CS 594 – Advanced Crypto (Spring 2026)

Alex Block

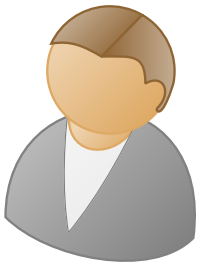
Lecture 2

January 21, 2026

Security Before Modern Cryptography

Security Before Modern Cryptography

Alice



Bob

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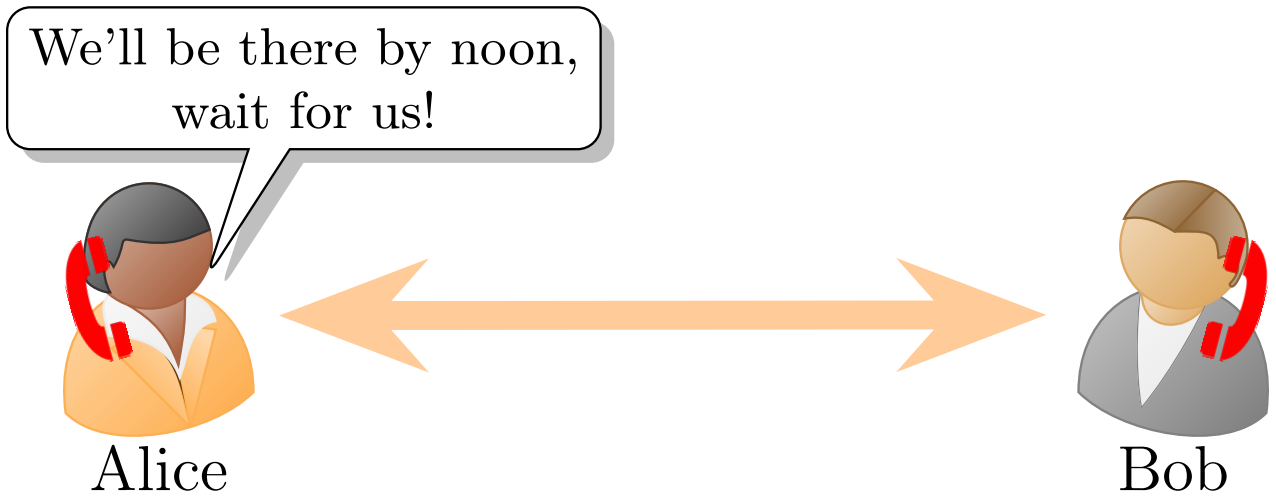

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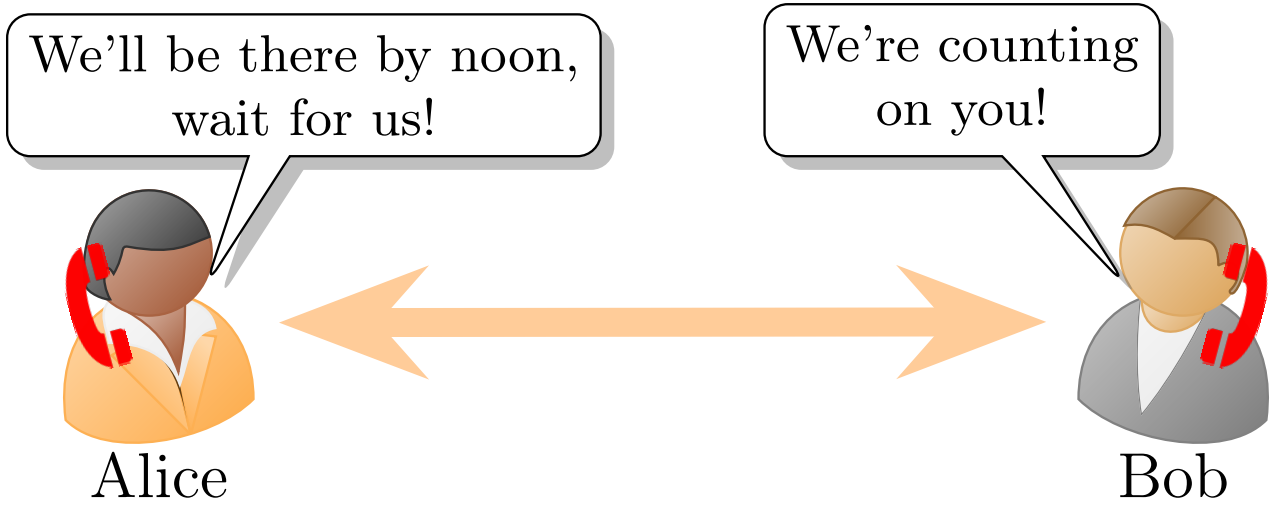

