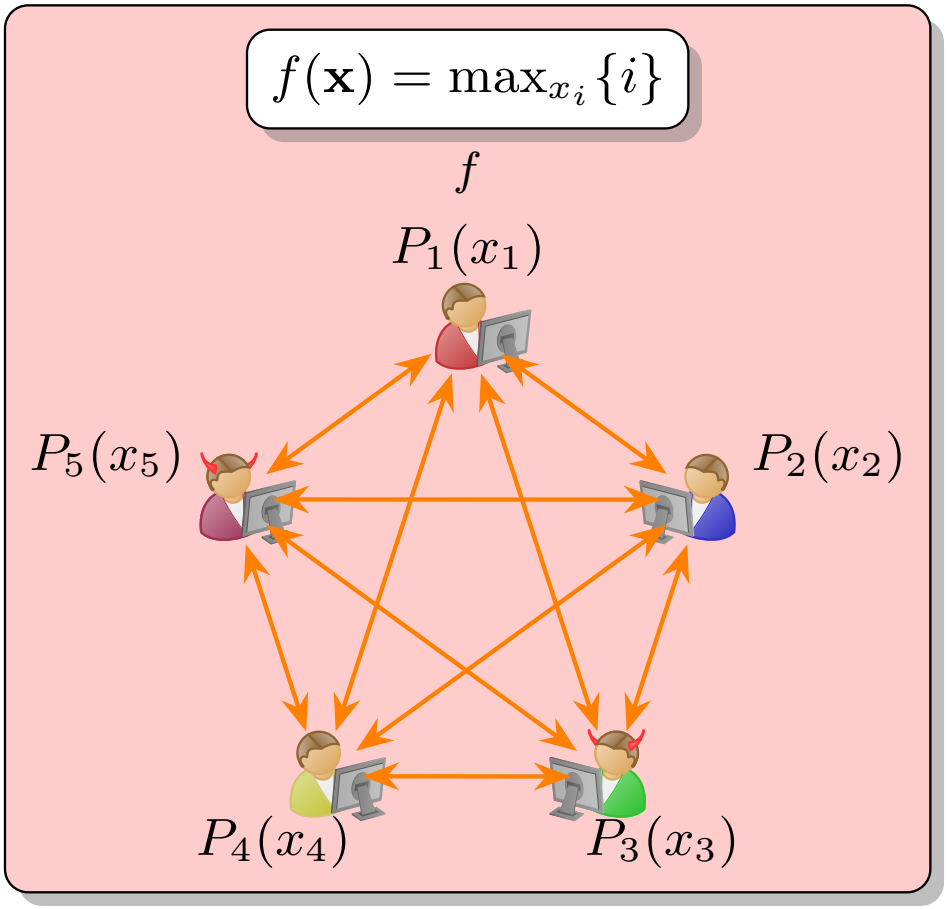

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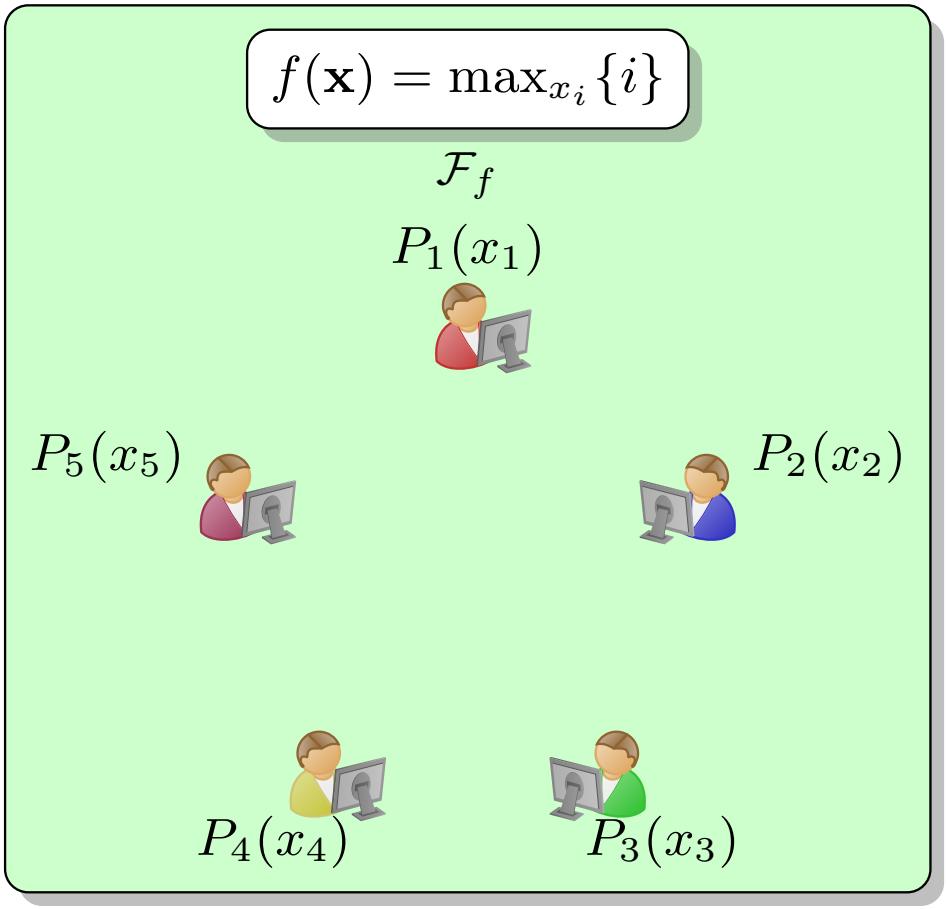

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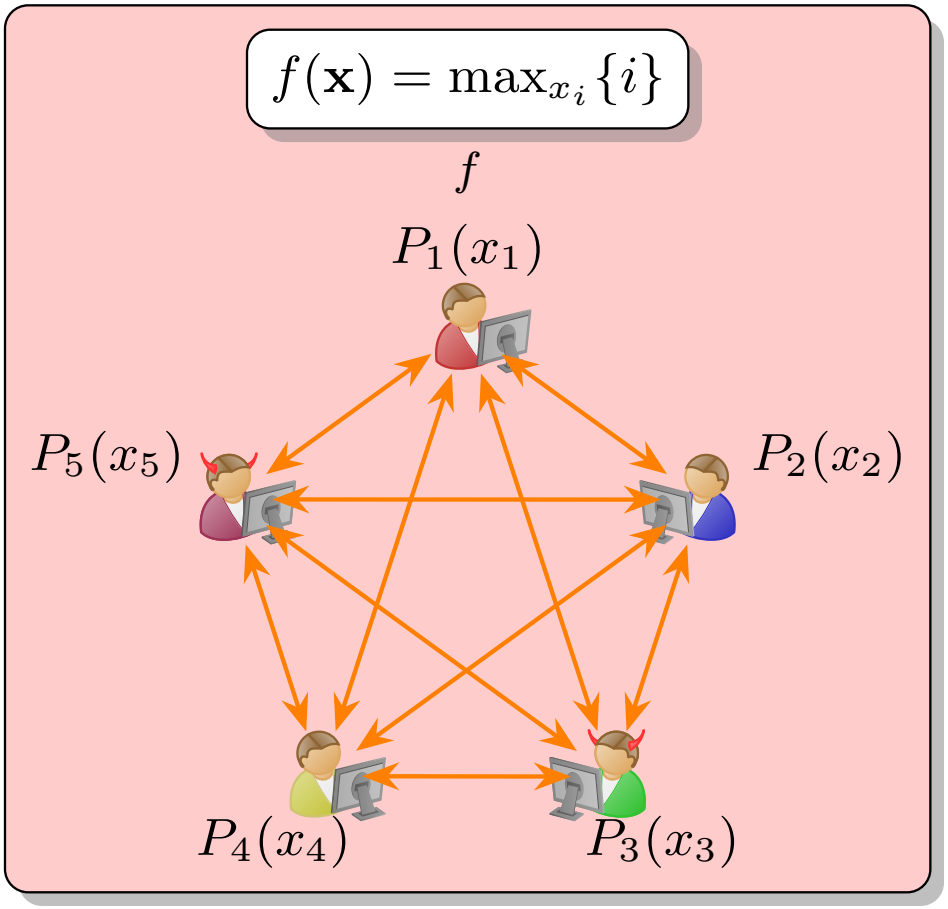

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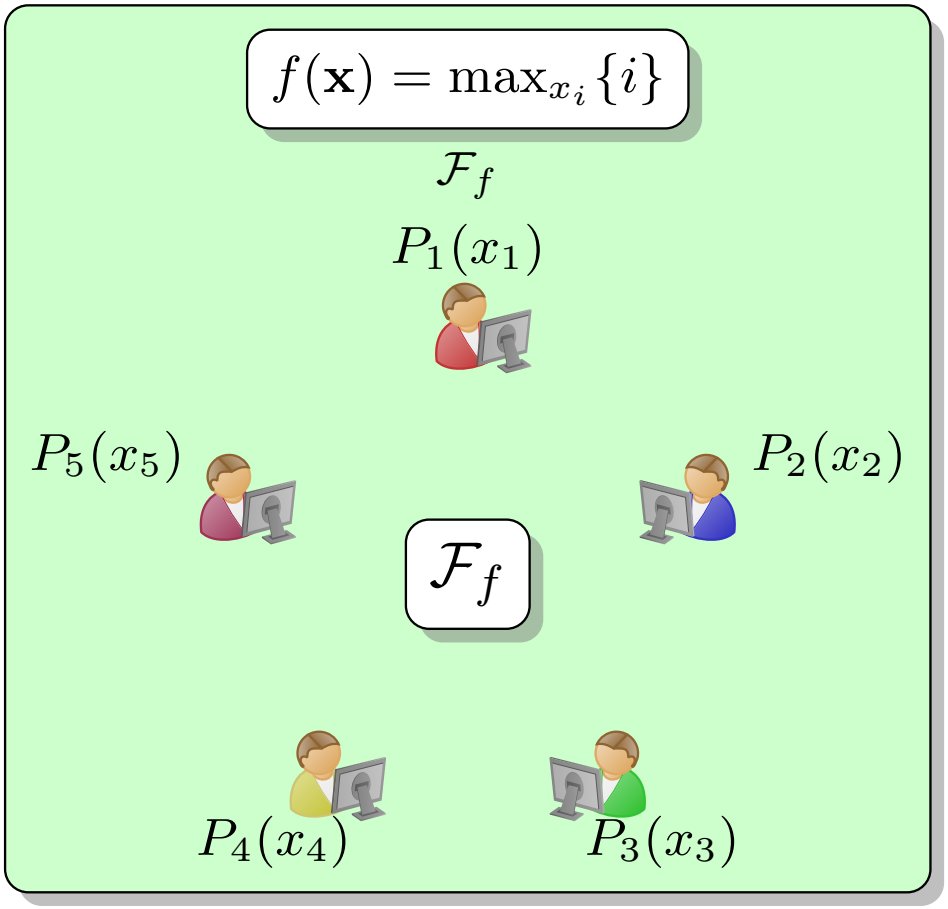

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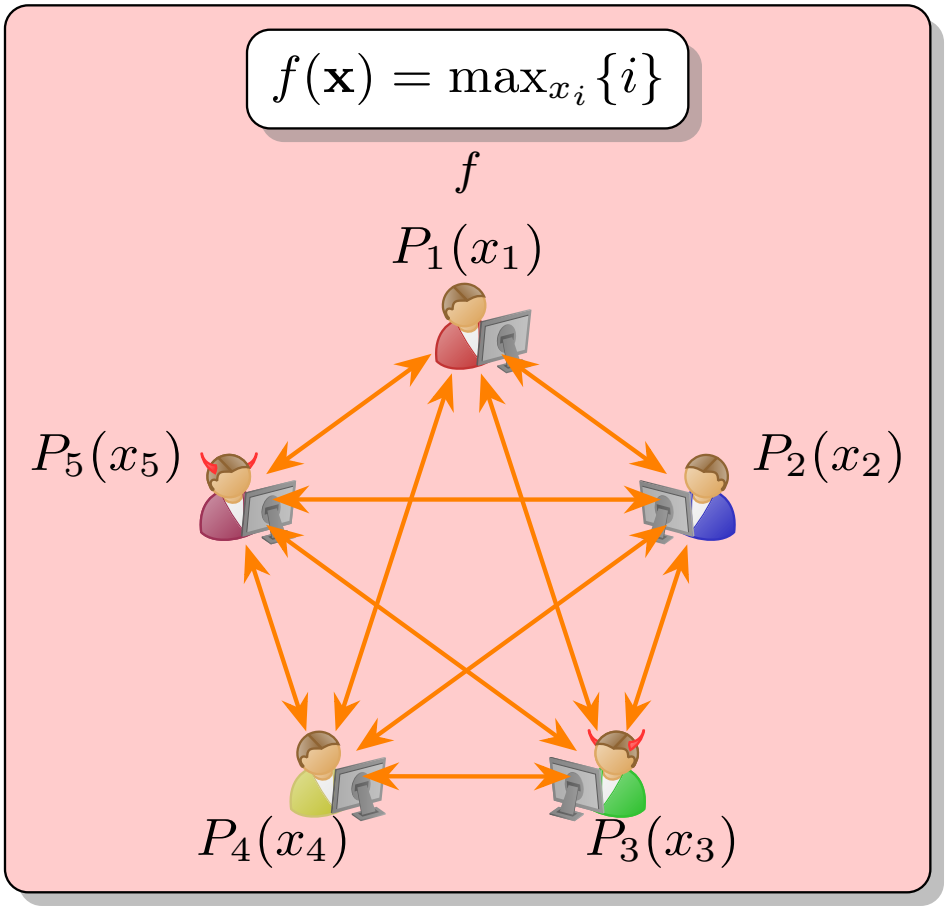

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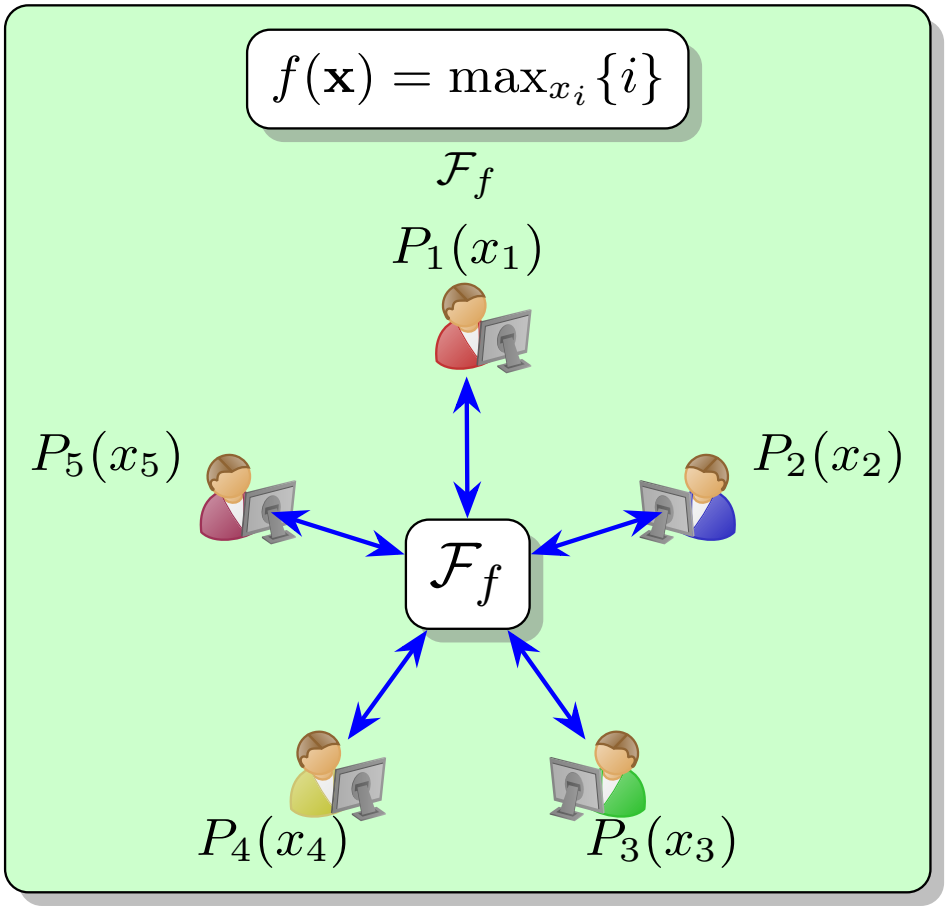

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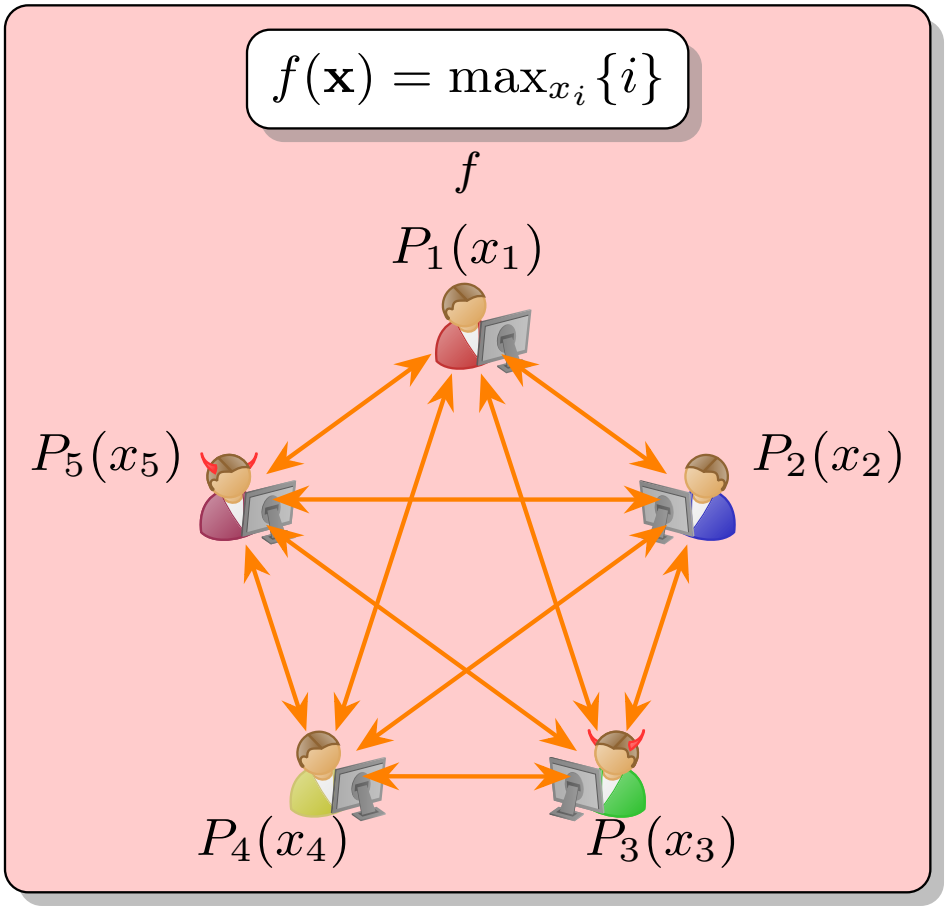

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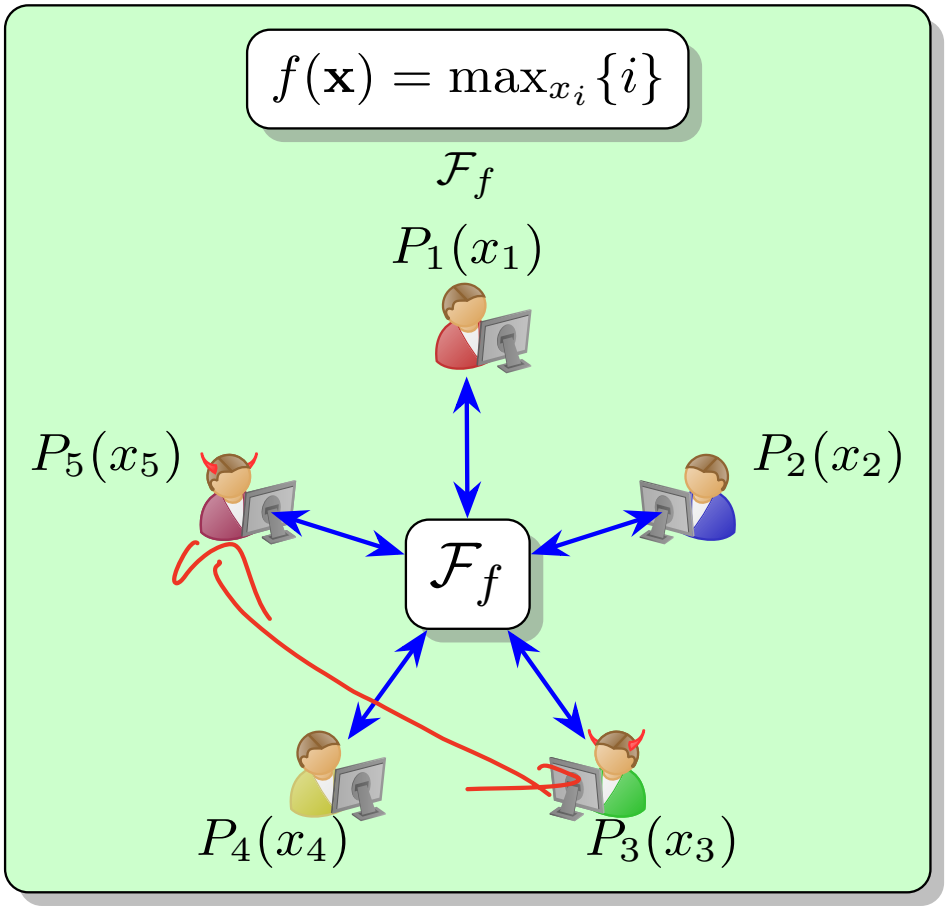
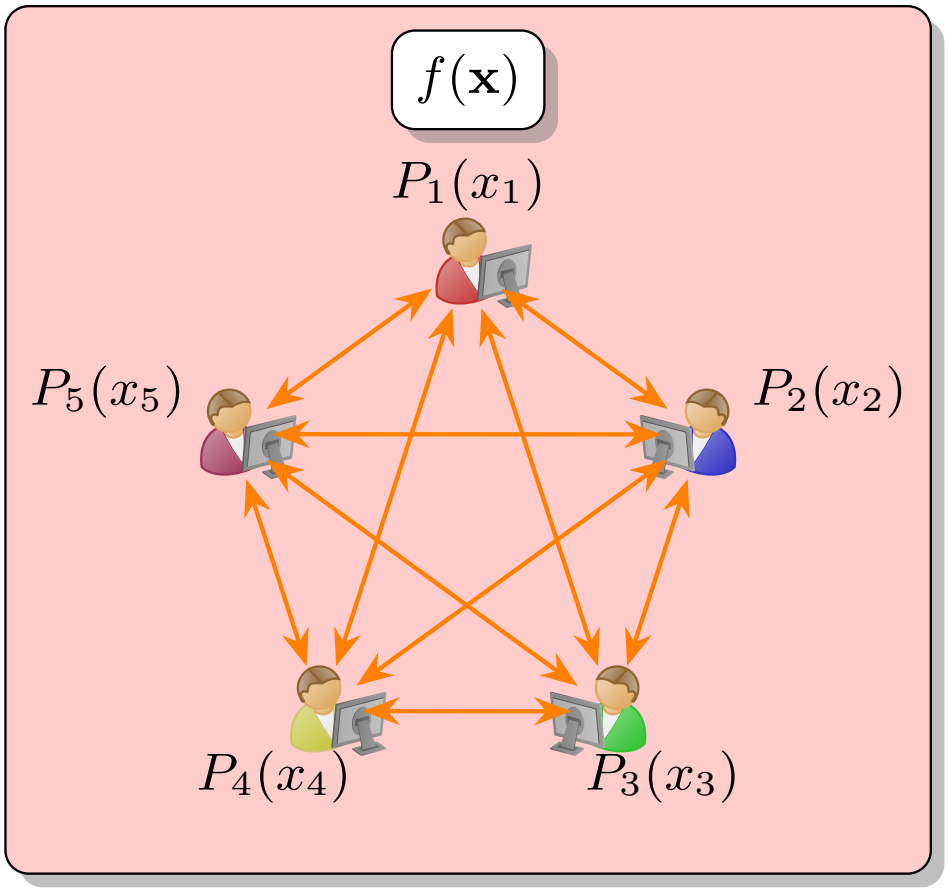
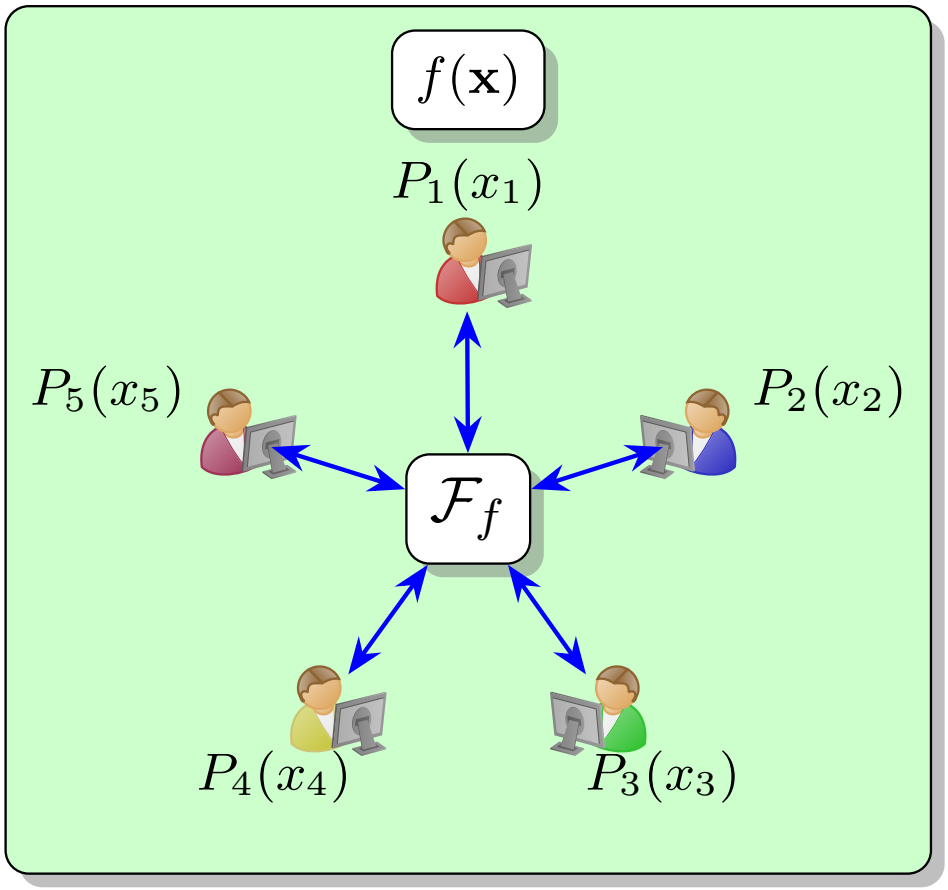


Ideal World

Semi-honest Security



$\mathsf{Real}_{\pi}(C, x_1, \ldots, x_m)$



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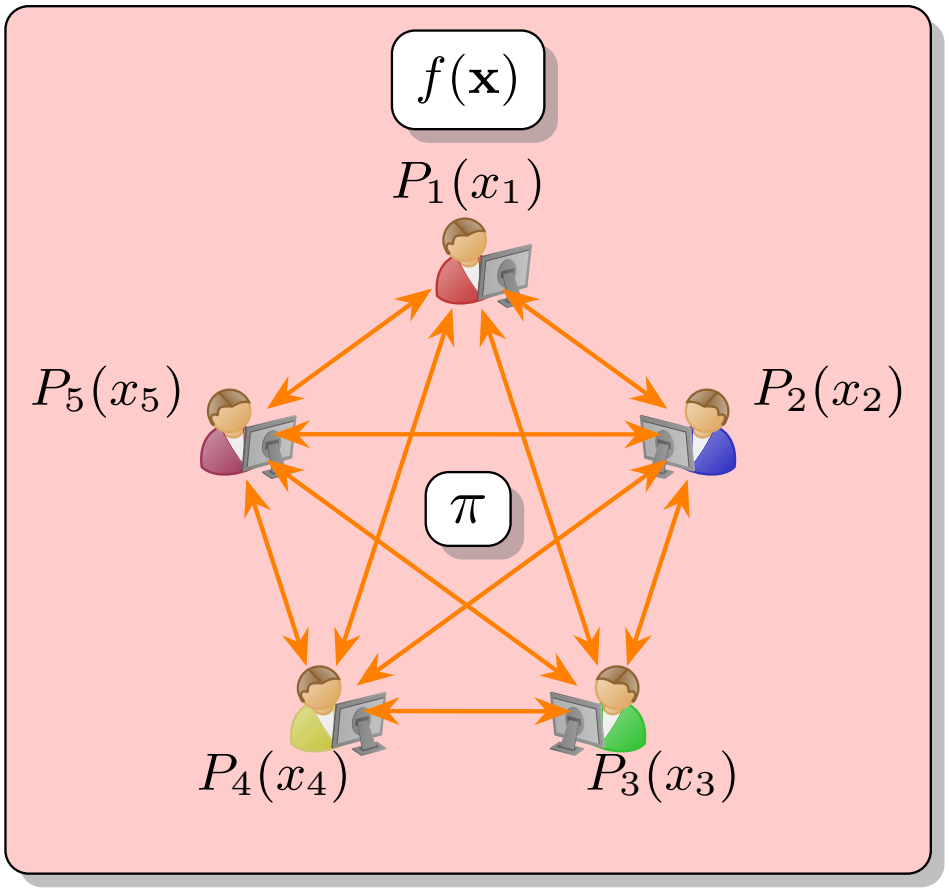

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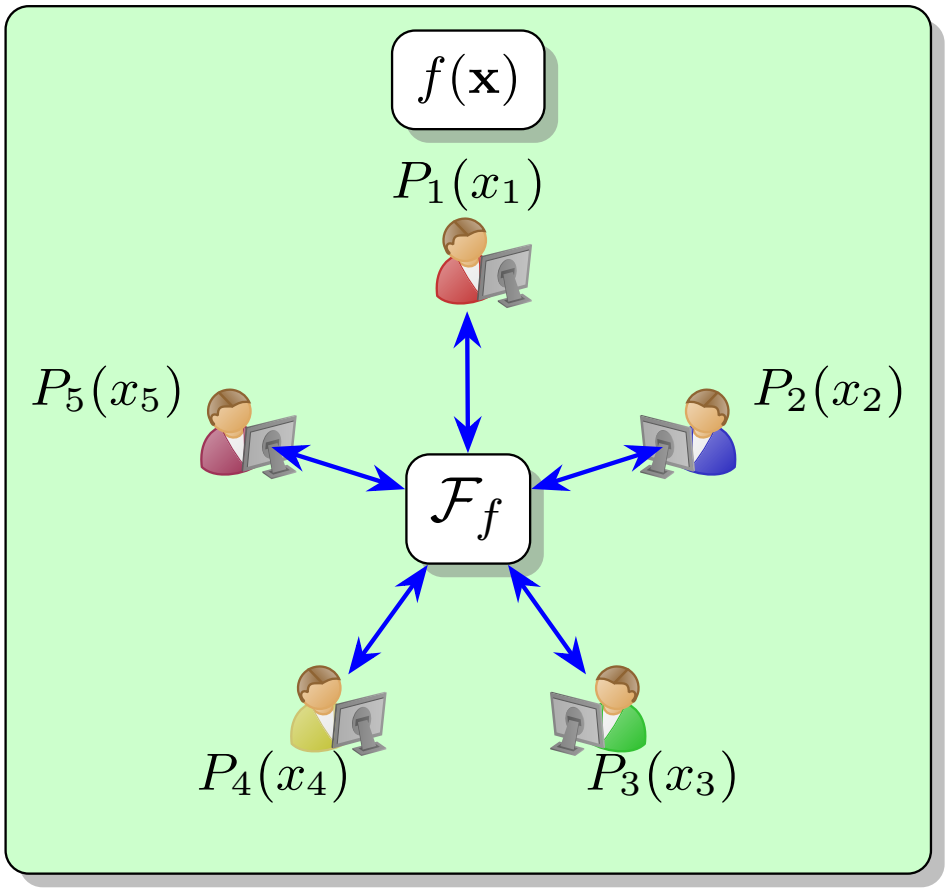

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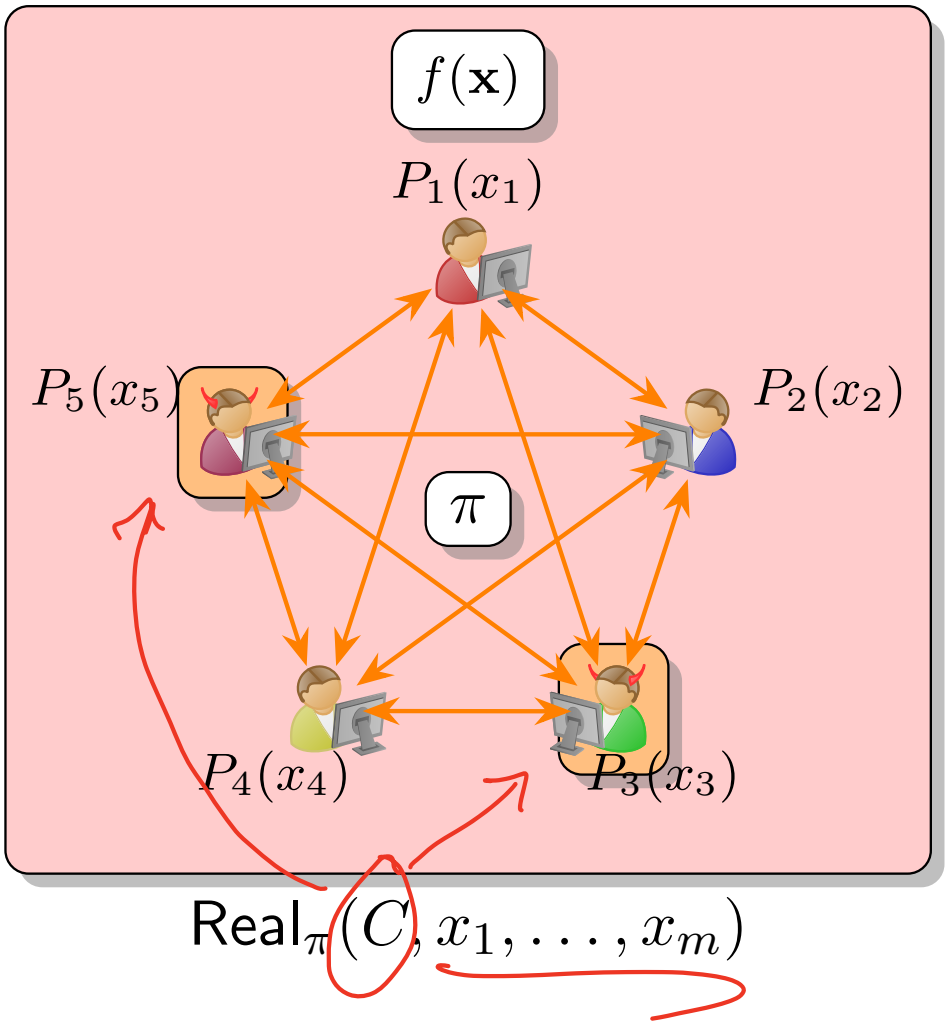

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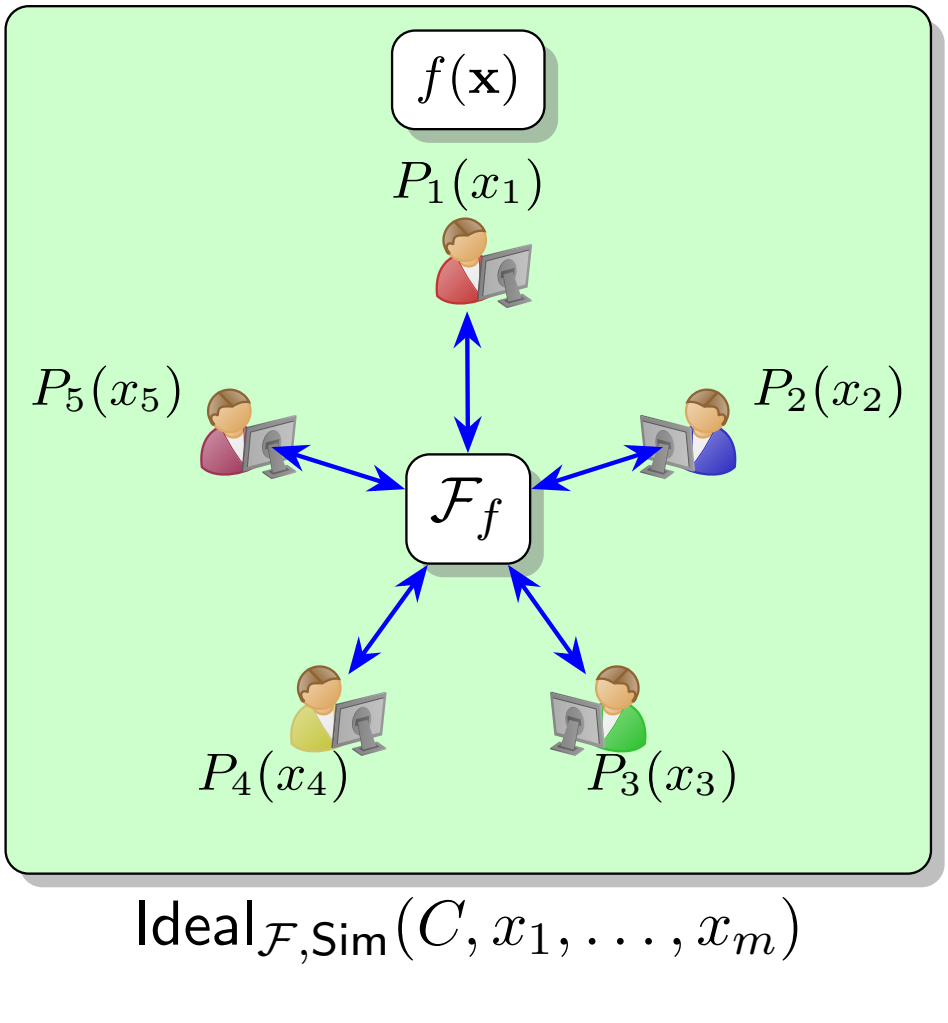

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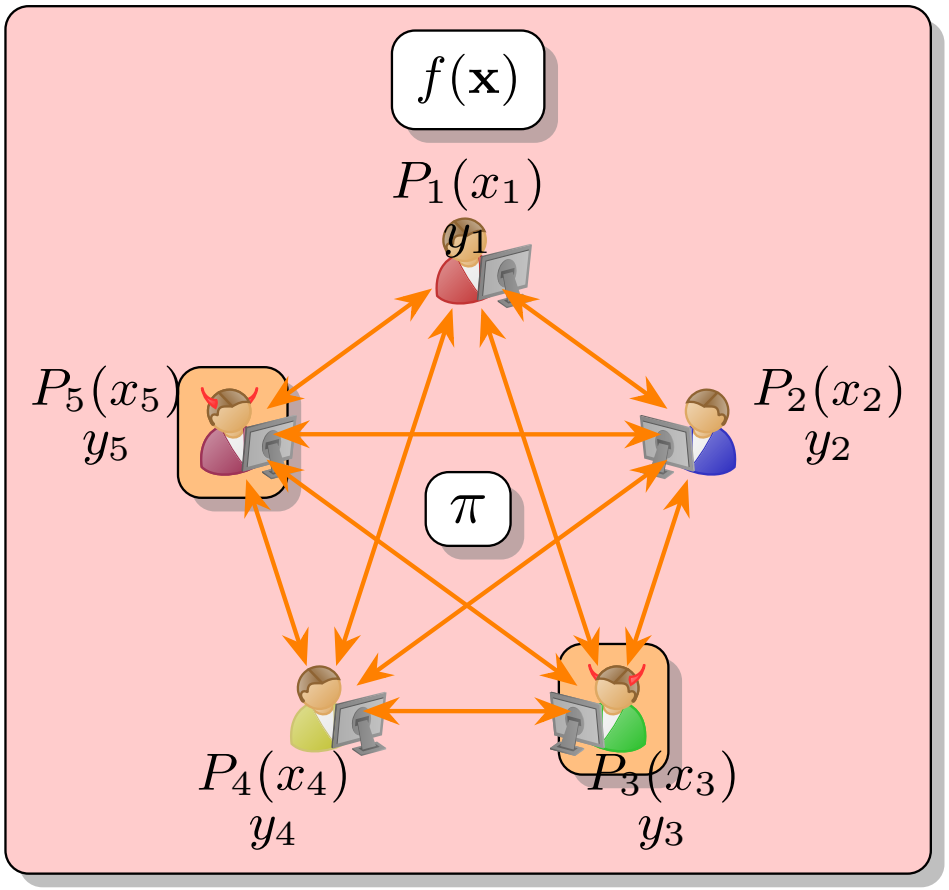

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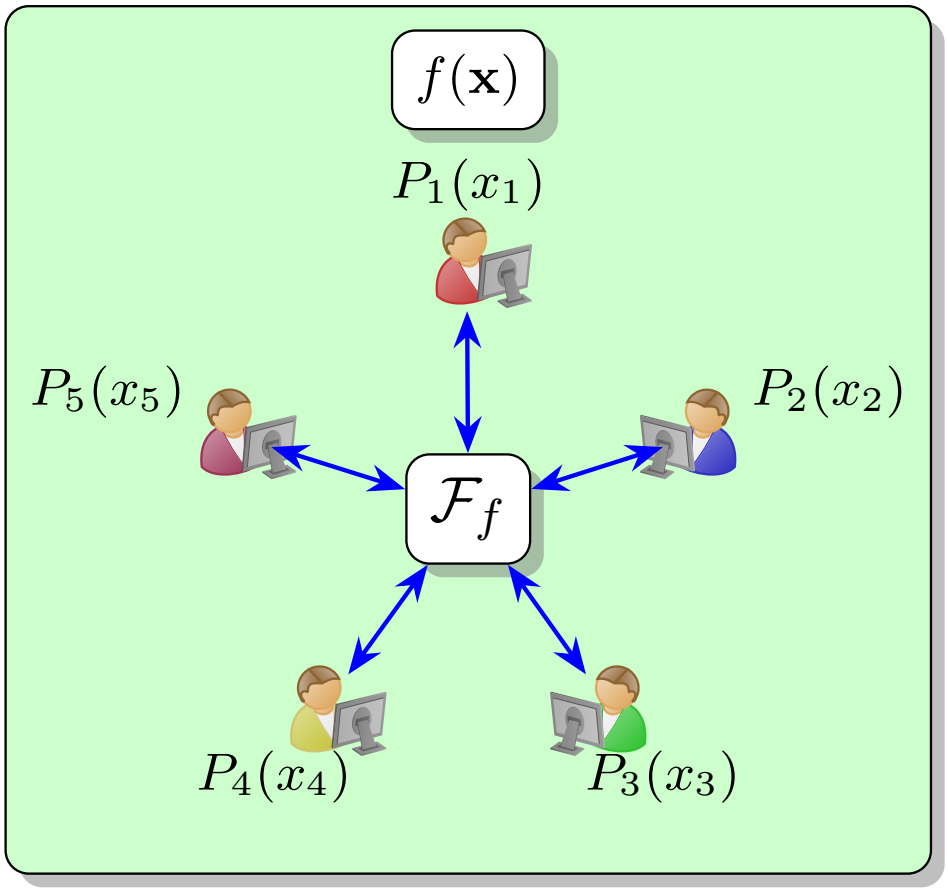

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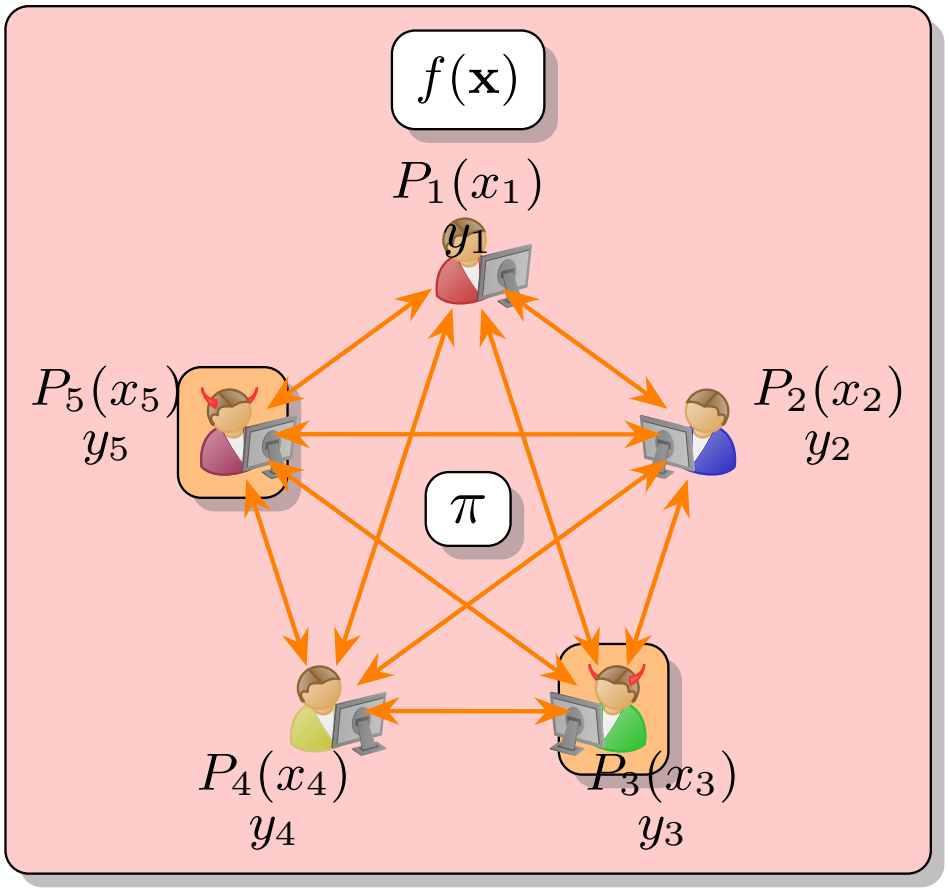

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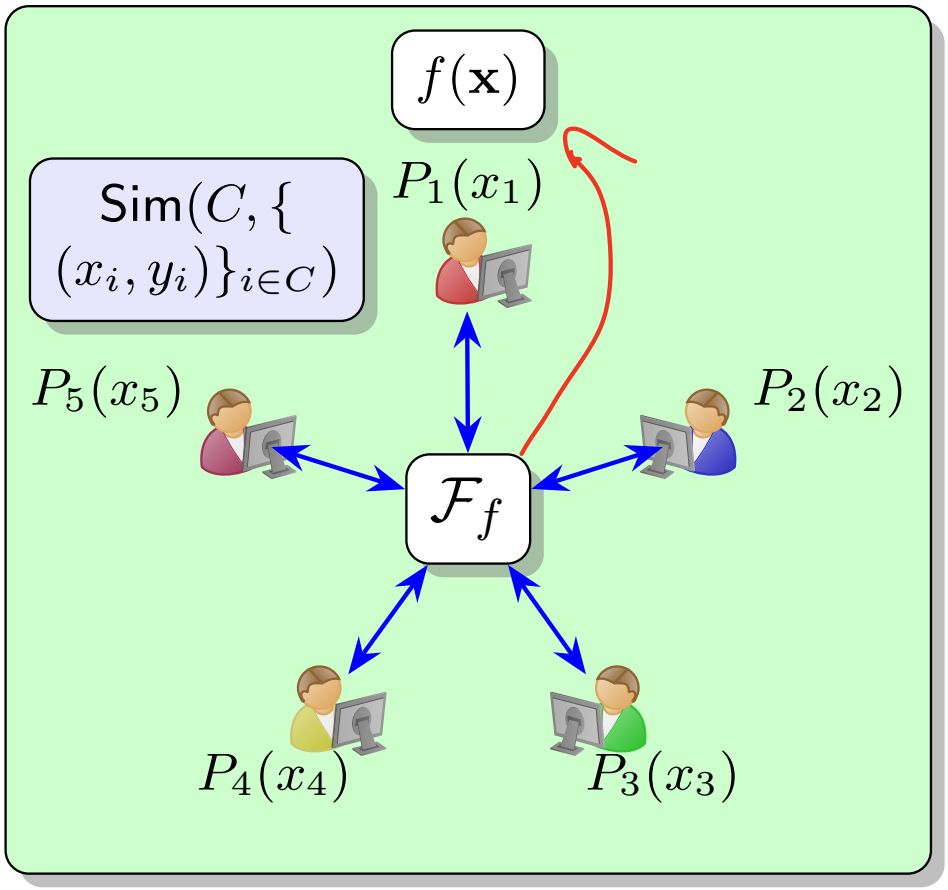