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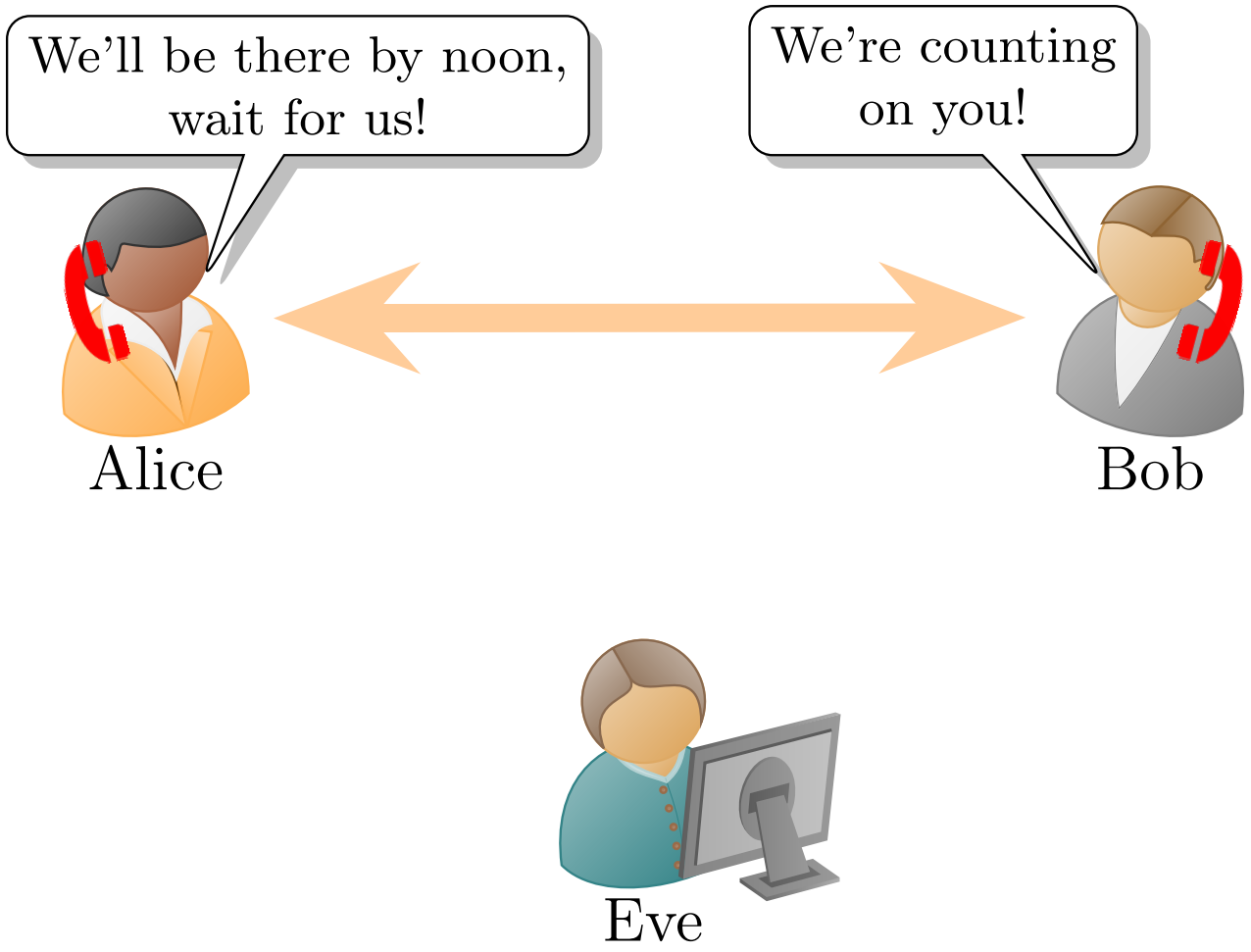

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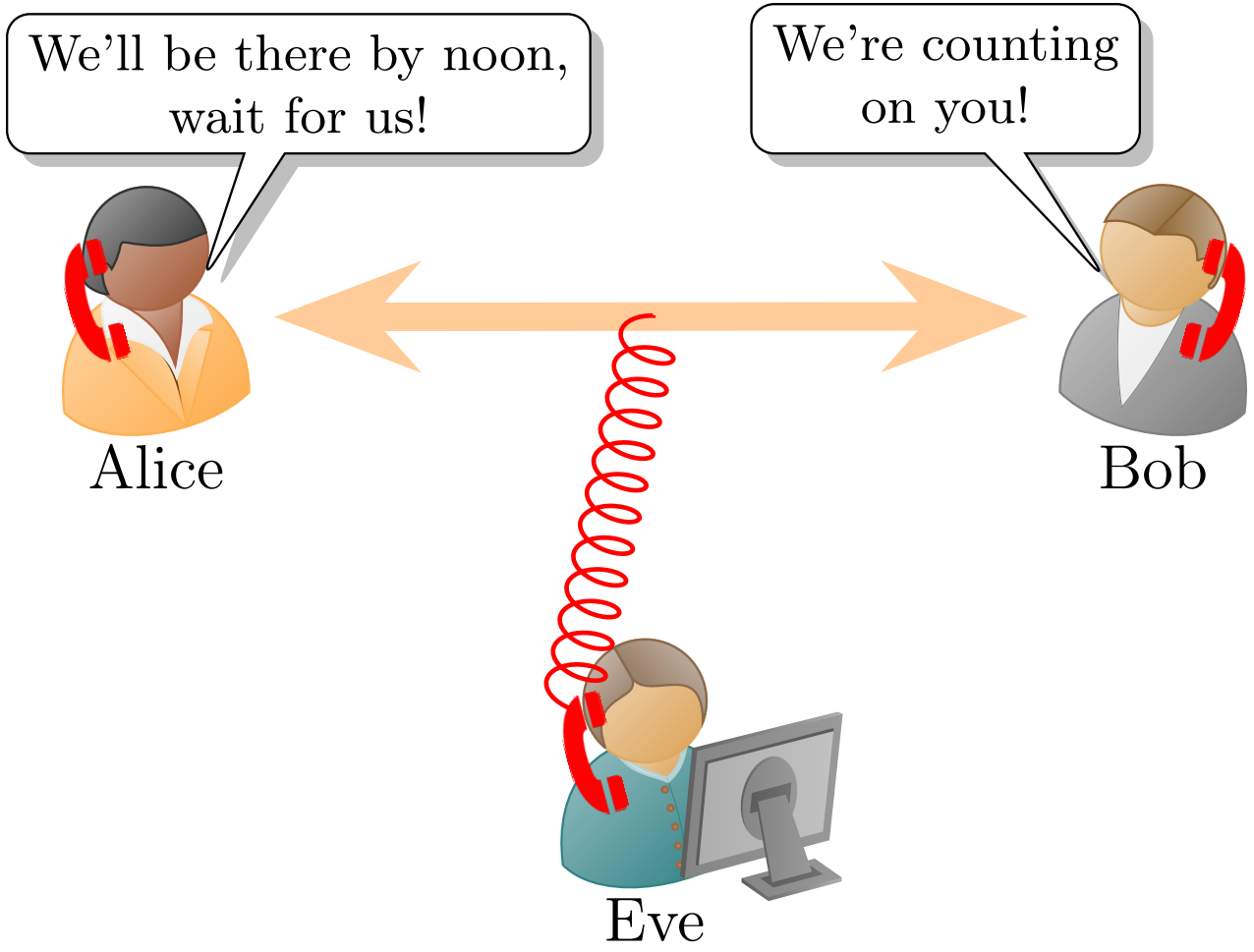