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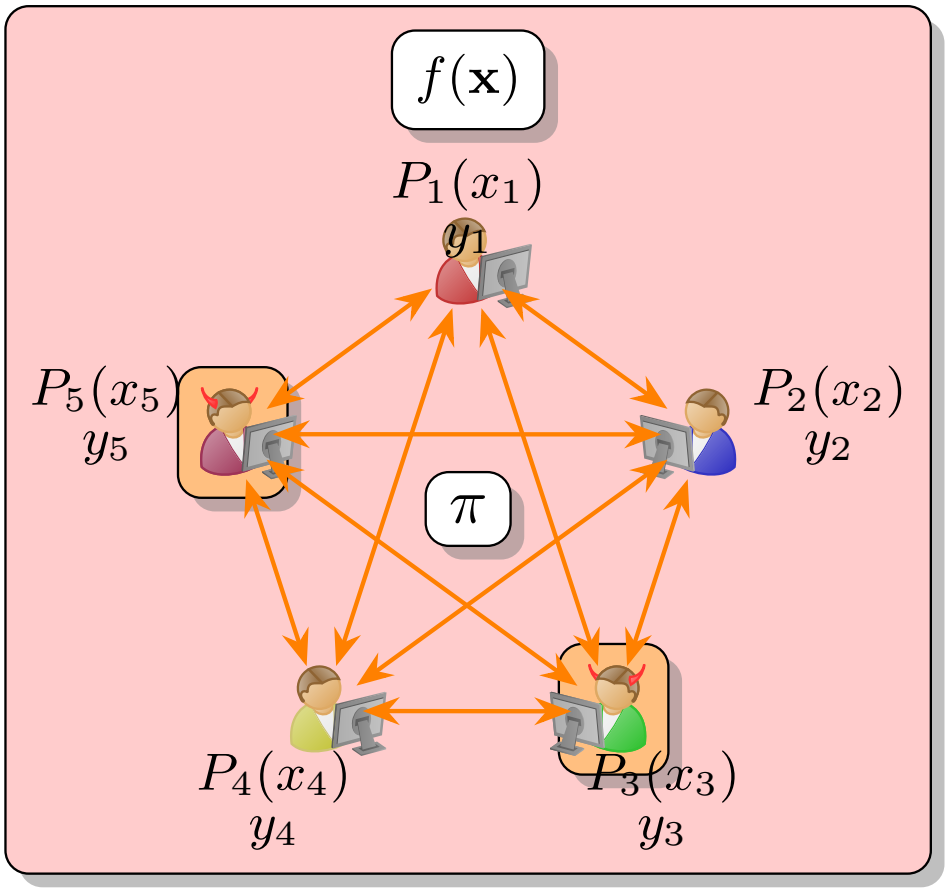

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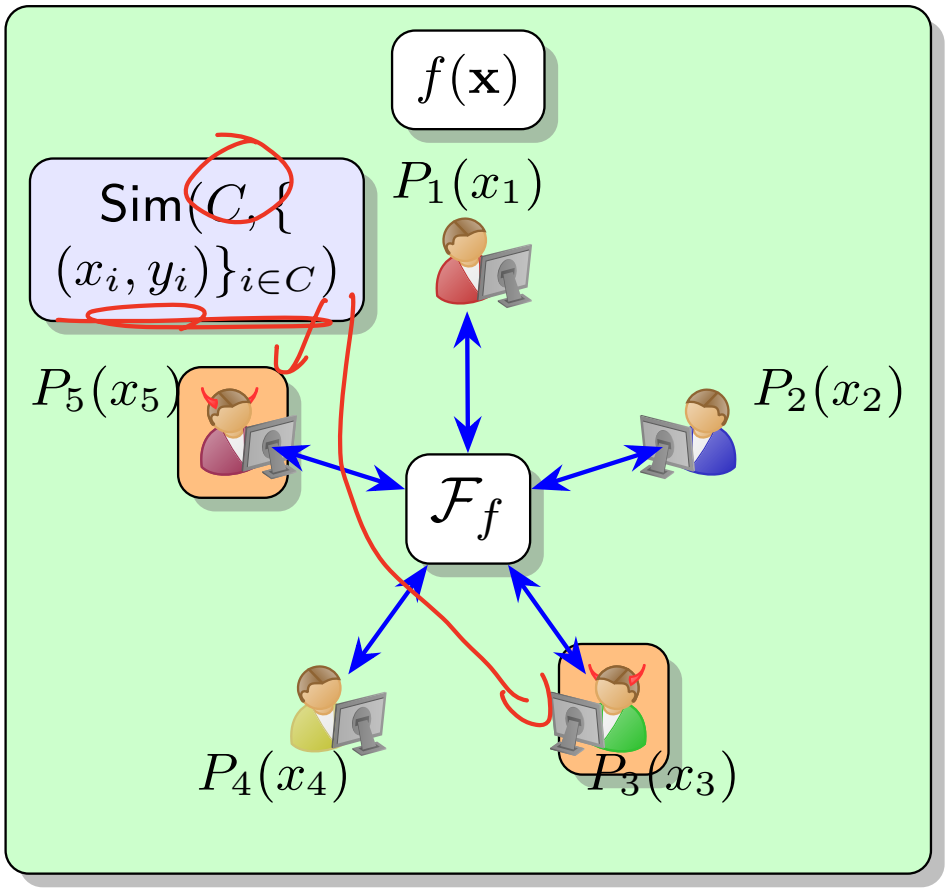

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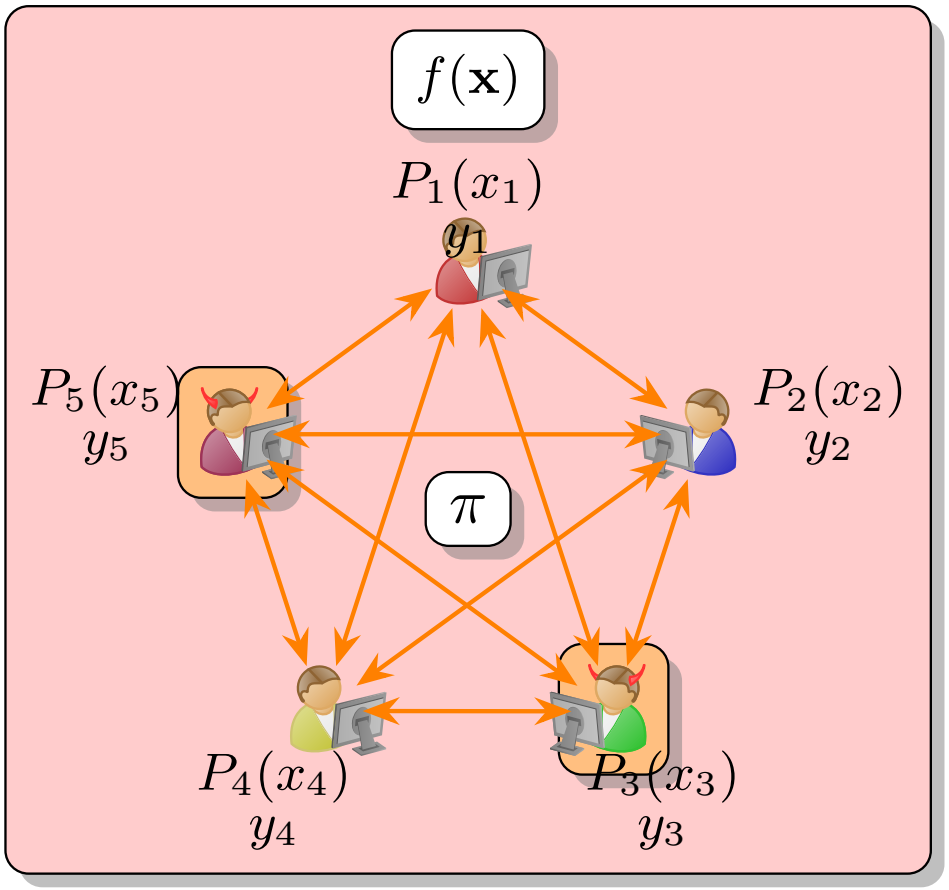

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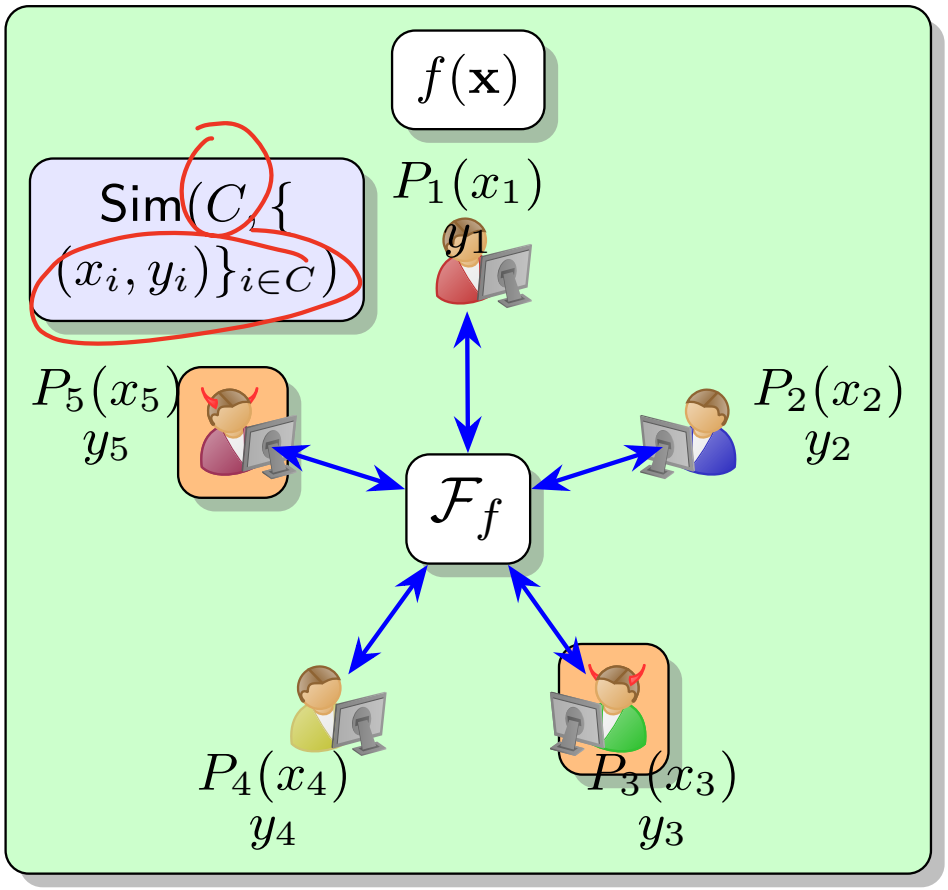

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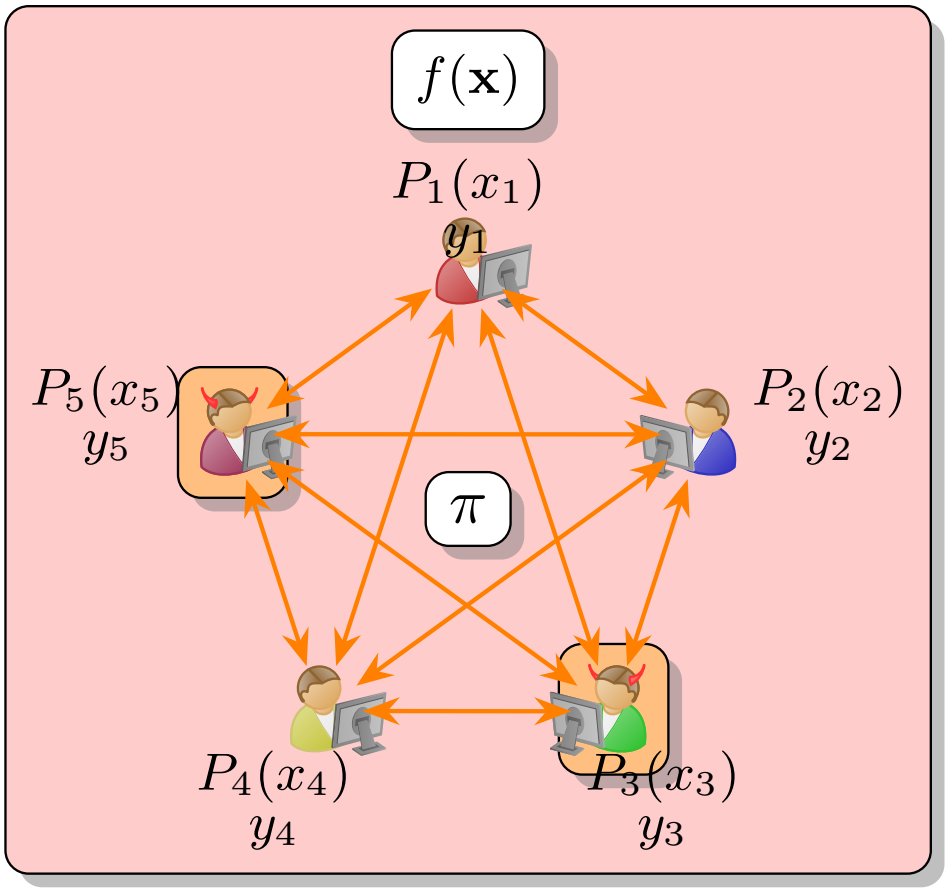

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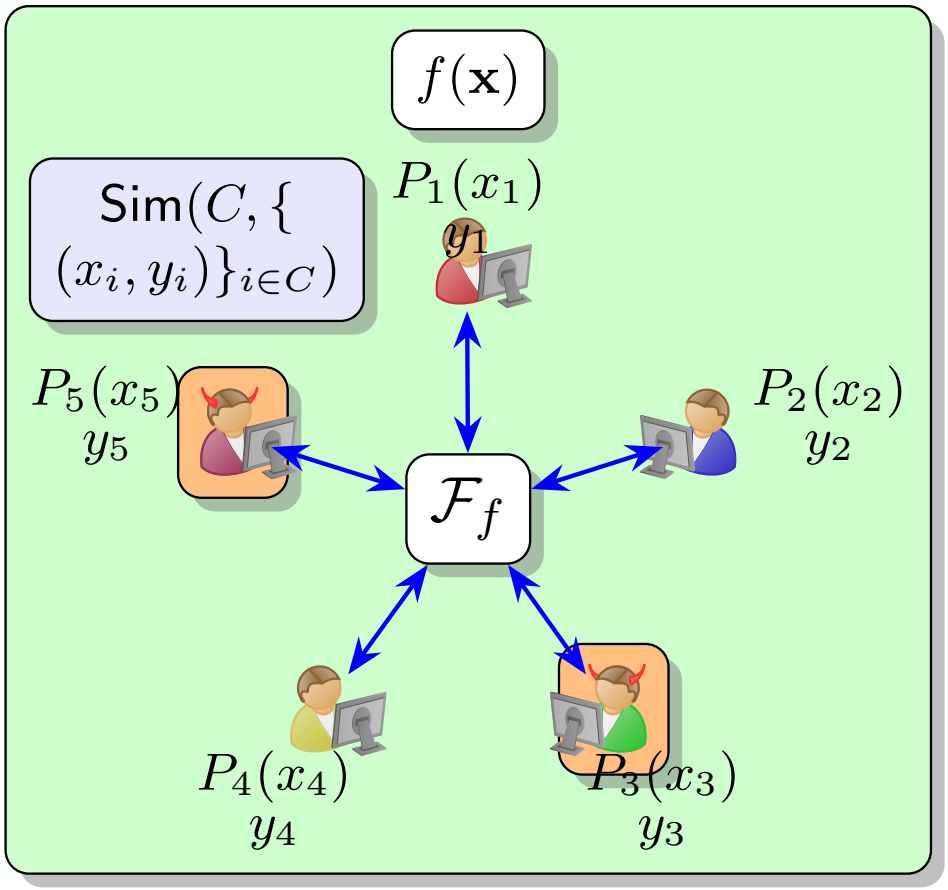

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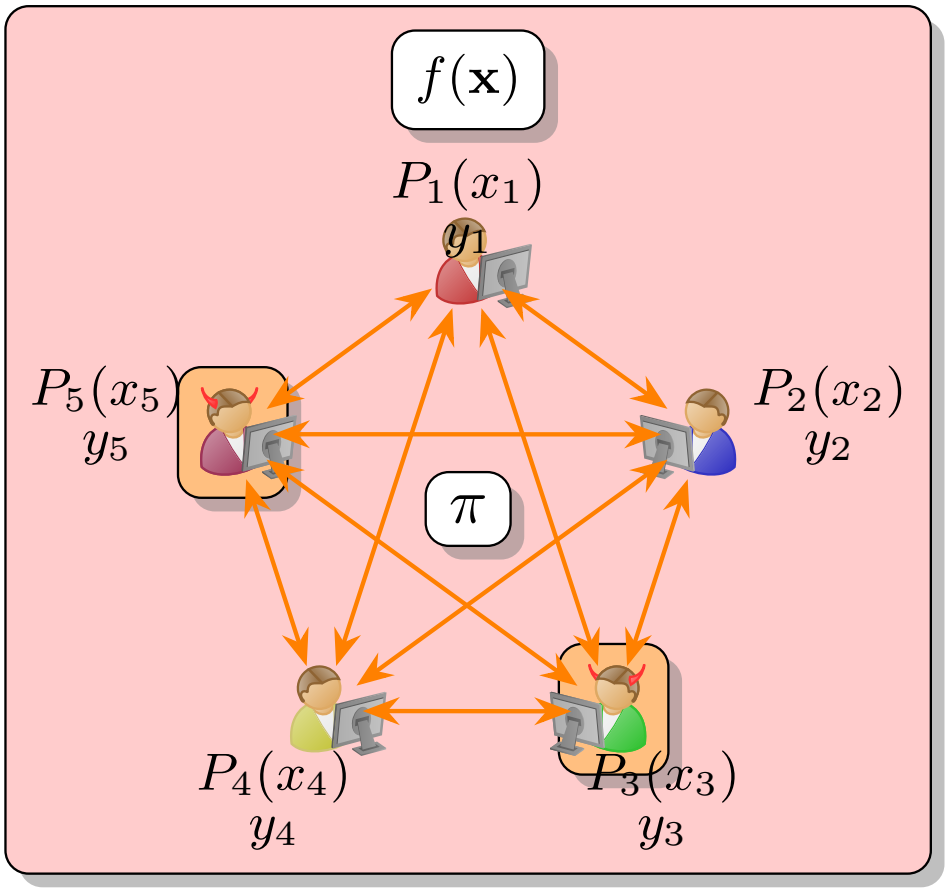
- Sim is Ideal World "adversary"

Semi-honest Security



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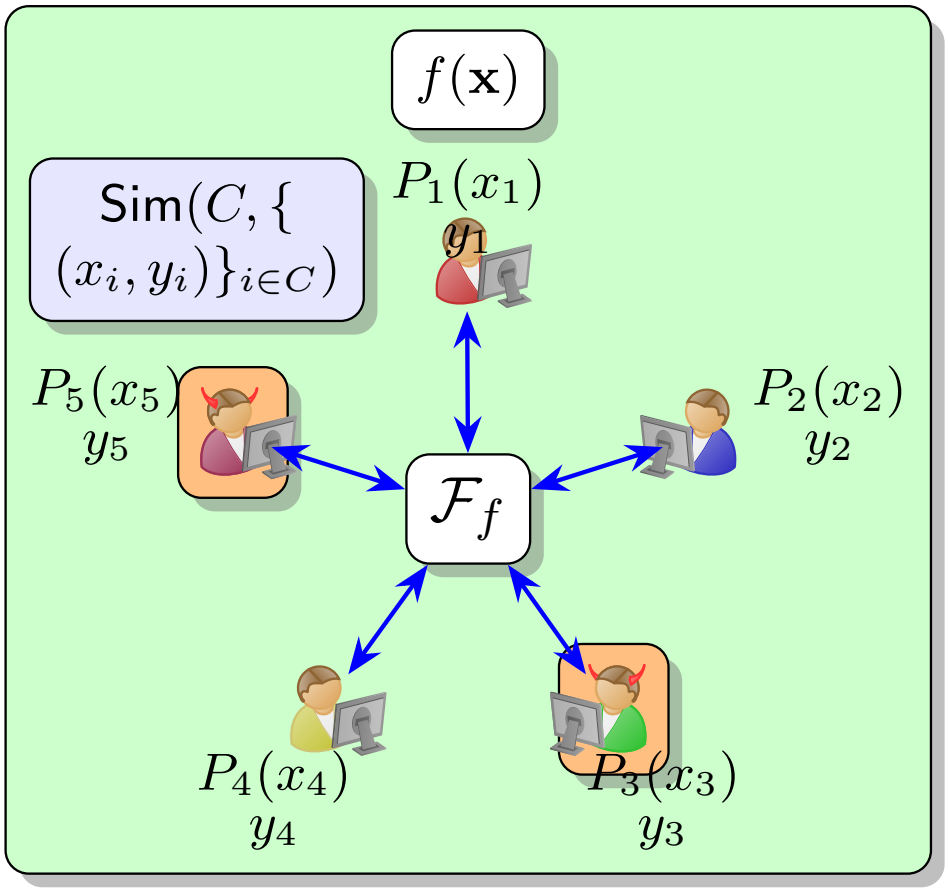

$\mathsf{Ideal}_{\mathcal{F},\mathsf{Sim}}(C, x_1, \ldots, x_m)$

- Sim is Ideal World "adversary"
- Tries to mimic (i.e., simulate) view of adversaries in Real World while living in Ideal World.

Semi-honest Secure MPC

- Let π be a protocol and $\mathcal{F}$ be a functionality.
- Let $C \subset [m]$ denote the set of corrupt parties in π.
- We say that π *securely realizes* $\mathcal{F}$ *in the presence of semi-honest adversaries* if there exists a simulator Sim such that $\forall C \subset [m]$ and all inputs $x_1 \ldots, x_m$, the distributions $\mathsf{Real}_\pi(C, x_1, \ldots, x_m)$ and $\mathsf{Ideal}_{\mathcal{F},\mathsf{Sim}}(C, x_1, \ldots, x_m)$ are indistinguishable.

$\mathsf{Real}_\pi(C, x_1, \ldots, x_m)$

- Run protocol π where each party P_i honestly behaves with input x_i.
- Let V_i denote the final view of P_i.
- Output $\{V_i : i \in C\}, (y_1, \ldots, y_m)$.

$\mathsf{Ideal}(C, x_1, \ldots, x_m)$

- Compute $(y_1, \ldots, y_m) \leftarrow \mathcal{F}(x_1, \ldots, x_m)$.
- Output $\mathsf{Sim}(C, \{(x_i, y_i) : i \in C\}), (y_1, \ldots, y_m)$

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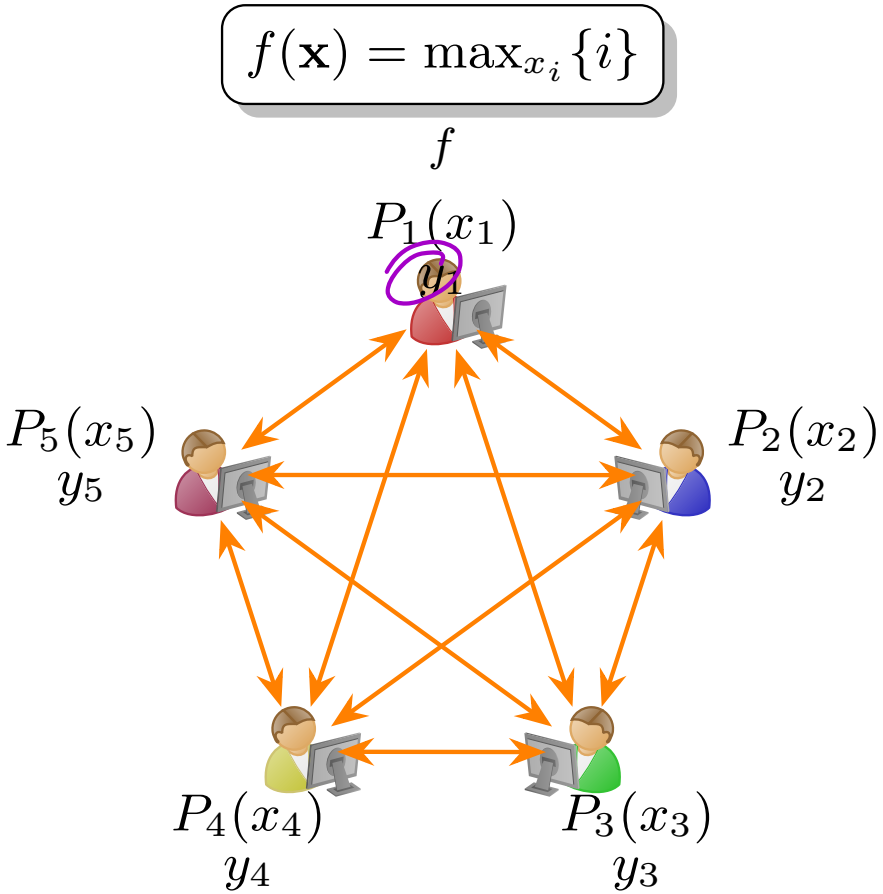
- Even semi-honest security has realistic real-world scenarios.
  - Security of protocols with parties who are trusted (or have incentives) to act honestly, but may become compromised later.