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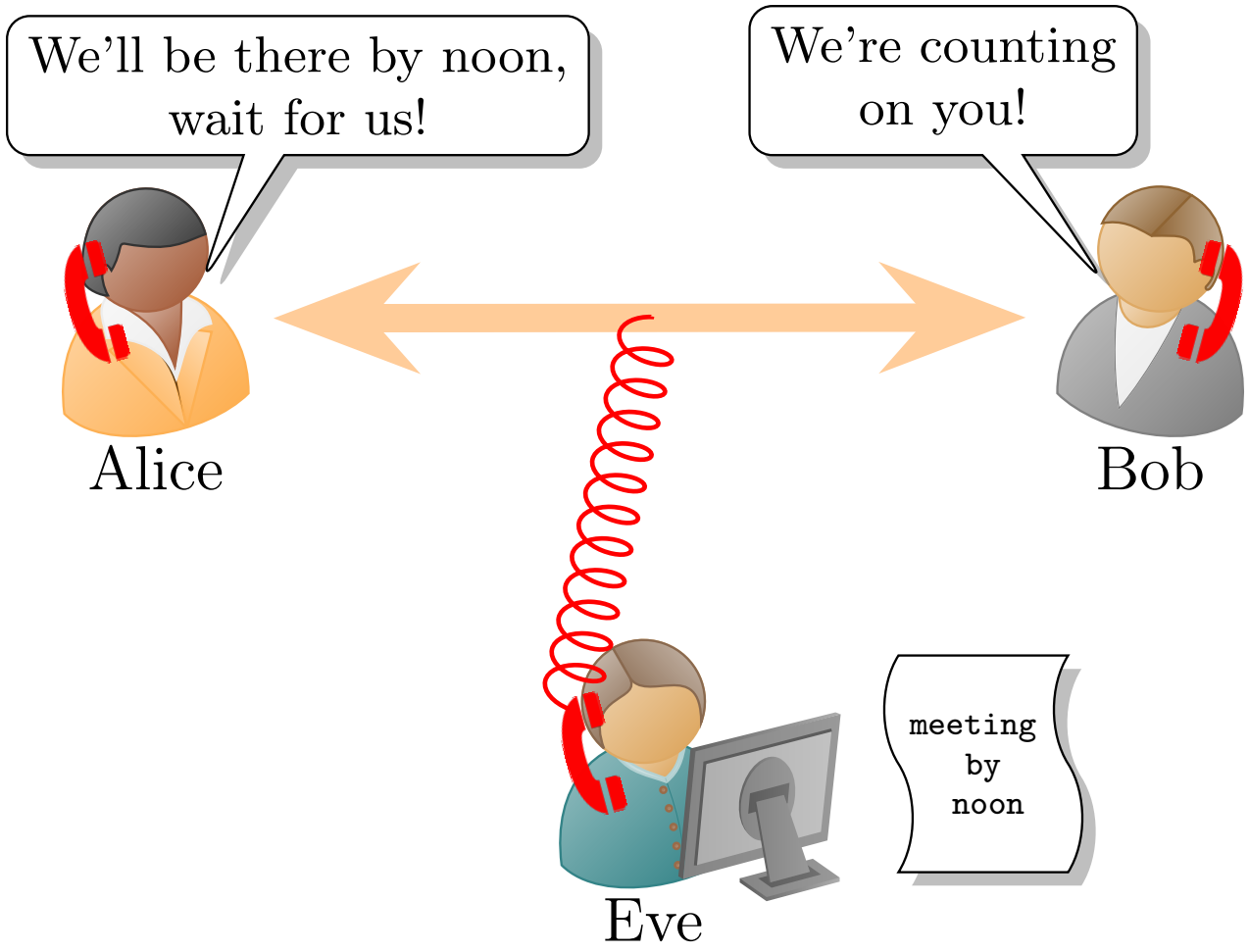

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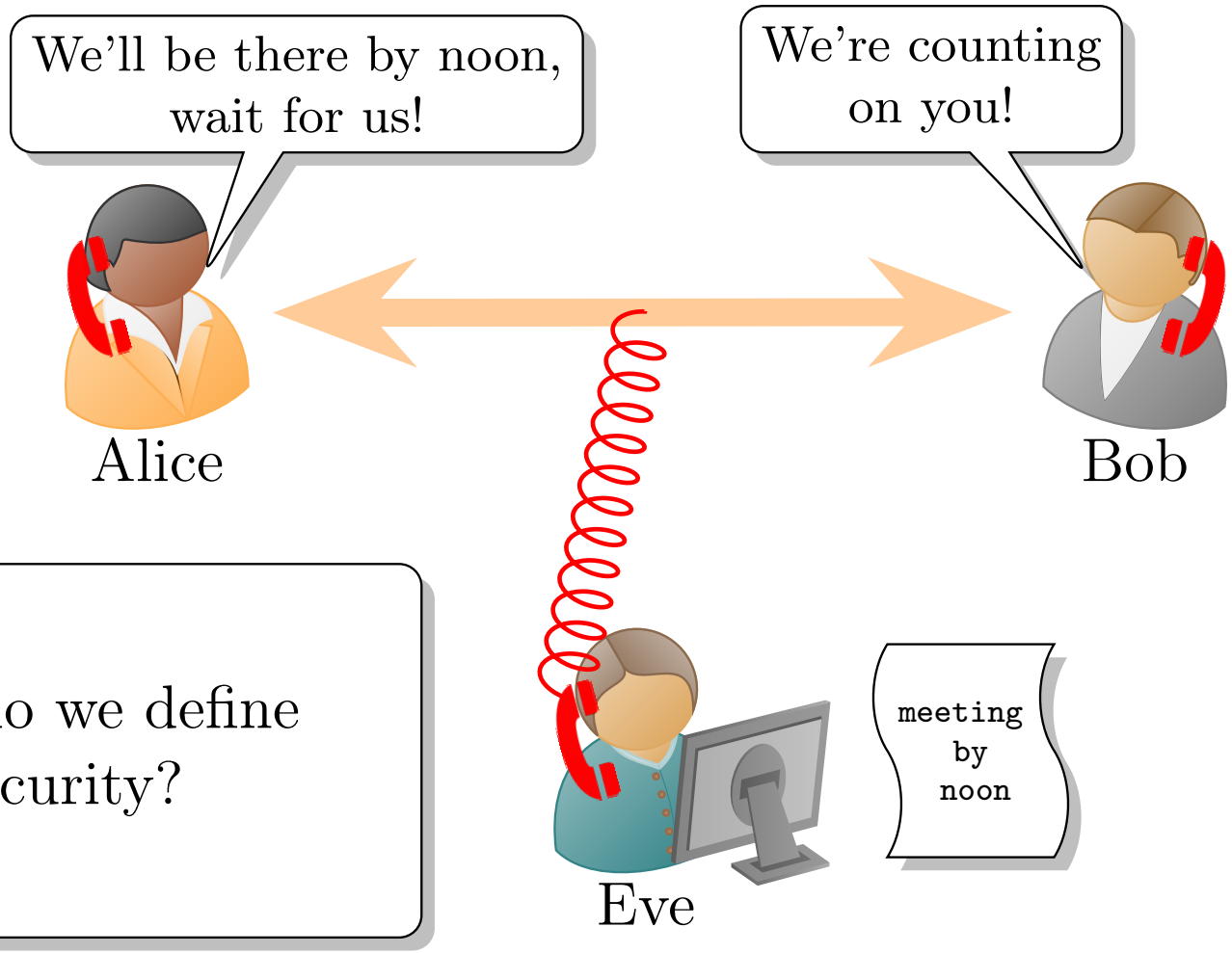
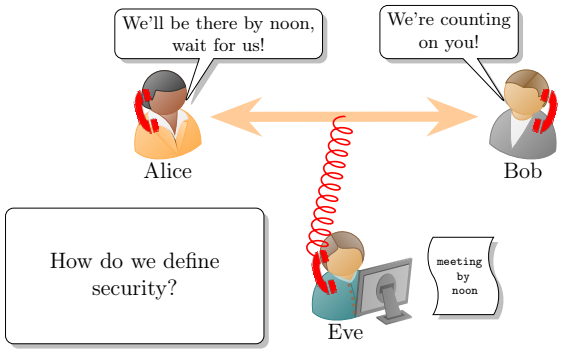


How do we define security?

Security Before Modern Cryptography



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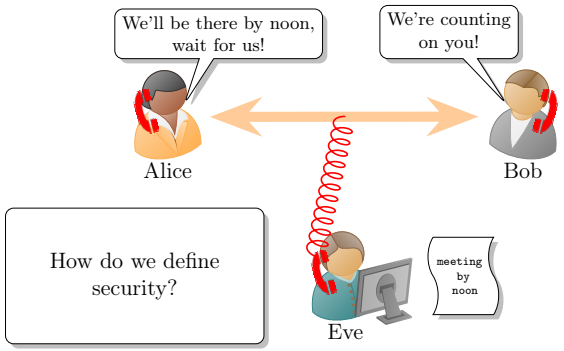

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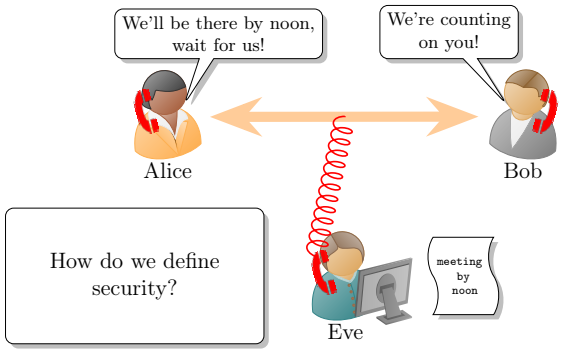

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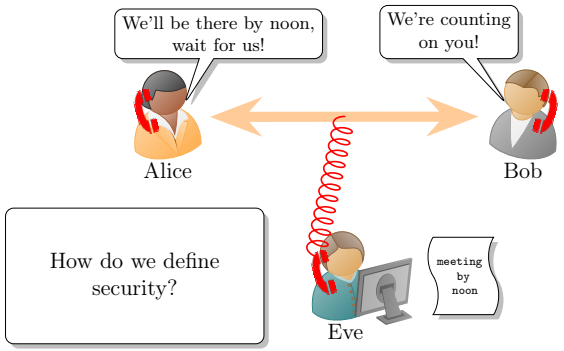