Malicious Corruptions



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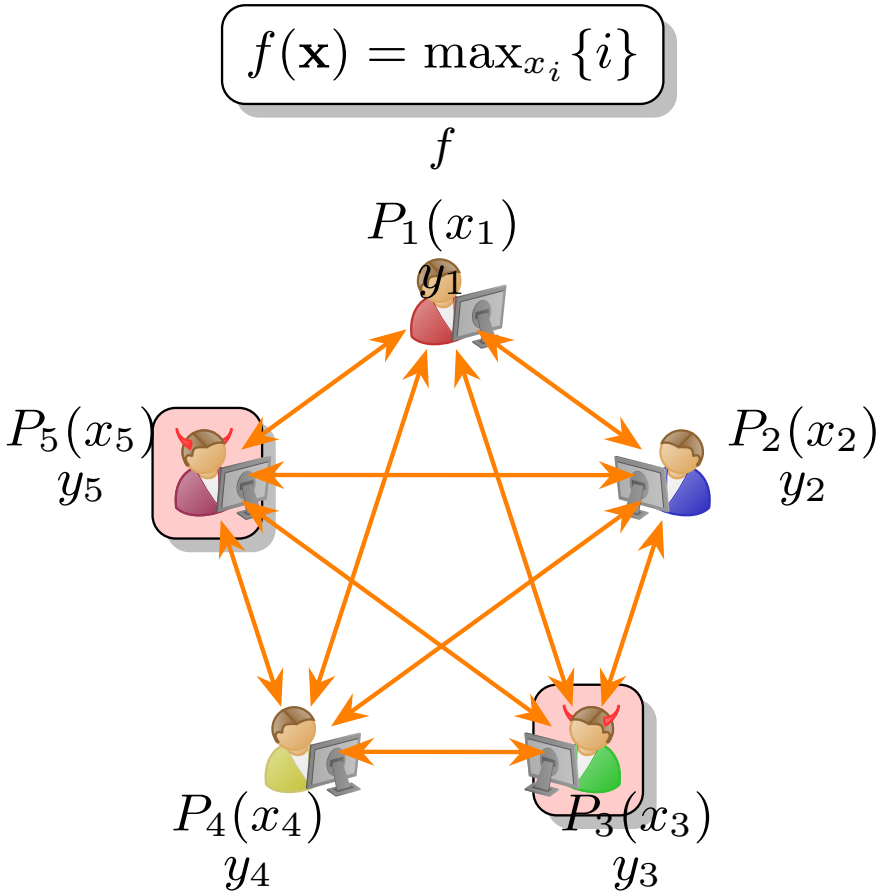

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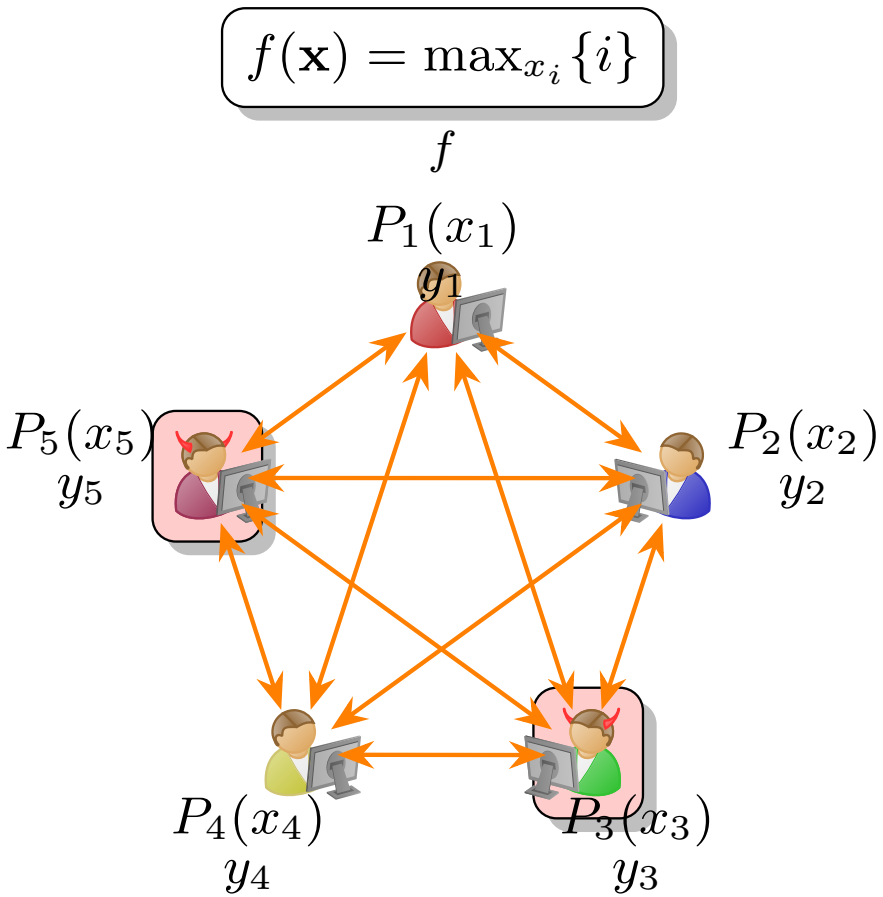

- Corrupt parties can deviate *arbitrarily* from protocol.

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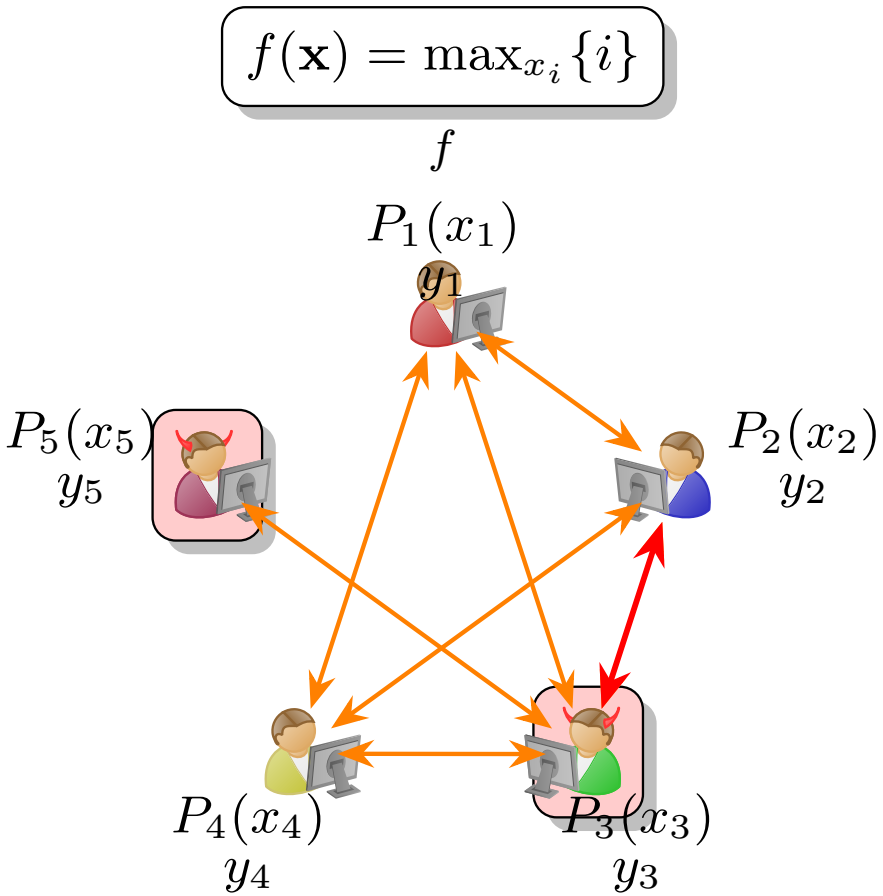

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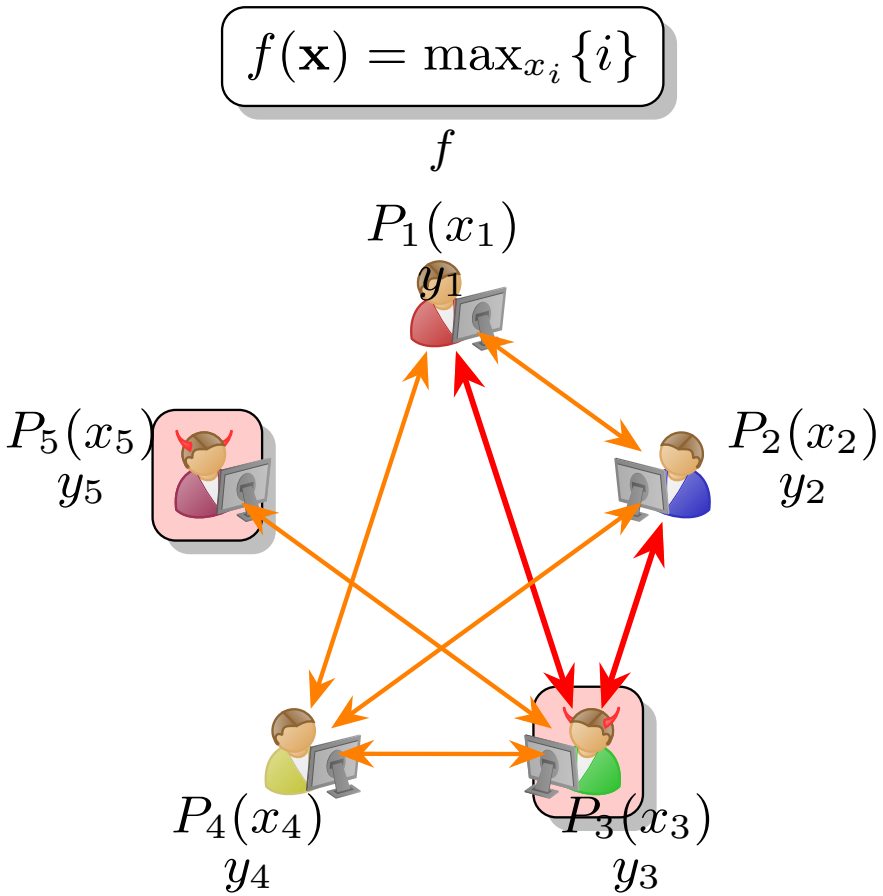

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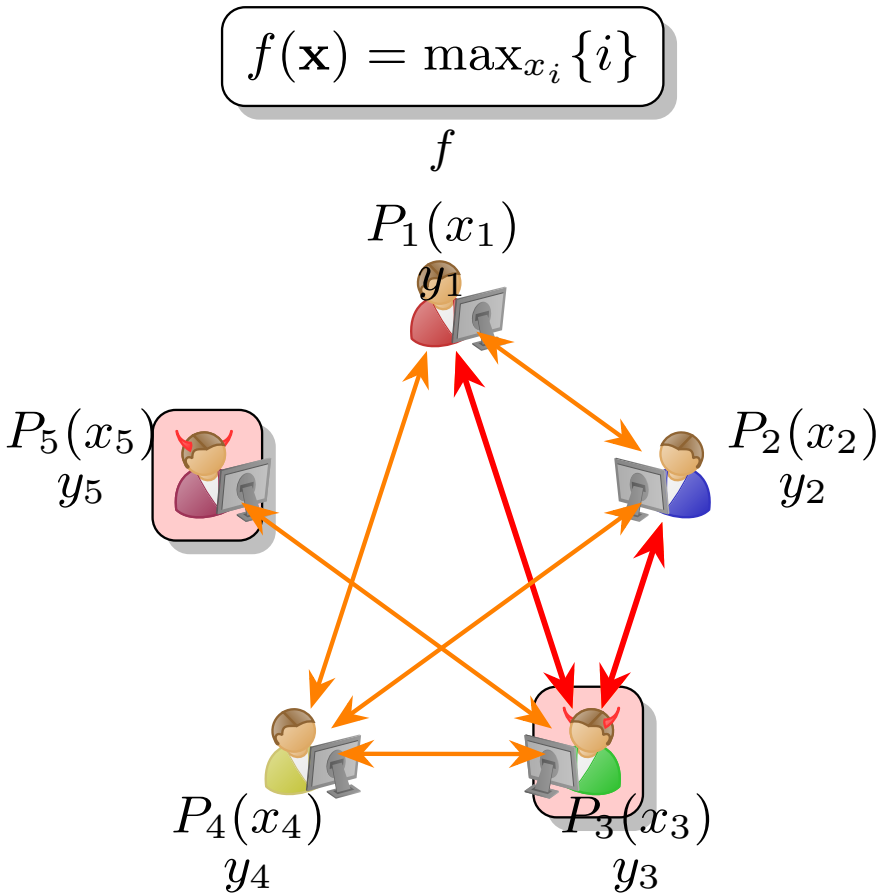

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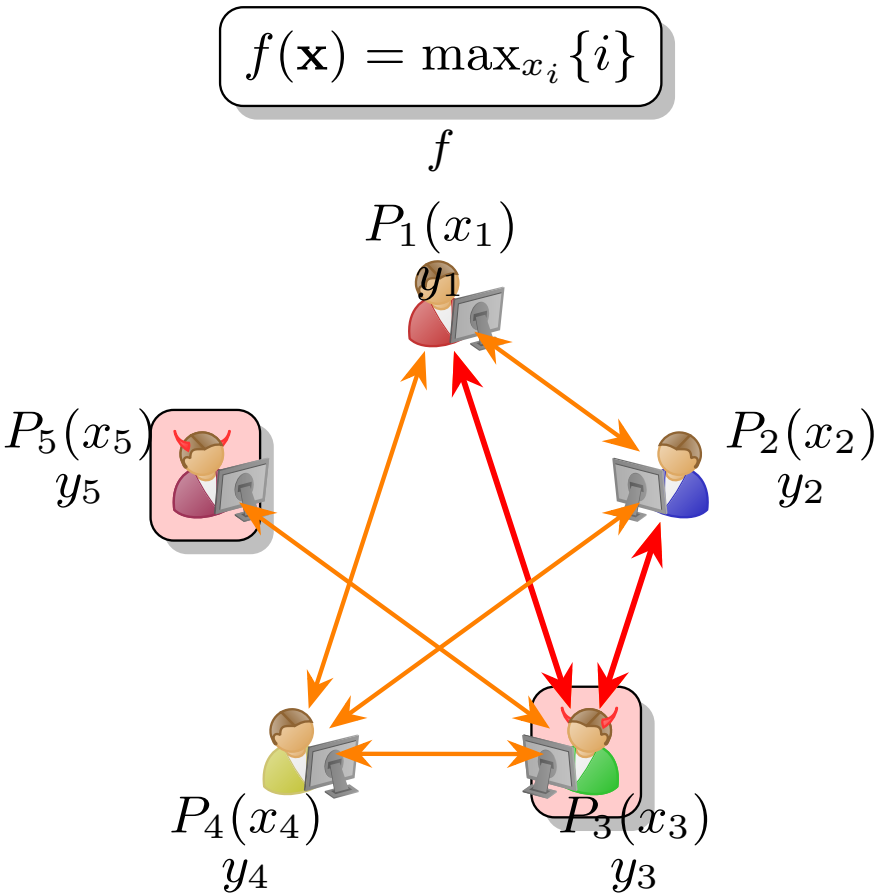

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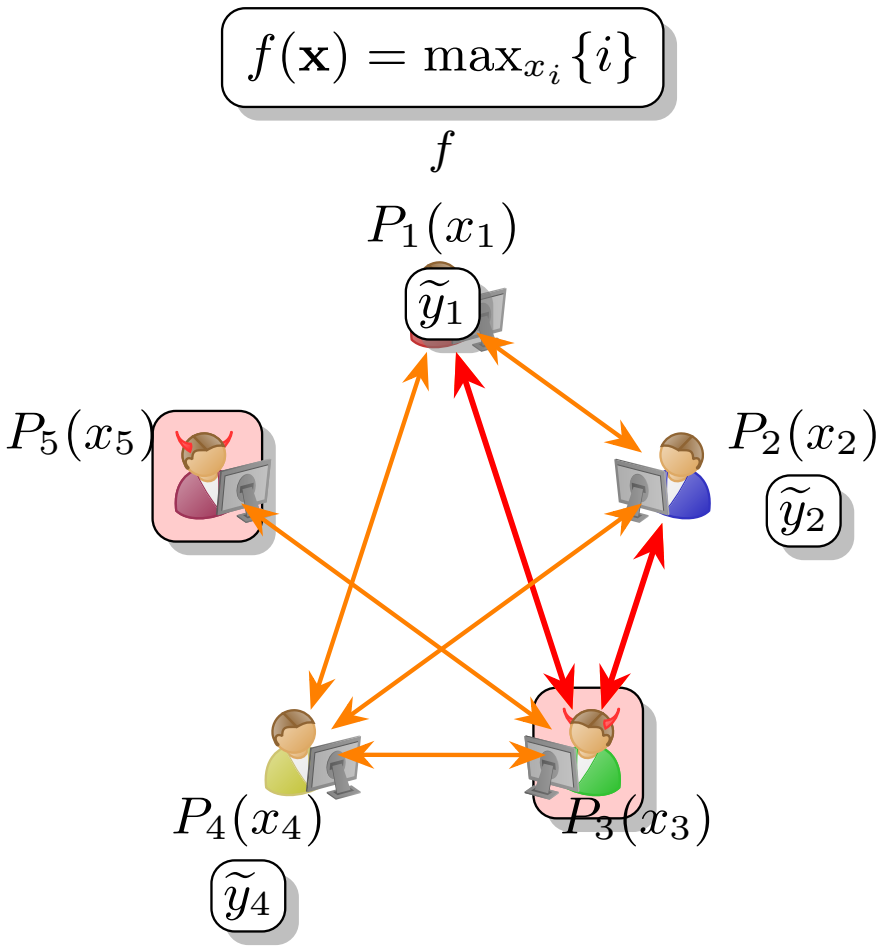

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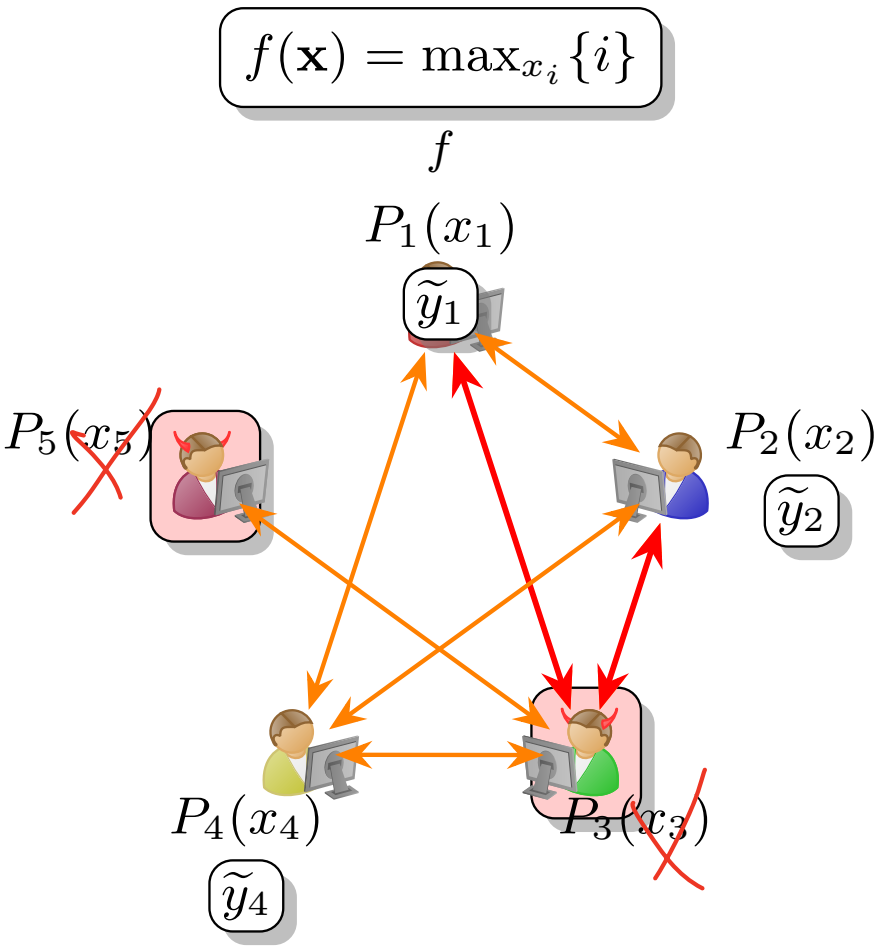

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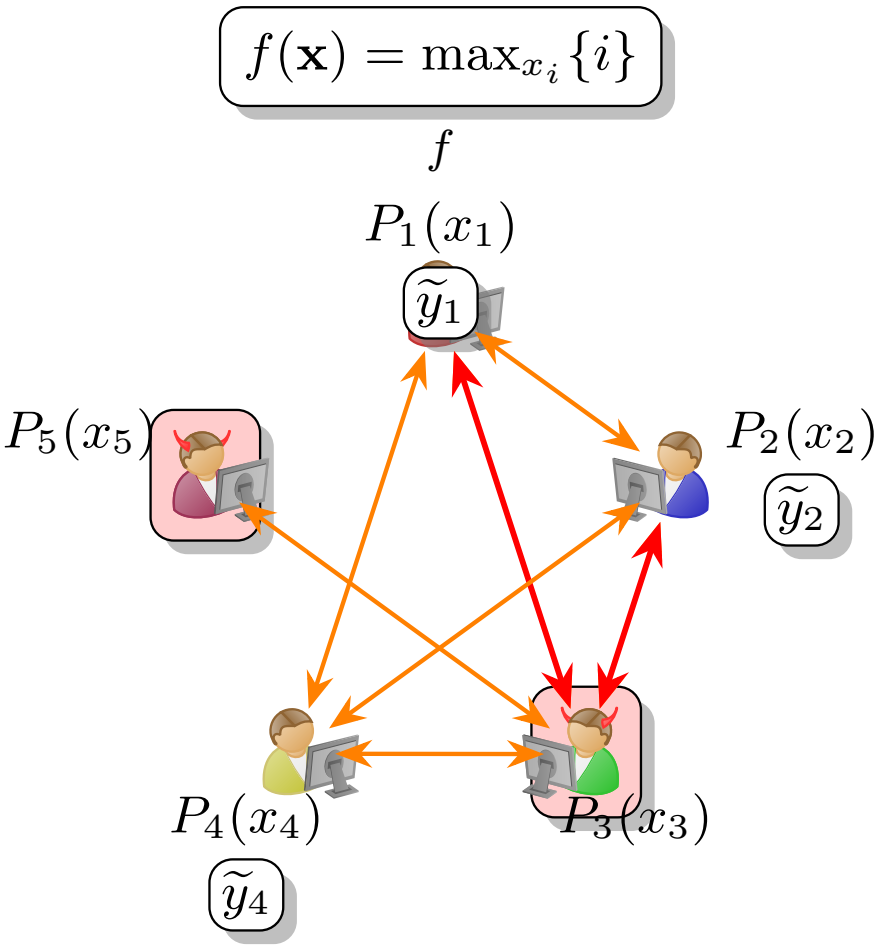

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  - What does Sim give the ideal functionality?