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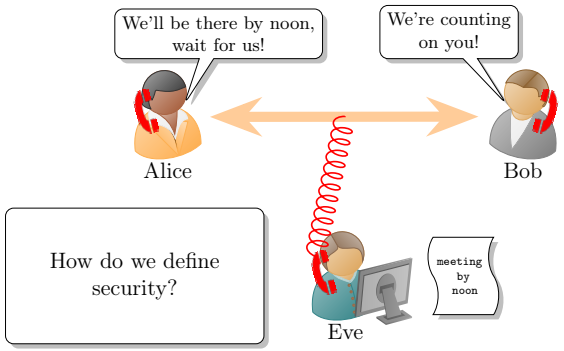

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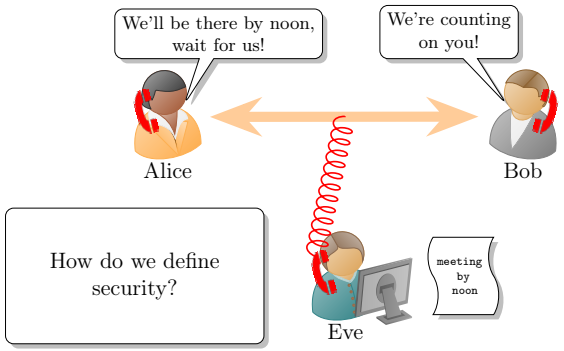
- What does "secure" even mean?



Core Principles of Modern Cryptography

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Formal Definitions

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(Hardness) Assumptions

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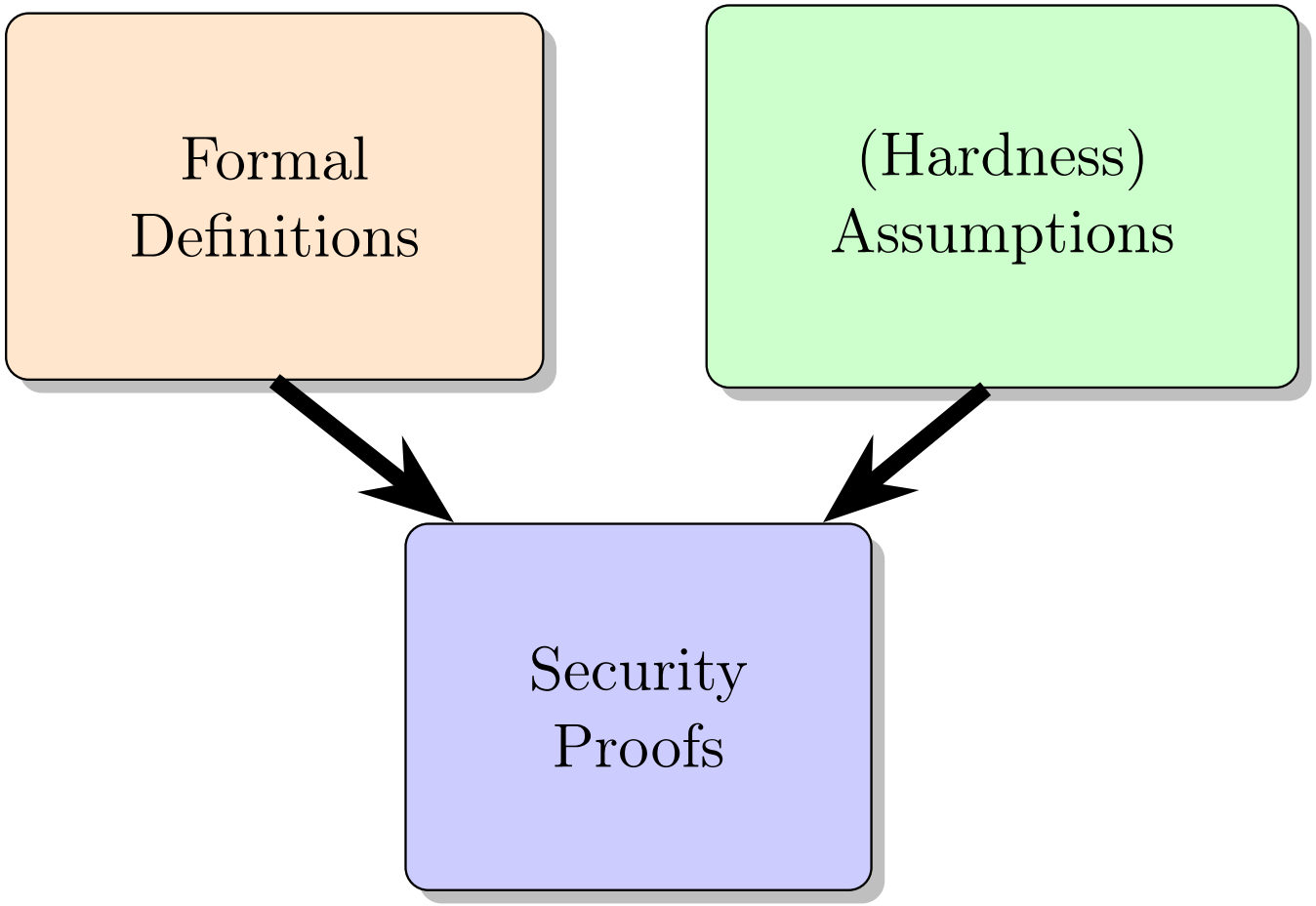

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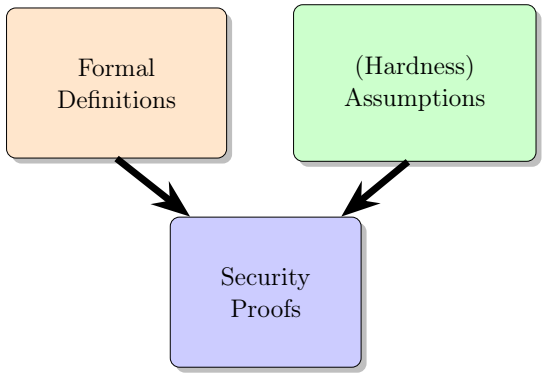
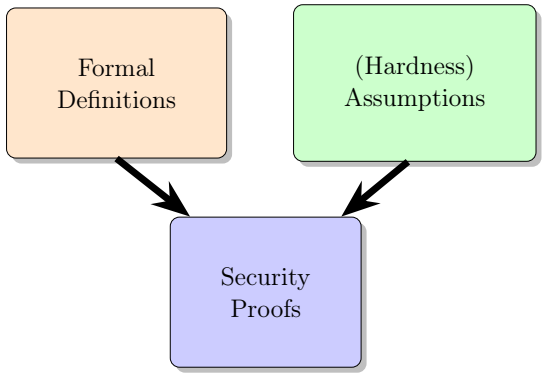


Core Principles of Modern Cryptography

- Precise, mathematical models and definitions for what "security" really means



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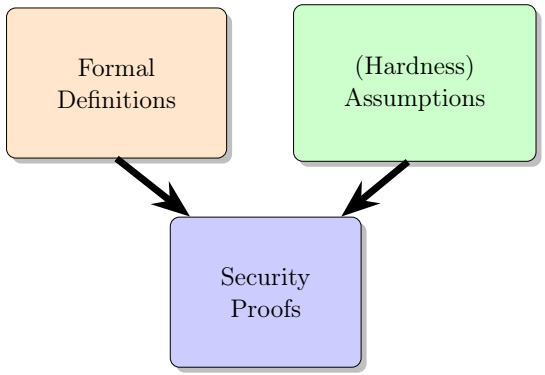

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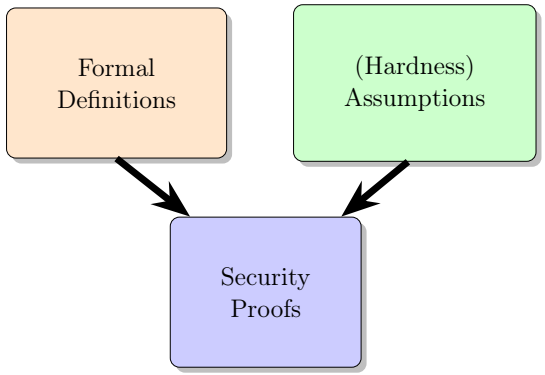

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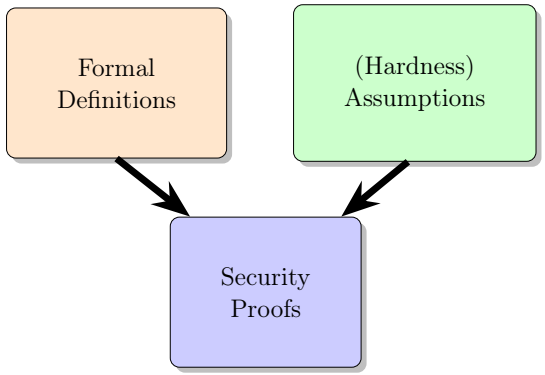