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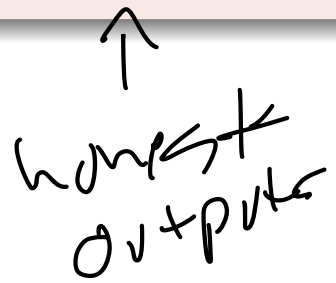

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Yao's Garbled Circuits

Building Block: Oblivious Transfer

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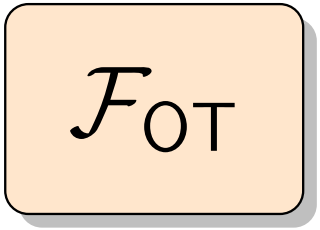

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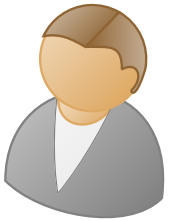

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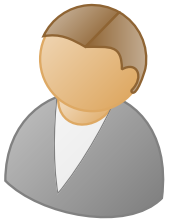

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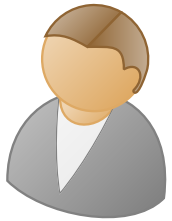

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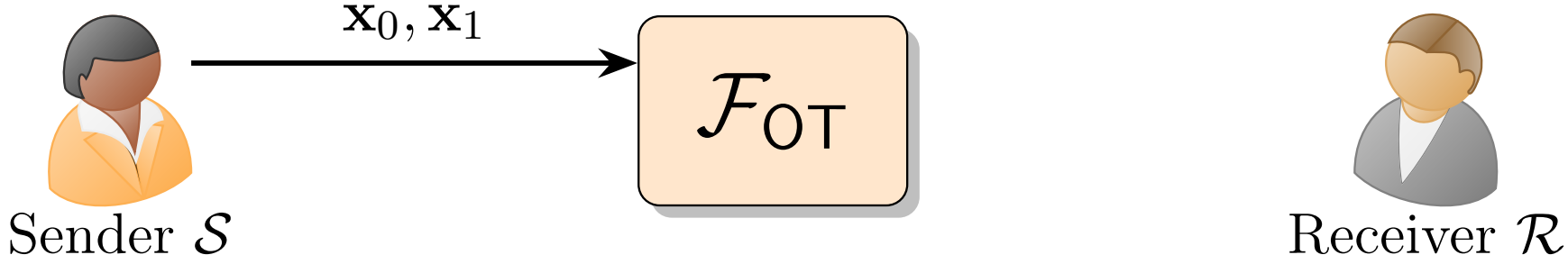

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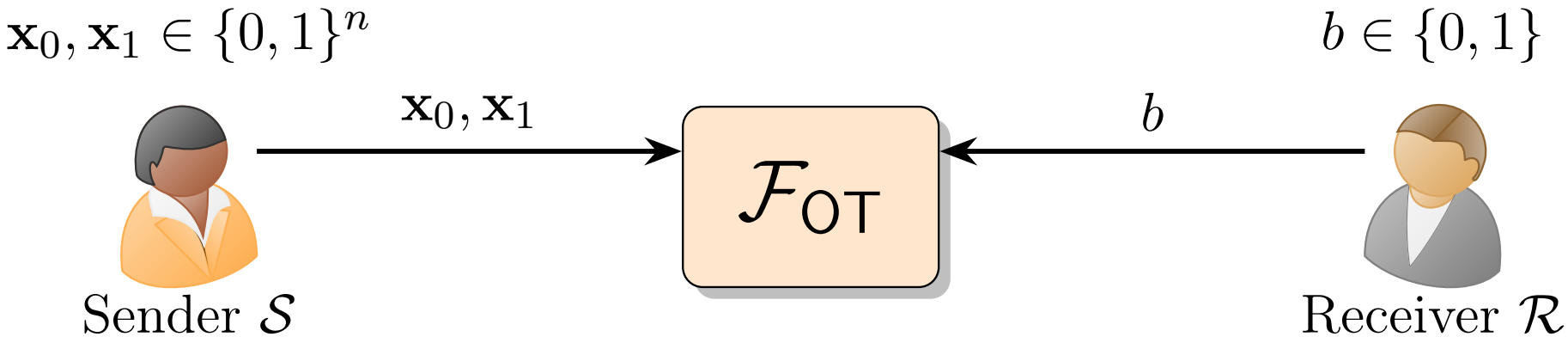

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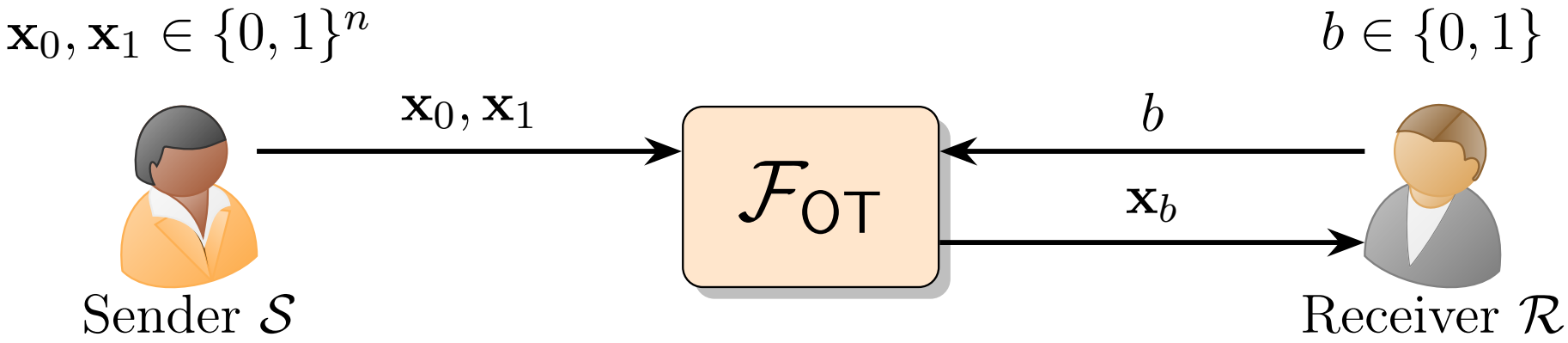

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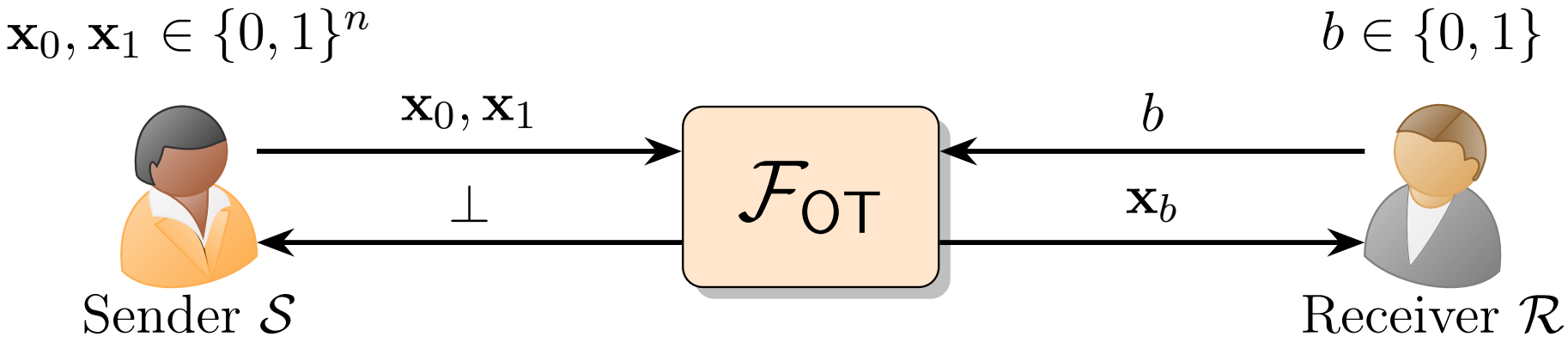

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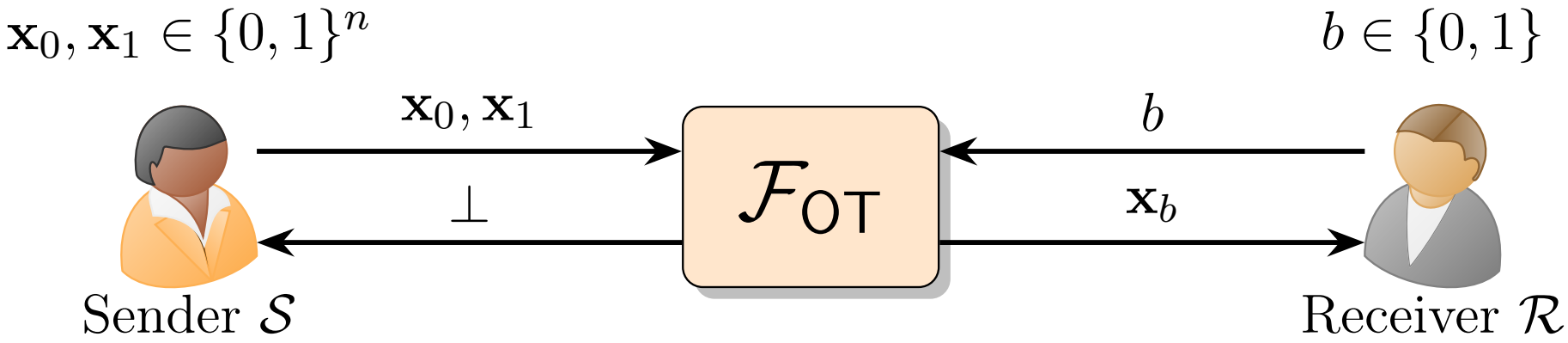

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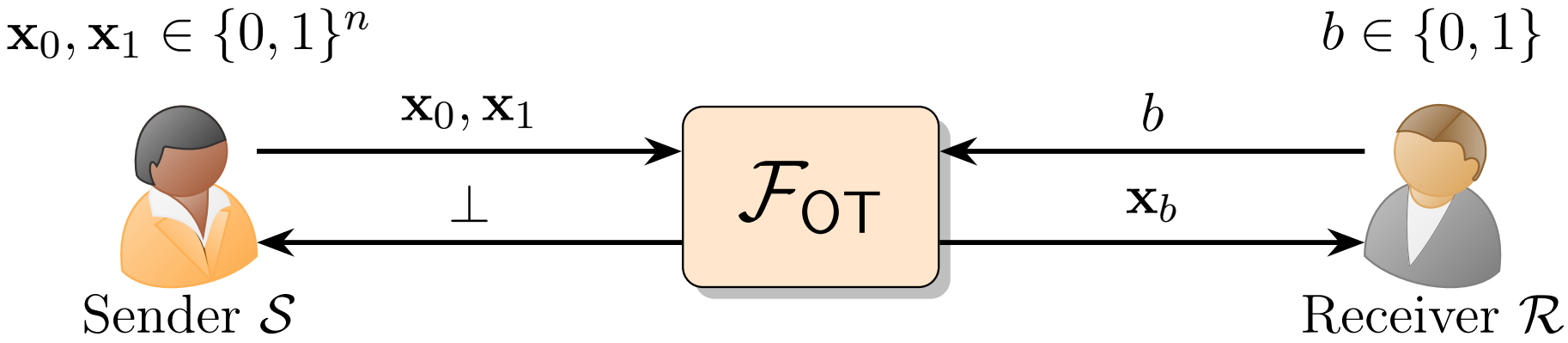

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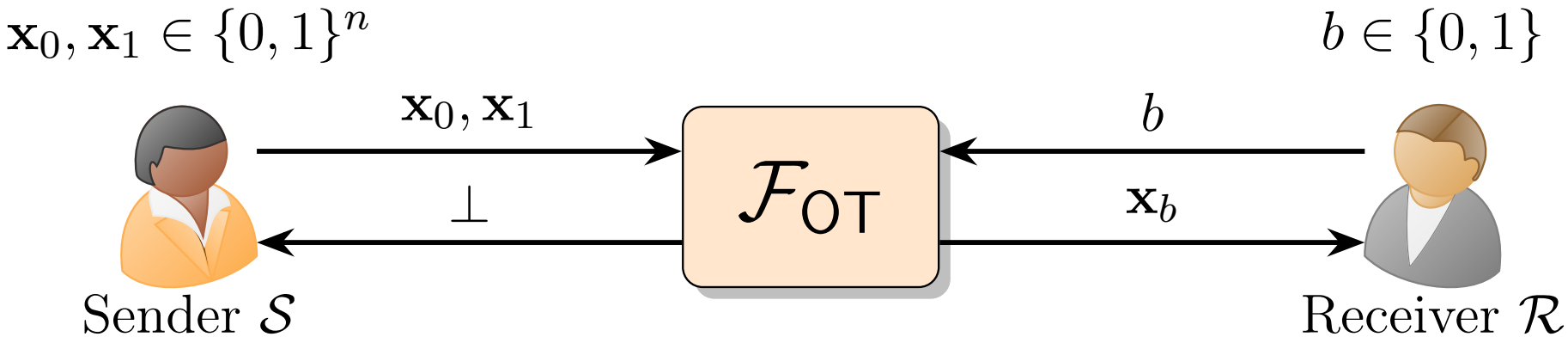

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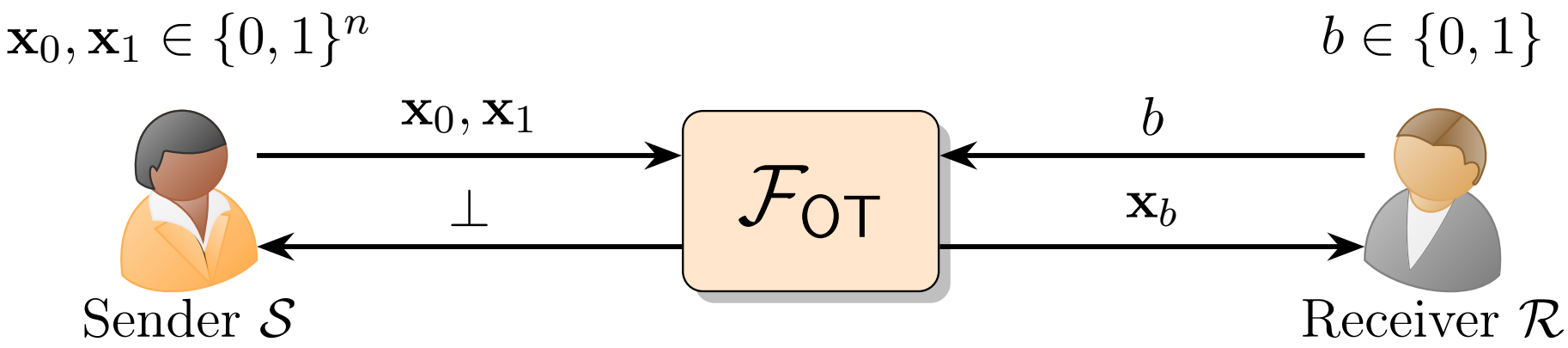

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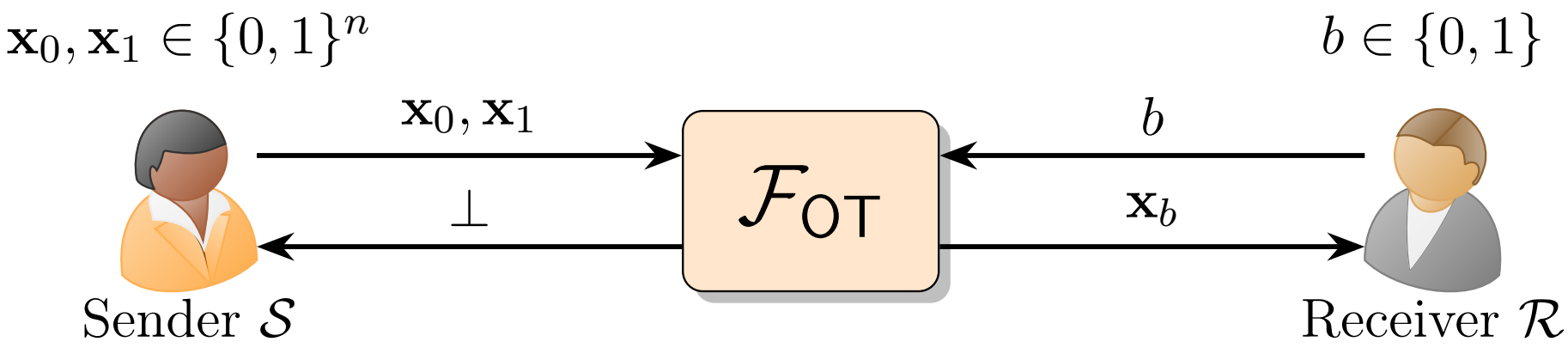

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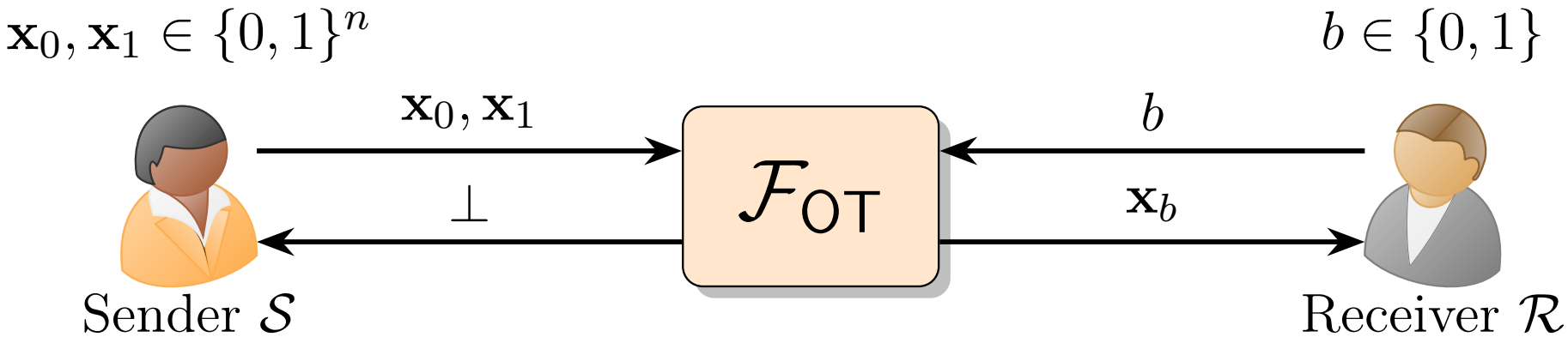

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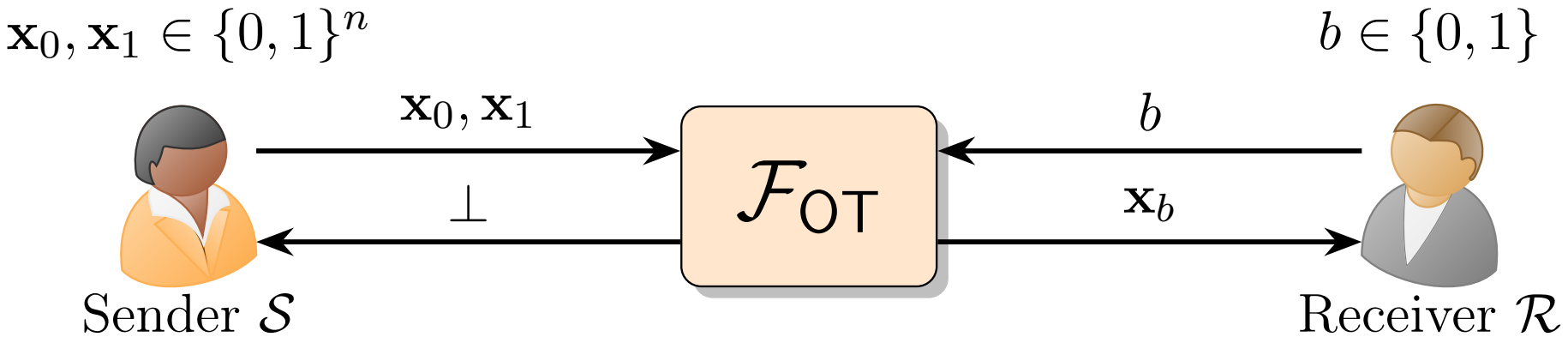

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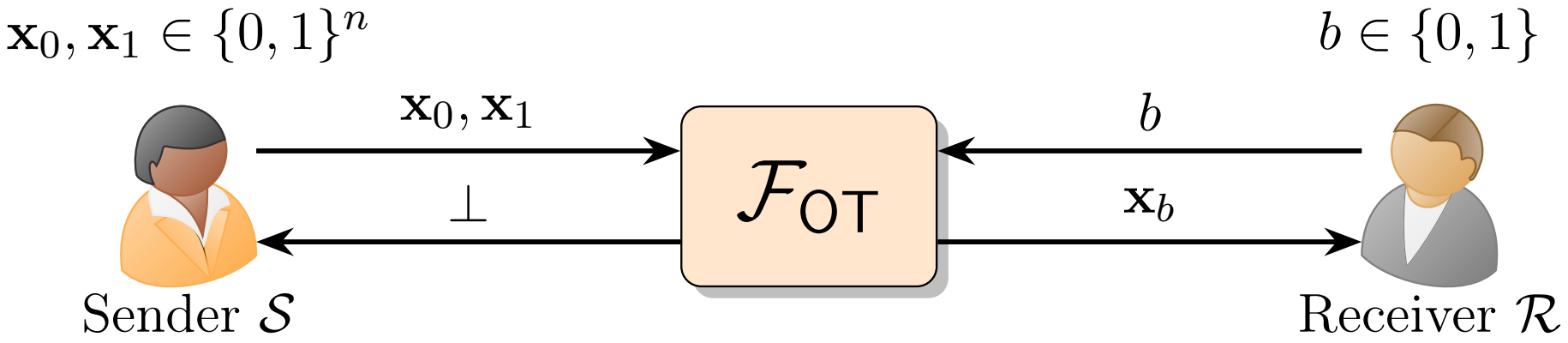

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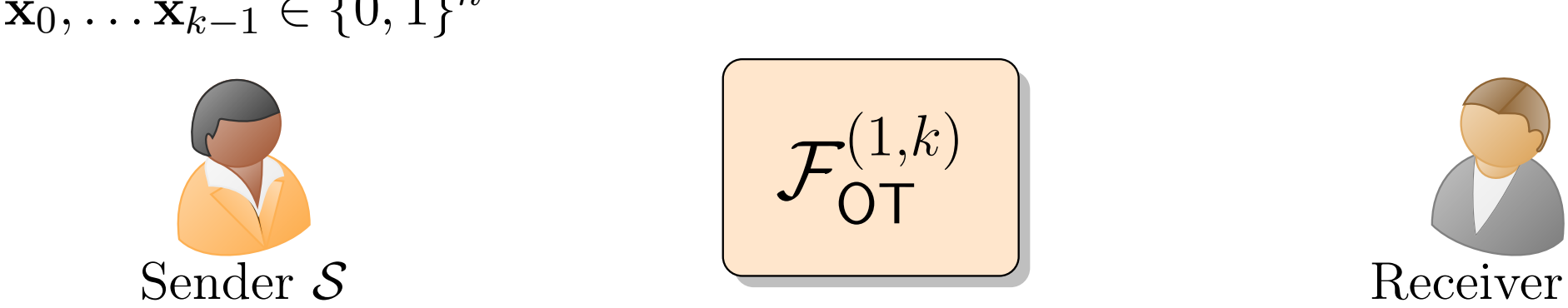

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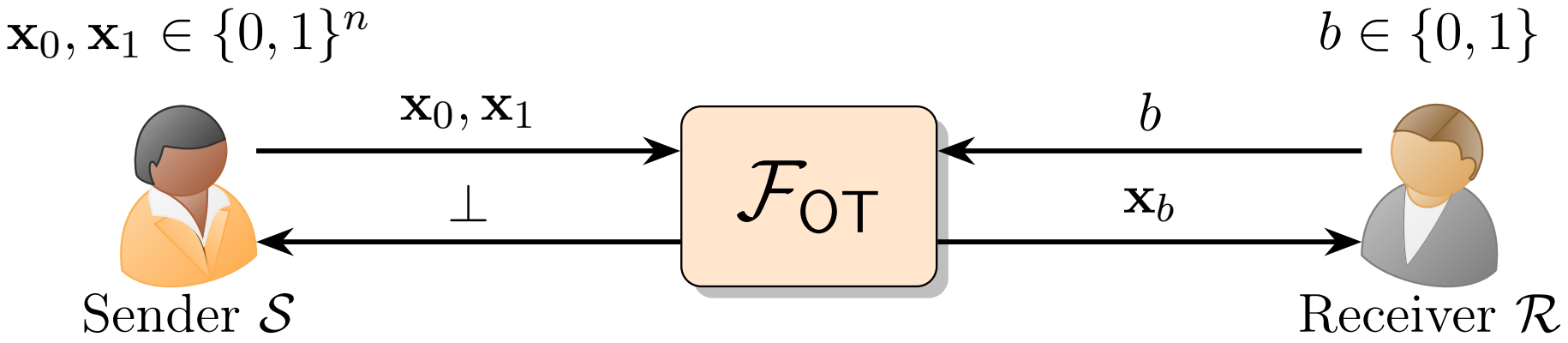

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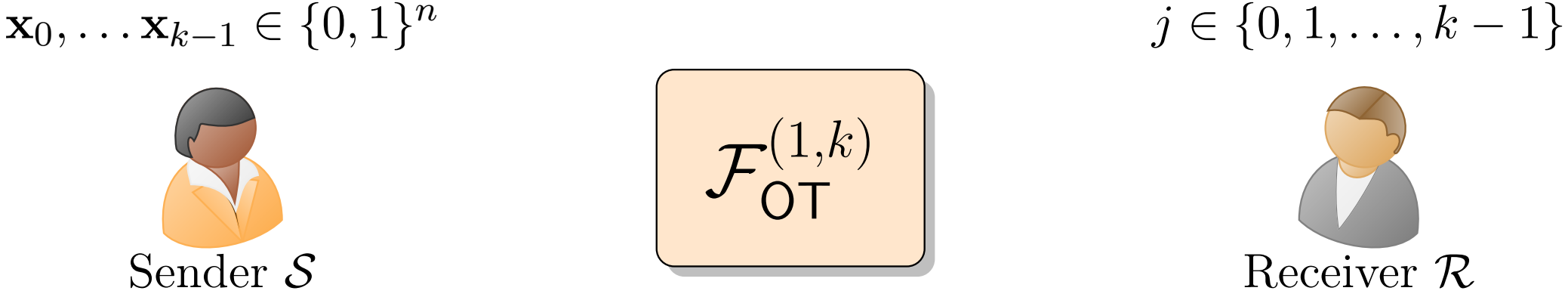

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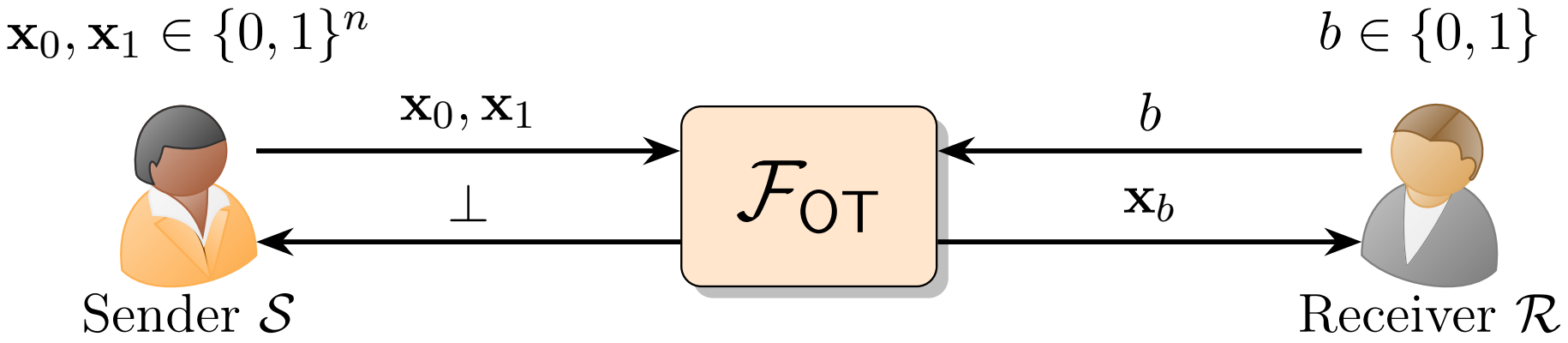

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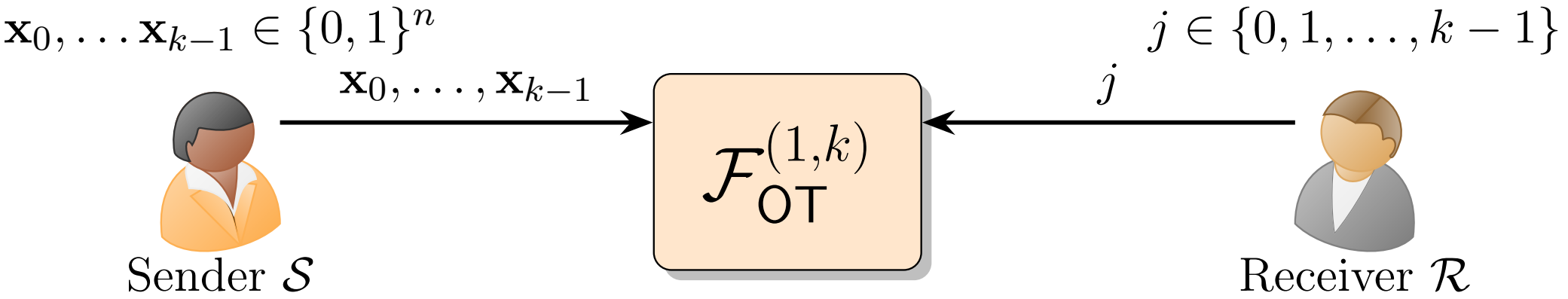

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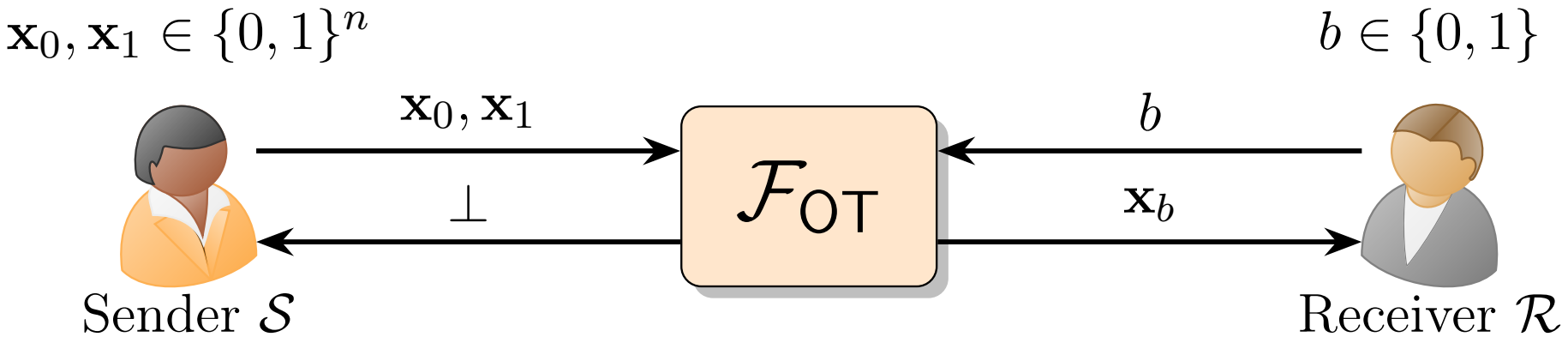

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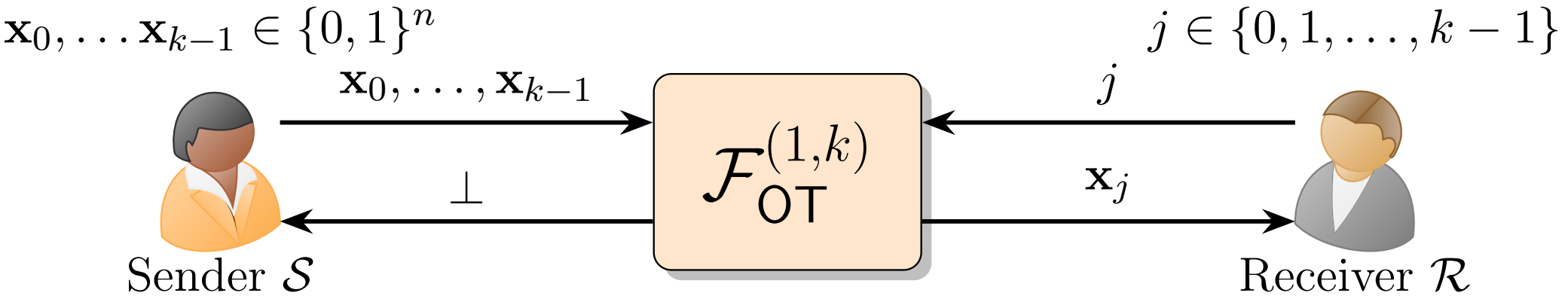

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- We will use OT as a crucial building block for Yao's Garbled Circuit Protocol.

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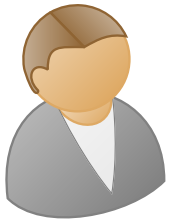

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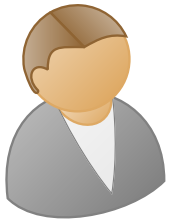

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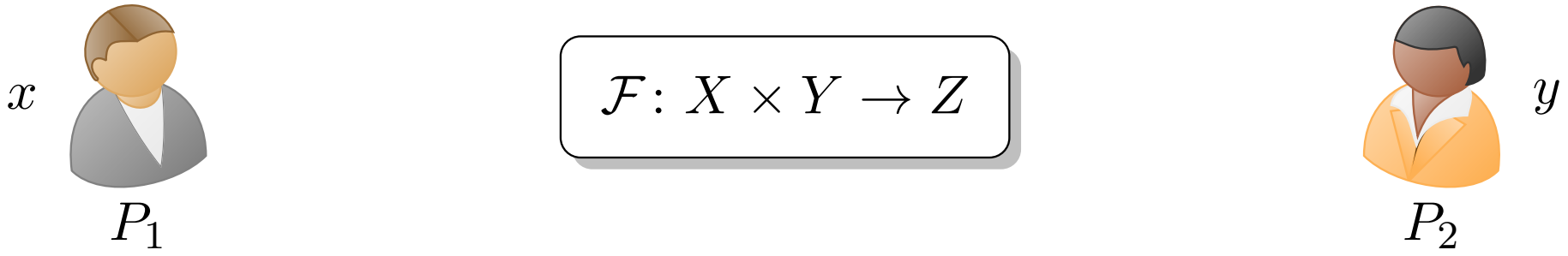

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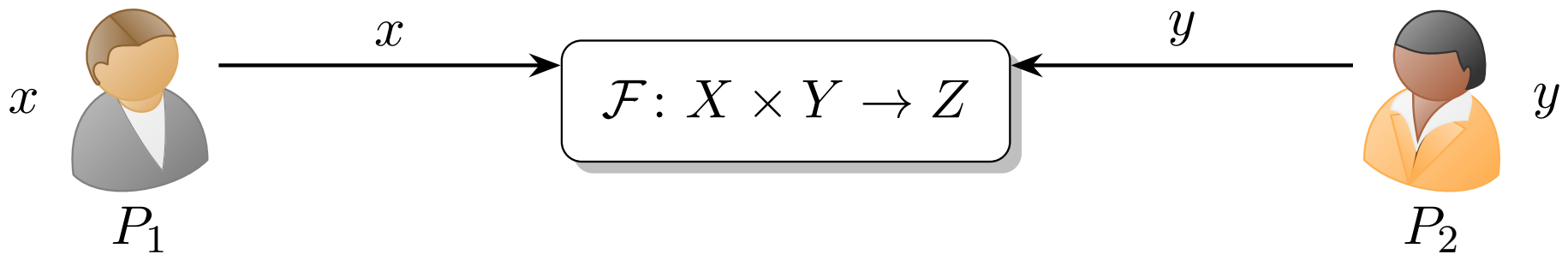

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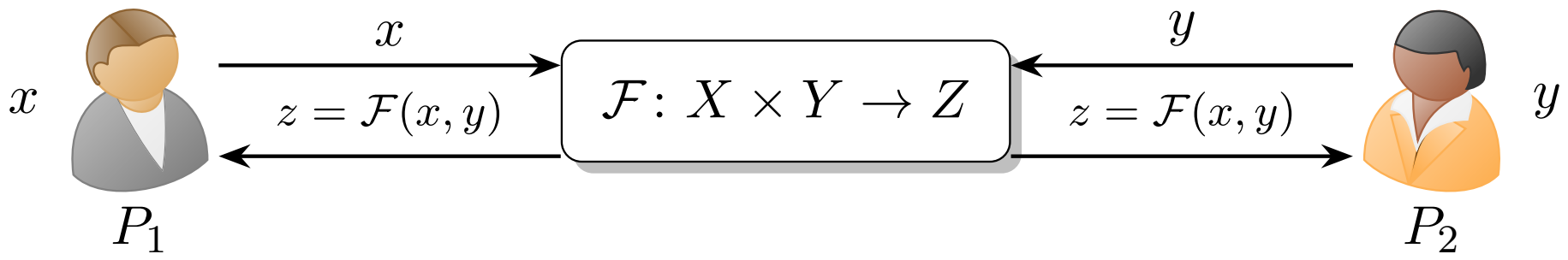

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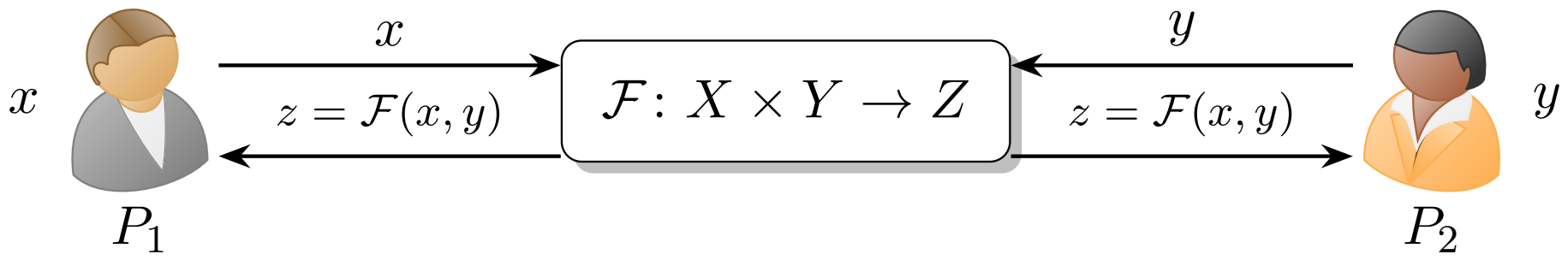

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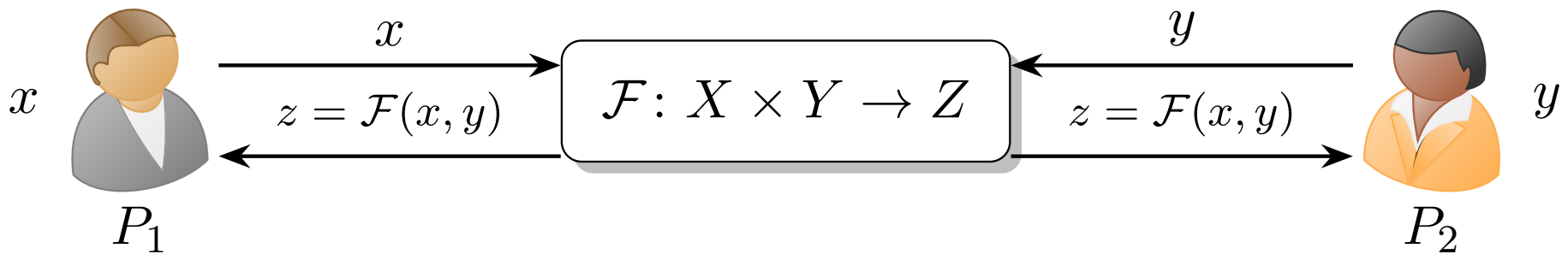

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- Final protocol will be *constant round*.

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	$\vdots$	
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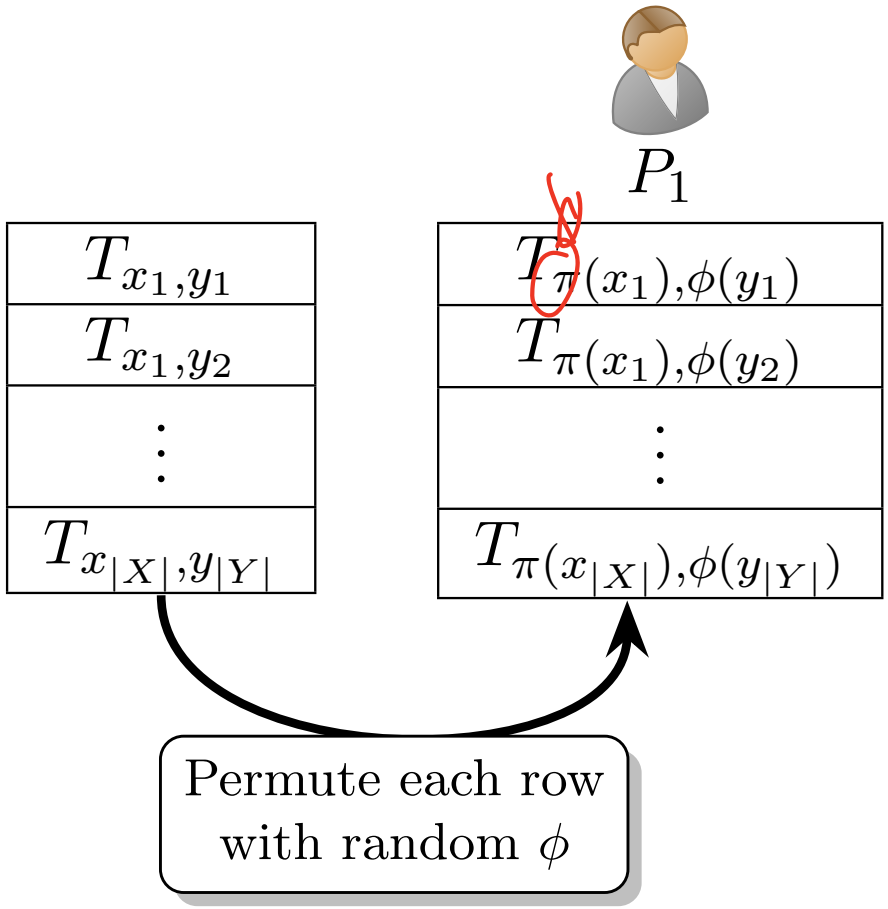

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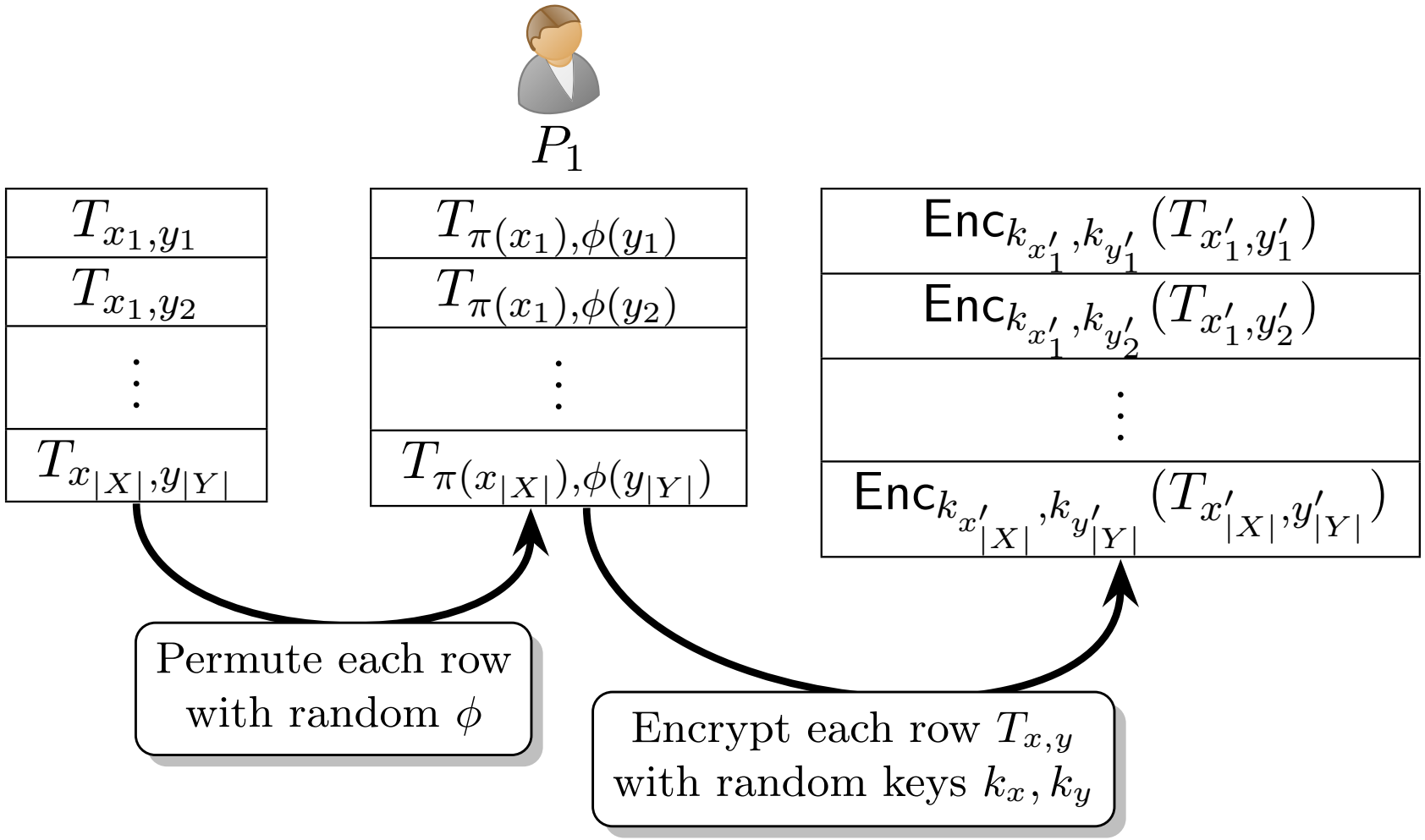

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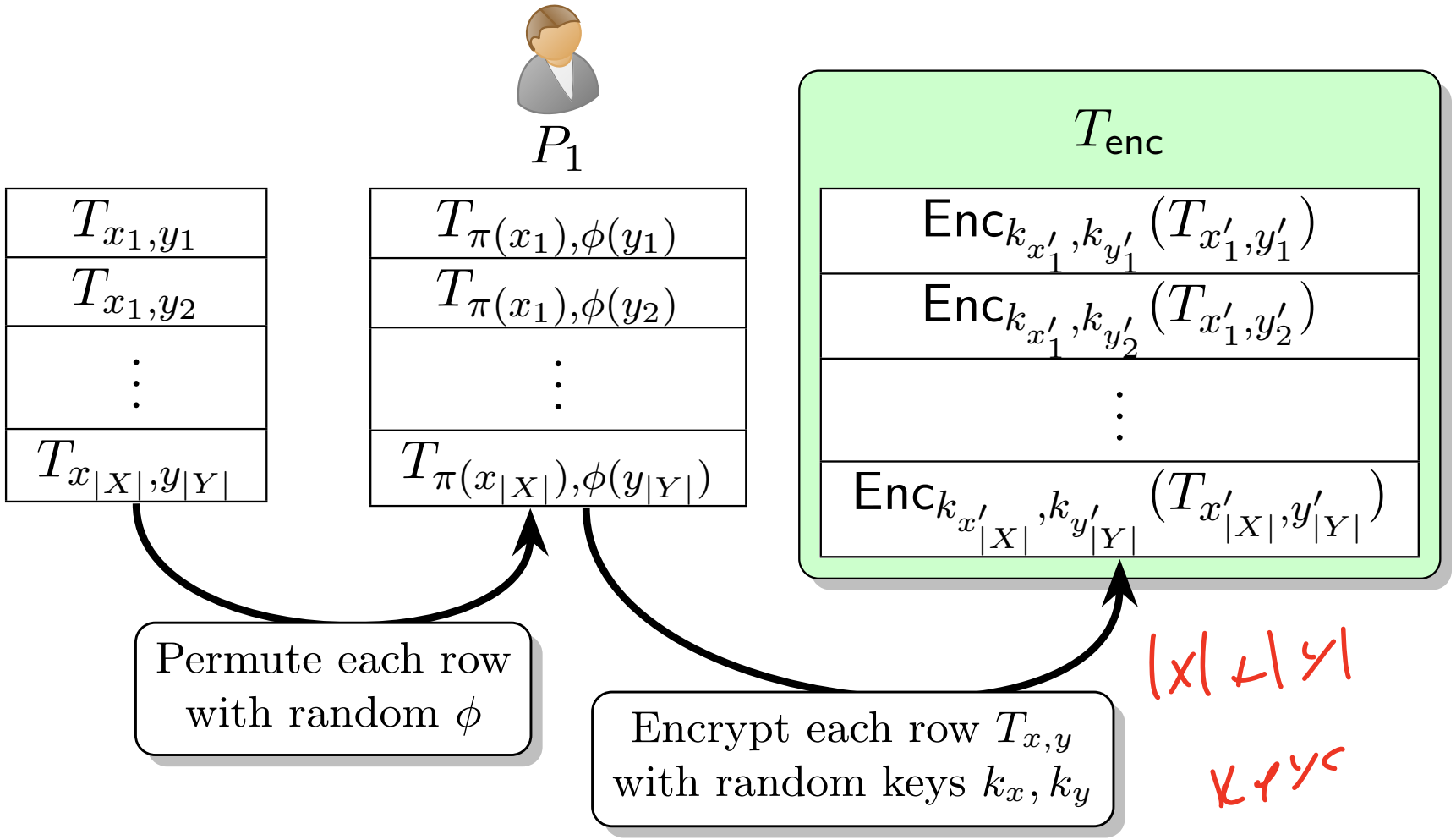

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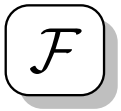

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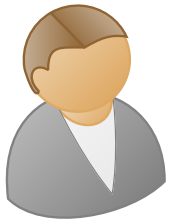

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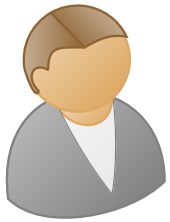

P_1

P_2

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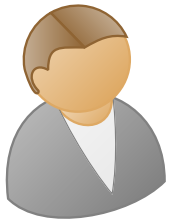
x_i



P_1

y_j

P_2

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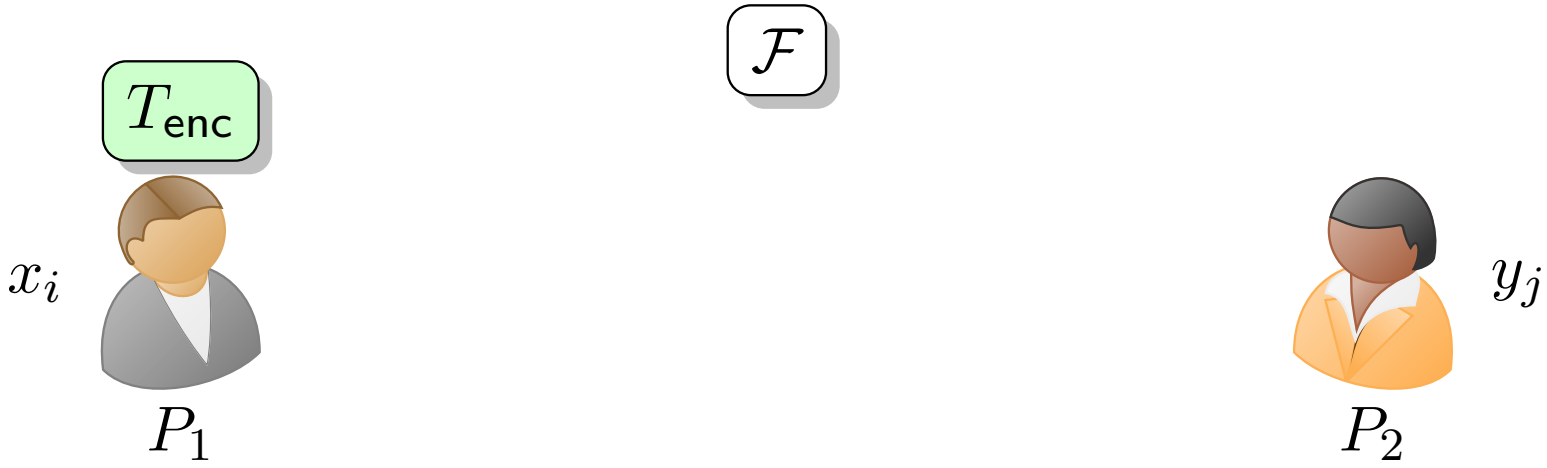

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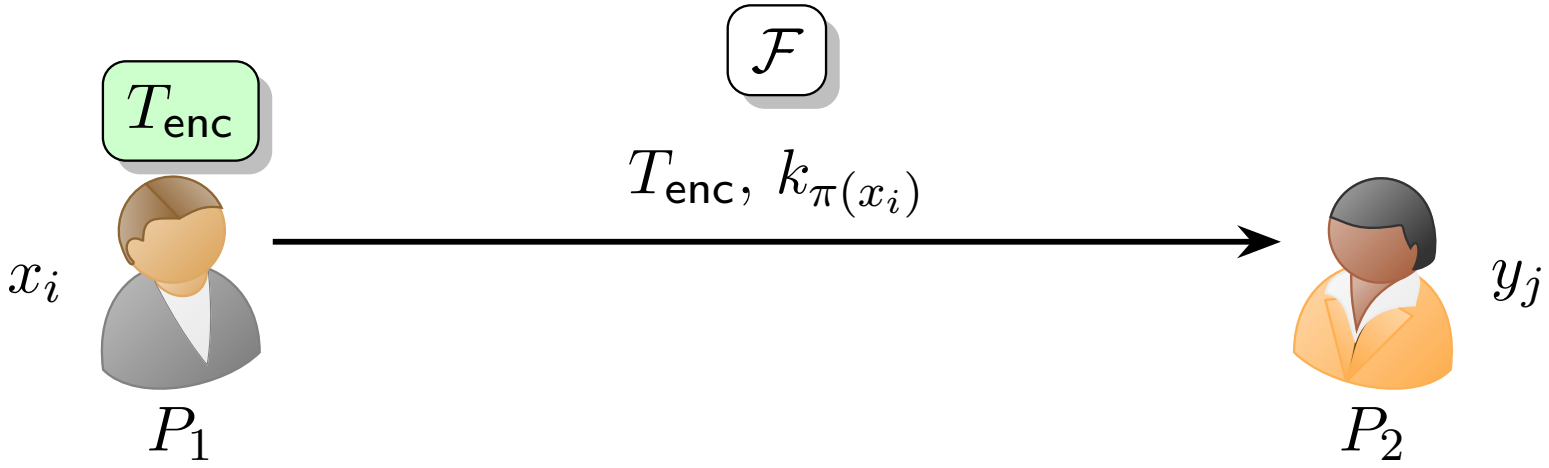

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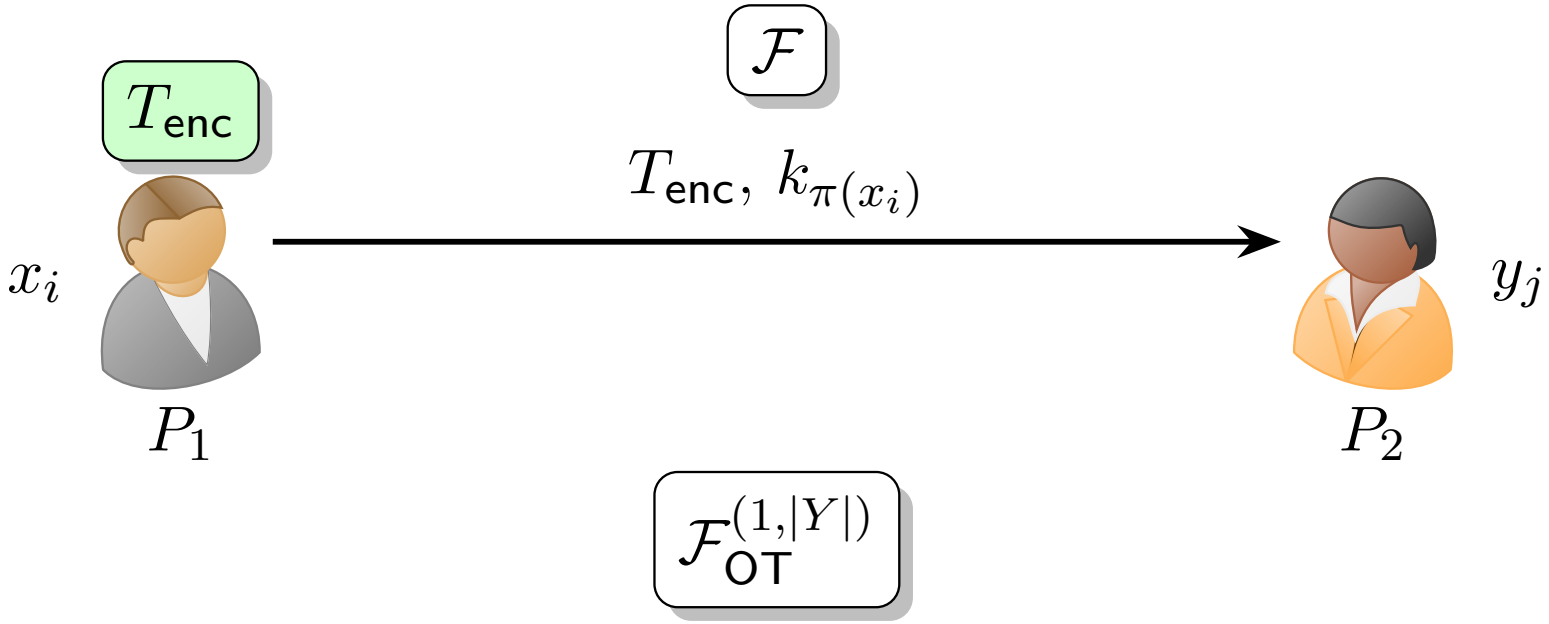

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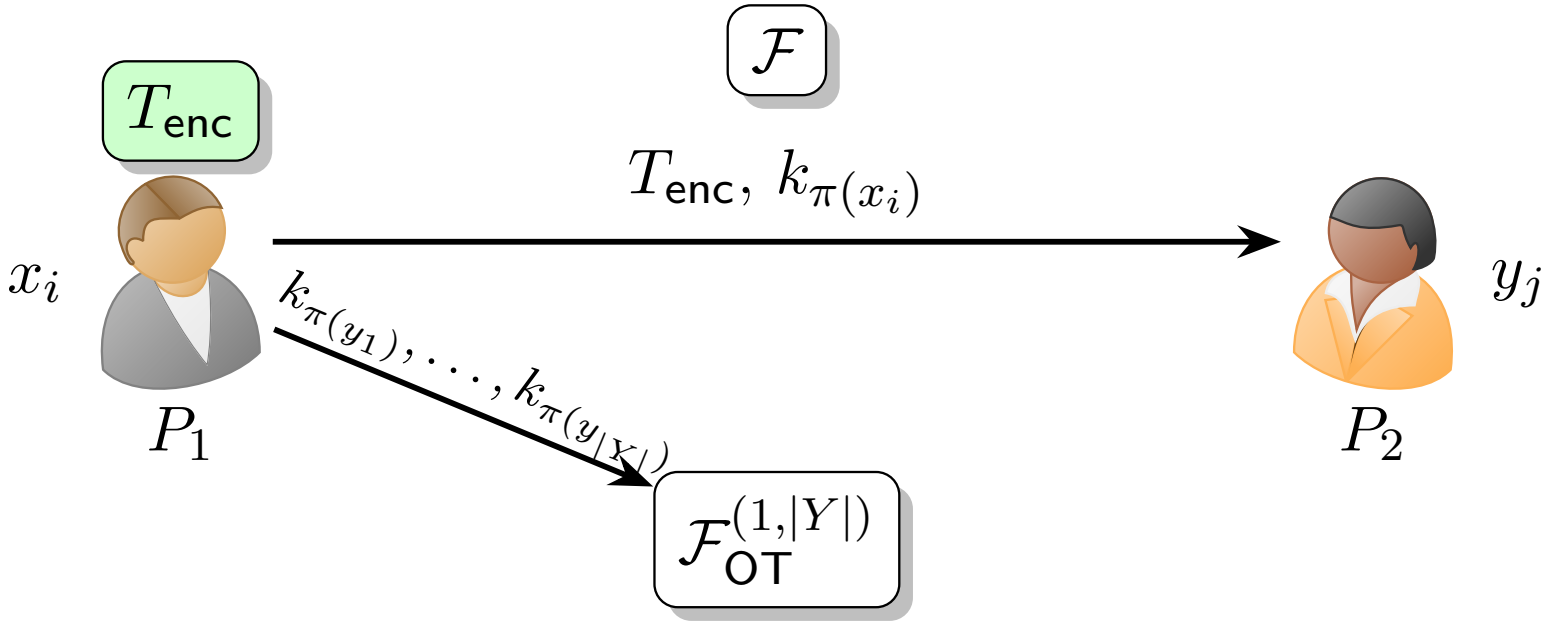

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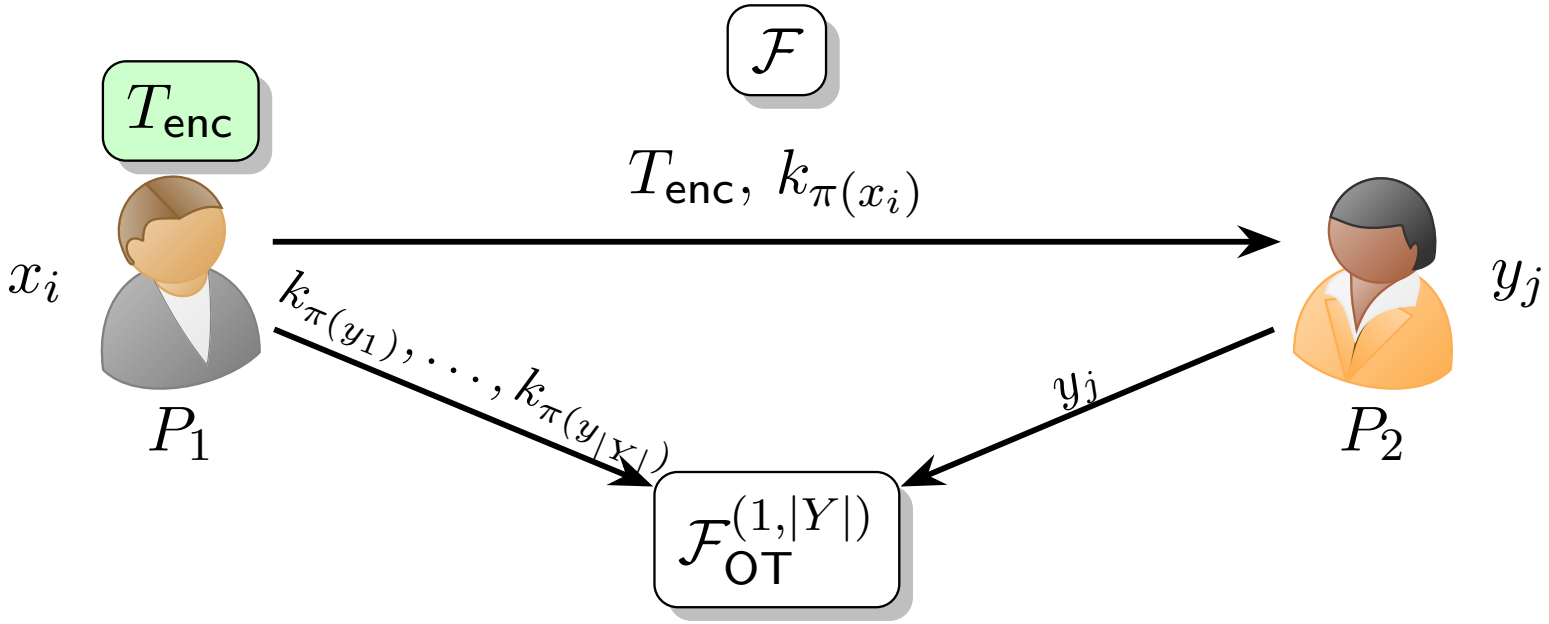

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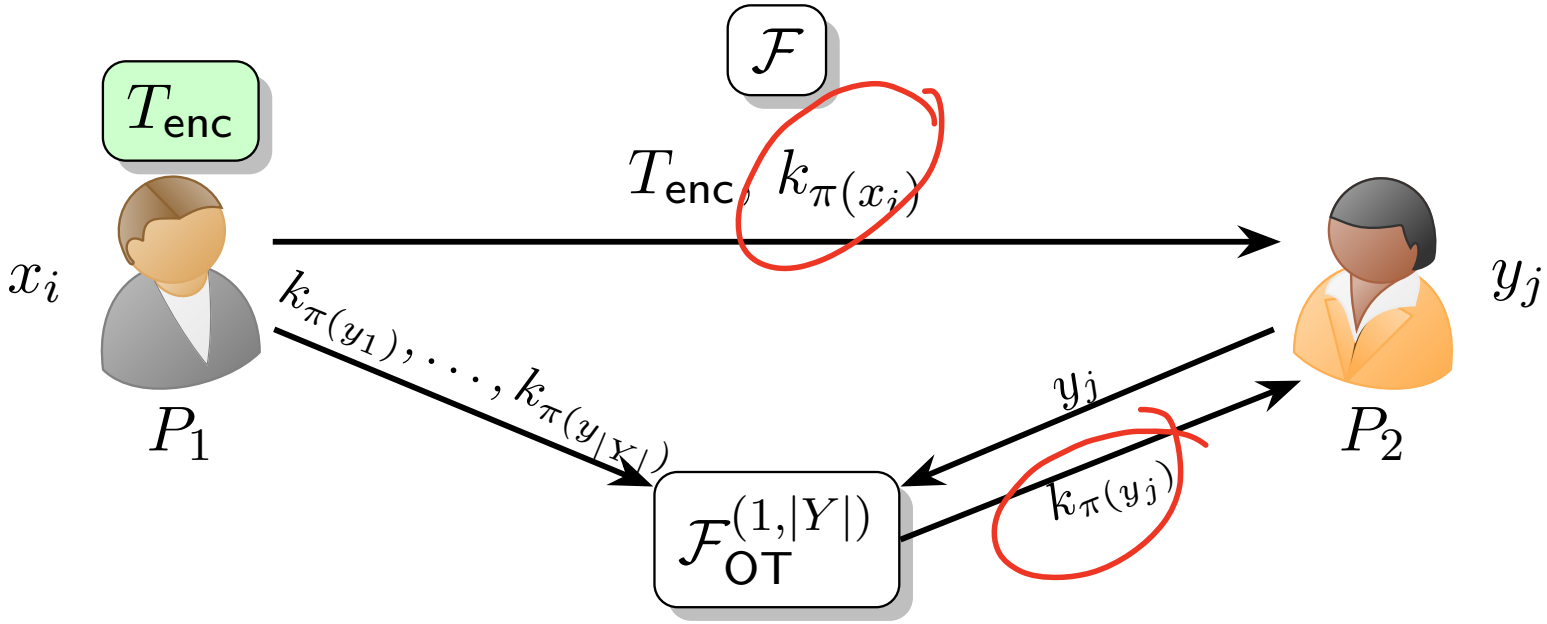

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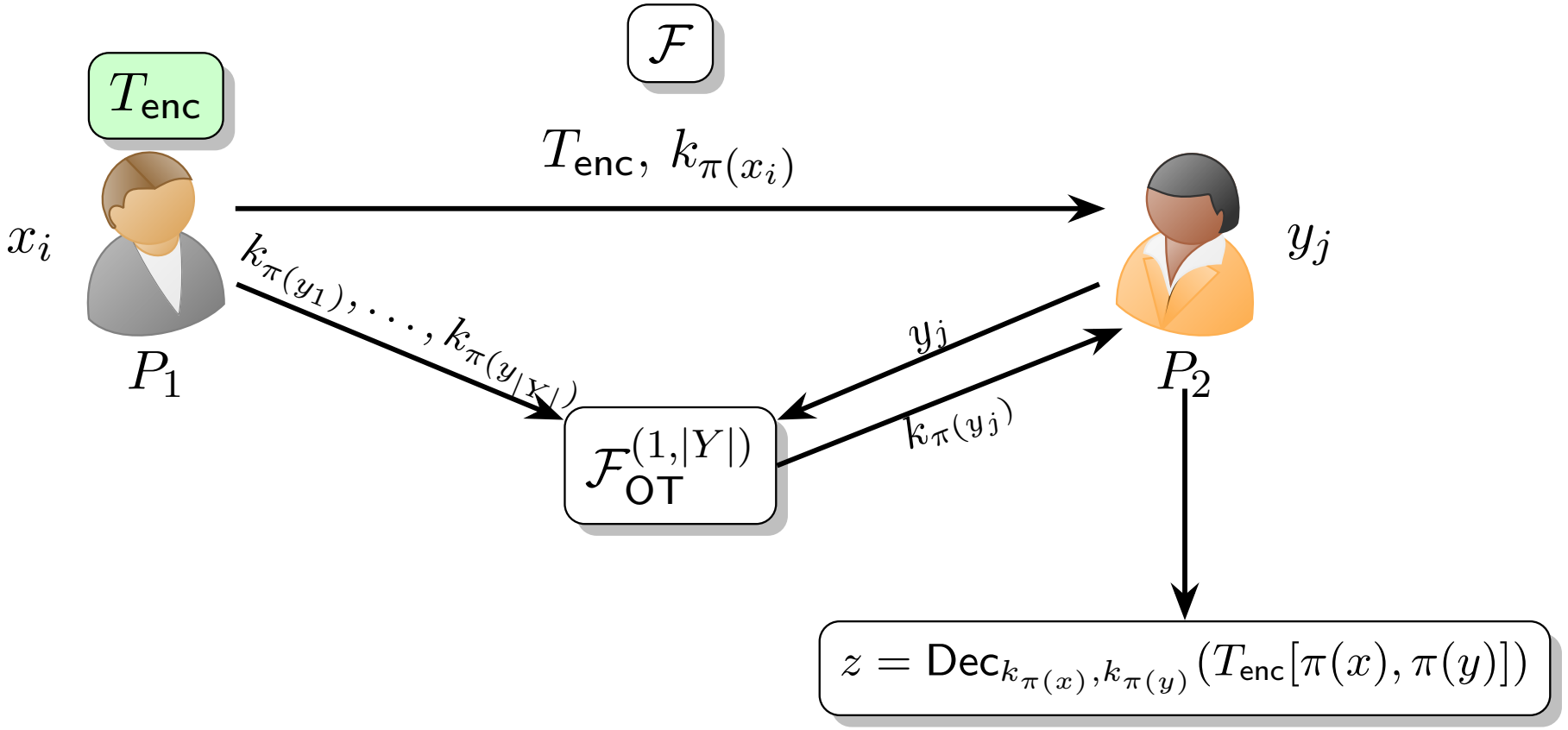

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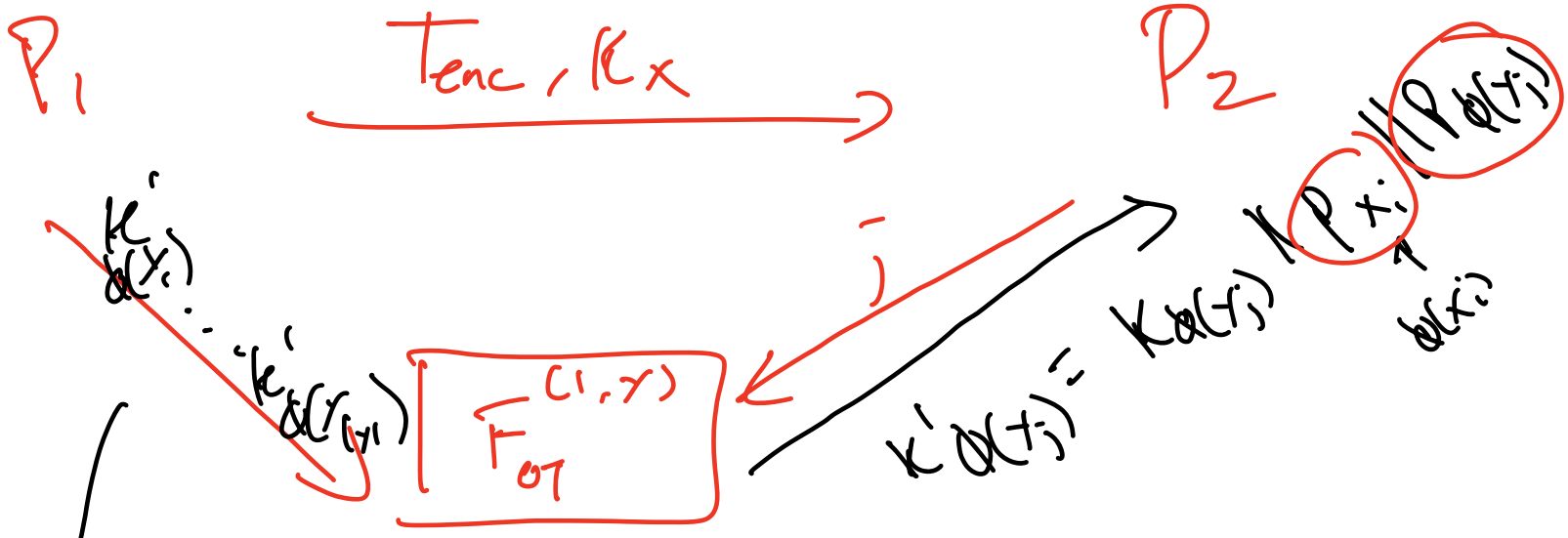

$$\hookrightarrow K'_Y = K_Y \| P_X \| P_T$$

$$s.t. \quad Q(T)\left[P_X, P_T\right] = F(X,Y)$$

$$P_{X_i} = Q(i) \quad (\ i\ )$$

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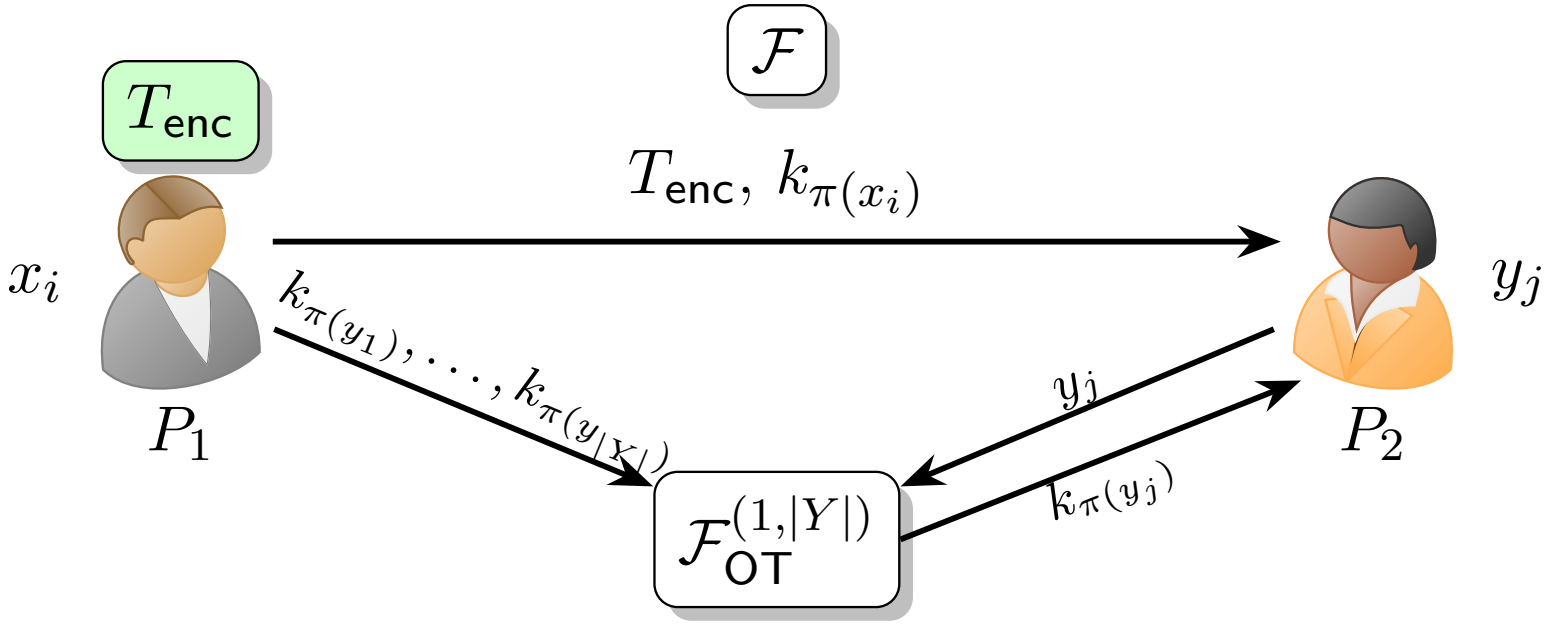
  - Knows exactly which row in T_{enc} to decrypt now!

Managing Look-up Table Size



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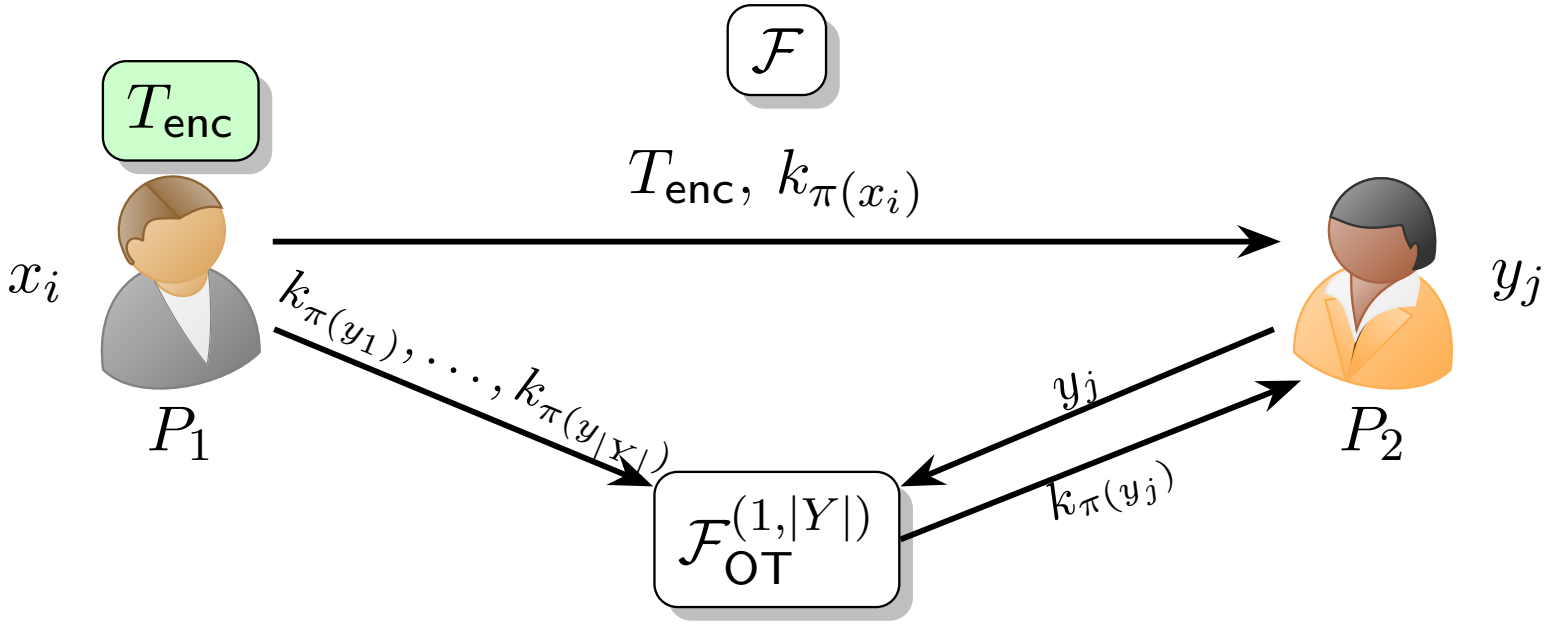

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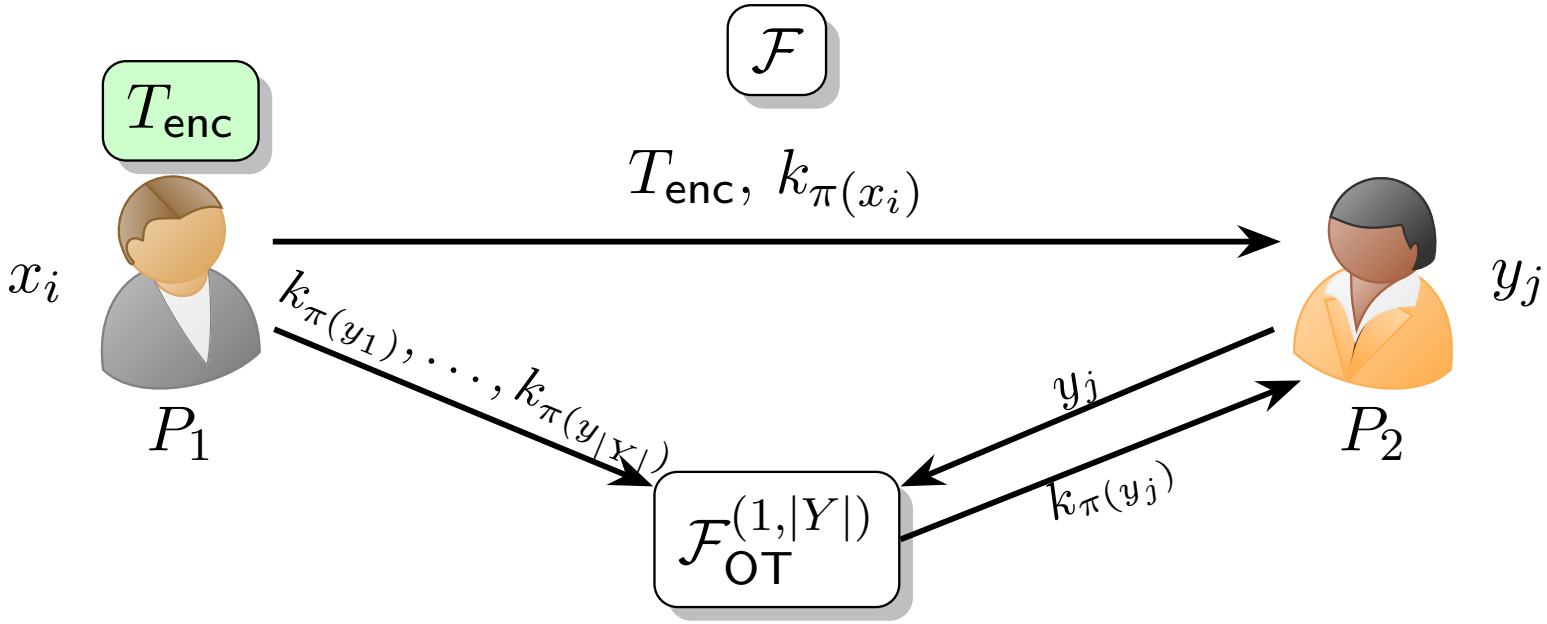

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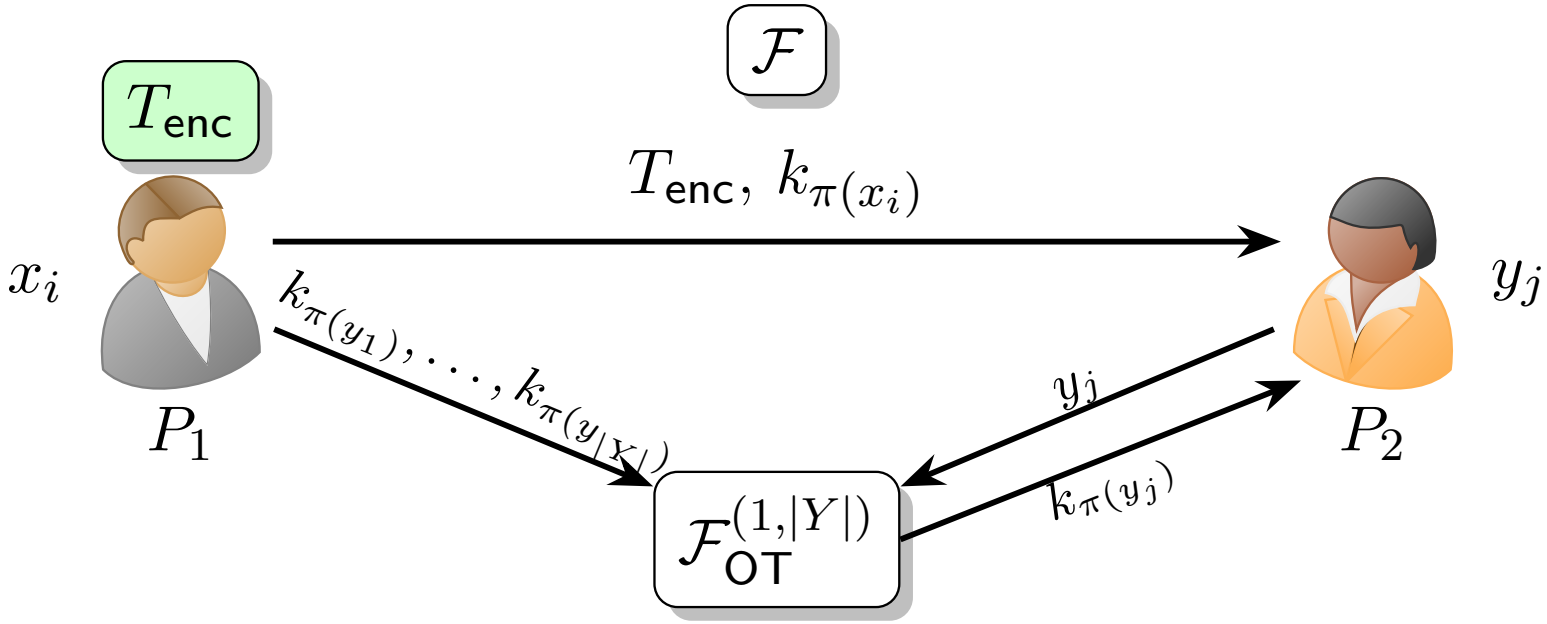

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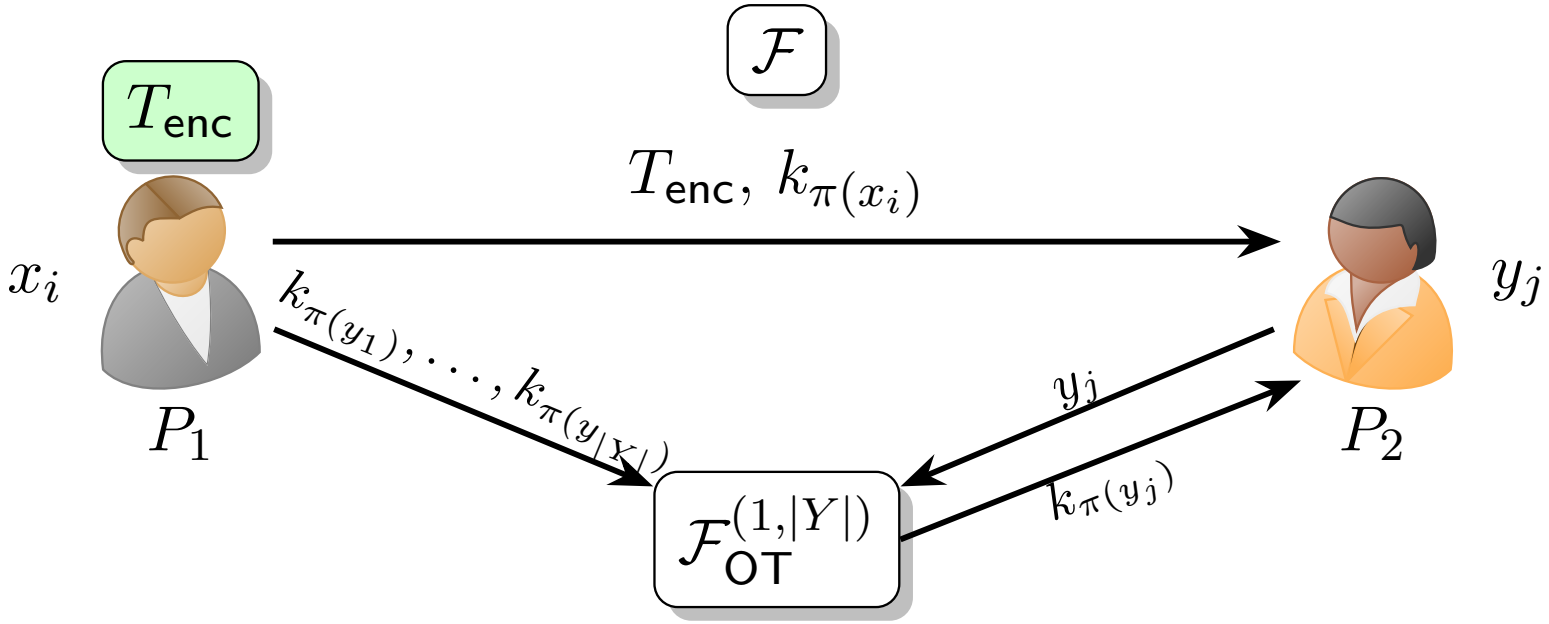

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- If $\mathcal{F}$ has a *small* (e.g., constant-sized) table, then this would be much better.

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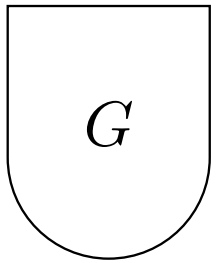

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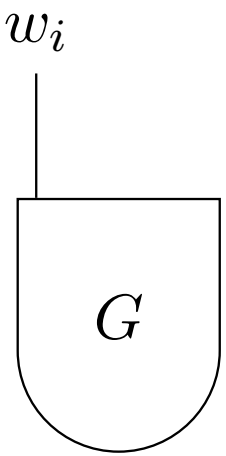

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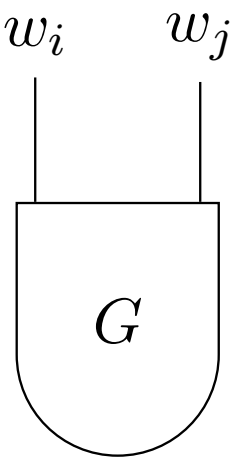

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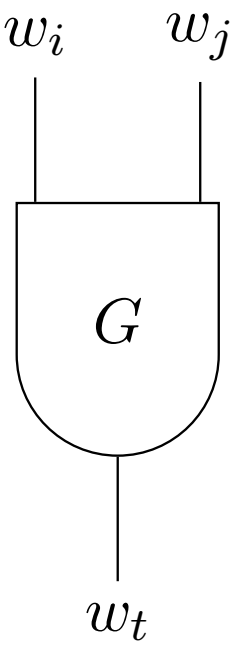

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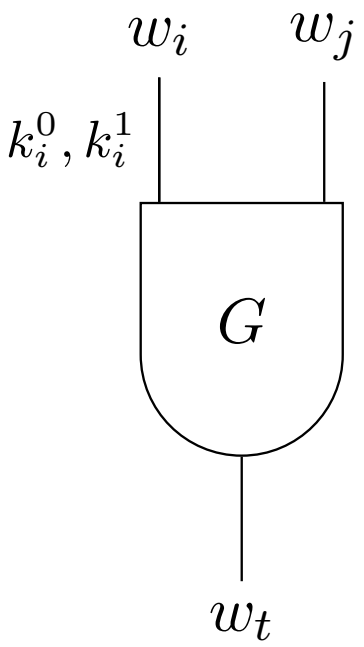

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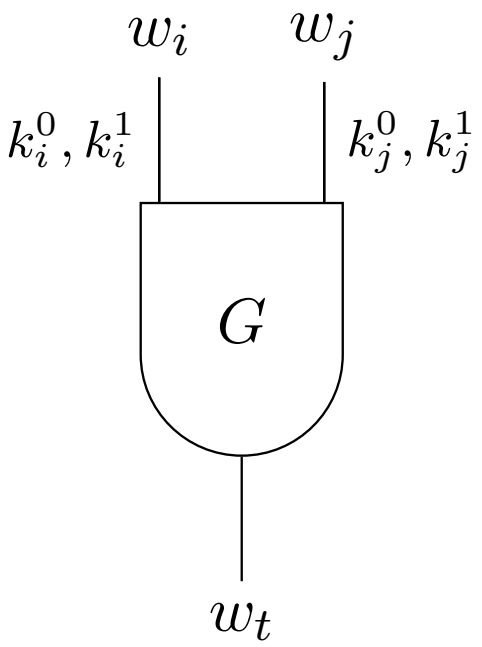

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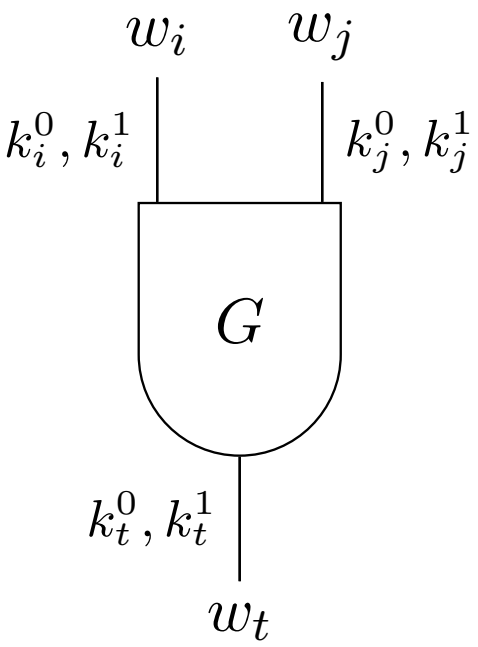

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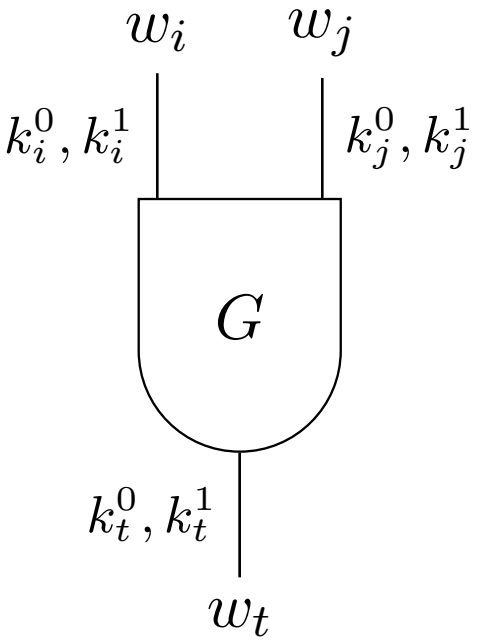

$$T_G = \begin{bmatrix} \mathsf{Enc}_{k_i^0,k_j^0}(k_t^{G(0,0)}) \\ \mathsf{Enc}_{k_i^0,k_j^1}(k_t^{G(0,1)}) \\ \mathsf{Enc}_{k_i^1,k_j^0}(k_t^{G(1,0)}) \\ \mathsf{Enc}_{k_i^1,k_j^1}(k_t^{G(1,1)}) \end{bmatrix}$$

AND

$$\begin{bmatrix} \mathsf{Enc}(k_t^0) \\ \mathsf{Enc}(k_t^0) \\ \mathsf{Enc}(k_t^0) \\ \mathsf{Enc}(k_t^1) \end{bmatrix}$$

$\updownarrow$

$Q_G(T_G)$

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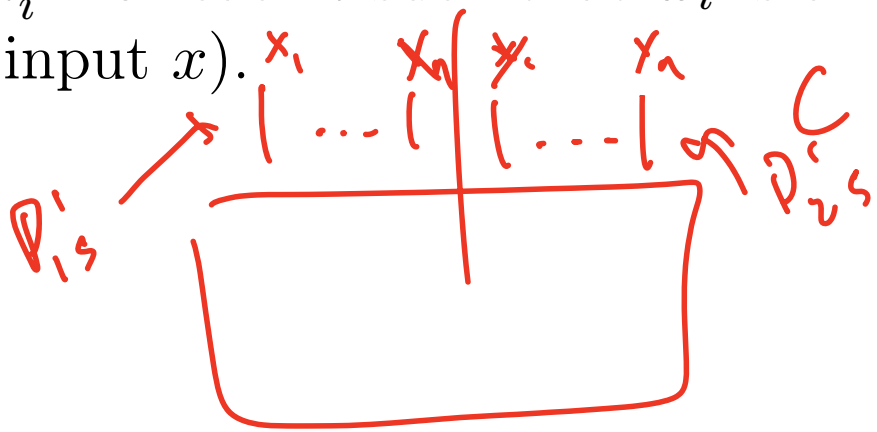
- P_1 then sends each (encrypted and permuted) T_G to P_2, along with key $k_i^{x_i}$ for each i such that w_i is an *input wire* (corresponding to P_1's input x).



$|C| = s$ $\quad \#T = s$

$C : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}^n$

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  - To save one round of communication, P_1 can instead send the decoding tables for each output wire w_ℓ (e.g., send $(0, k_\ell^0), (1, k_\ell^1)$).

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$O(\lambda(s+n))$ bits

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- Finally, P_2 outputs s_{out} bits.

Next Time: the GMW Protocol