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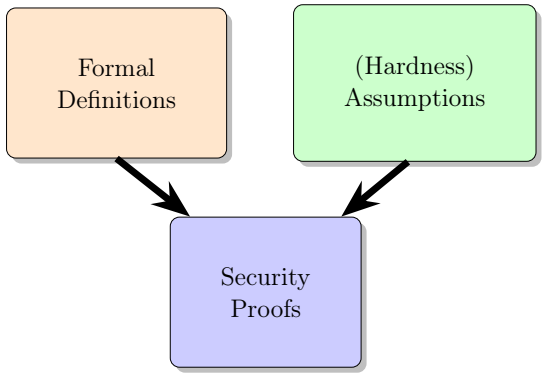
- Proof pattern: Construction X satisfies Security Definition Y, unless Hard Problem/Secure Construction Z is broken



Core Principles of Modern Cryptography

Pros

- Rigor
- Immune to tech improvements
- Robustness
- Tunable

.fast?

Cons

- Tech improvements (Quantum)
- Implementations
  - ↳ Legacy compat.
  - ↳ Impractical in real world

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Pros and Cons do not necessarily work against each other!

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- Asymptotic: trying to catch-all for all possible adversaries (for some class of adversaries)

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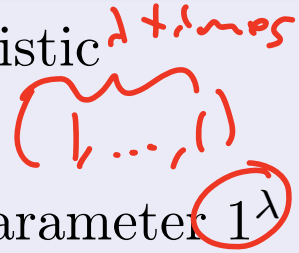

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Probabilistic

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We require that for every $\lambda \in \mathbb{Z}^+$, every $k \in \mathsf{Gen}(1^\lambda)$, and every $m \in \{0,1\}^*$, it holds that $\mathsf{Verify}_k(m, \mathsf{Mac}_k(m)) = 1$.

same message

= key

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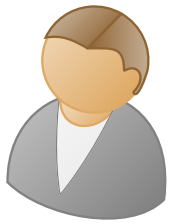

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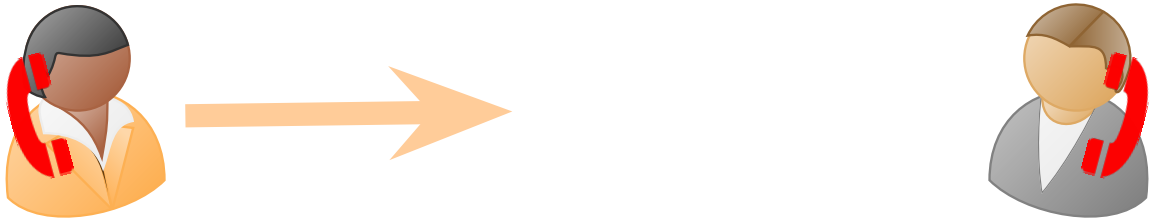

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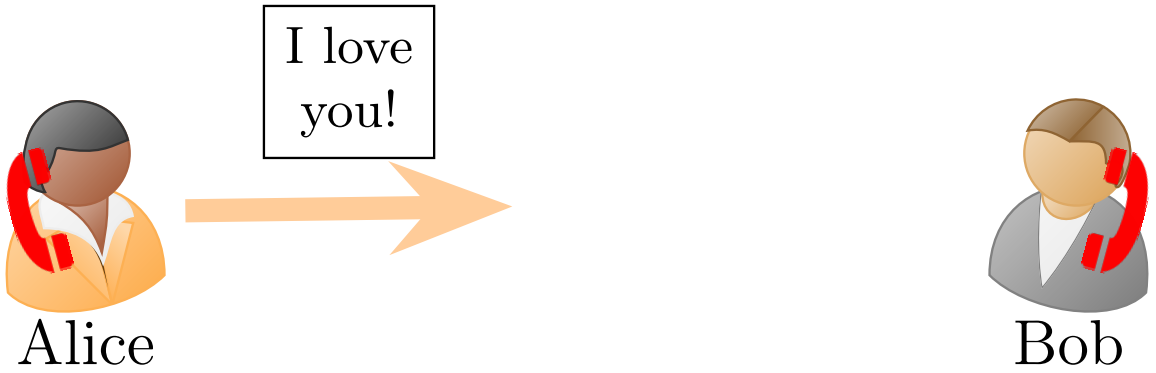

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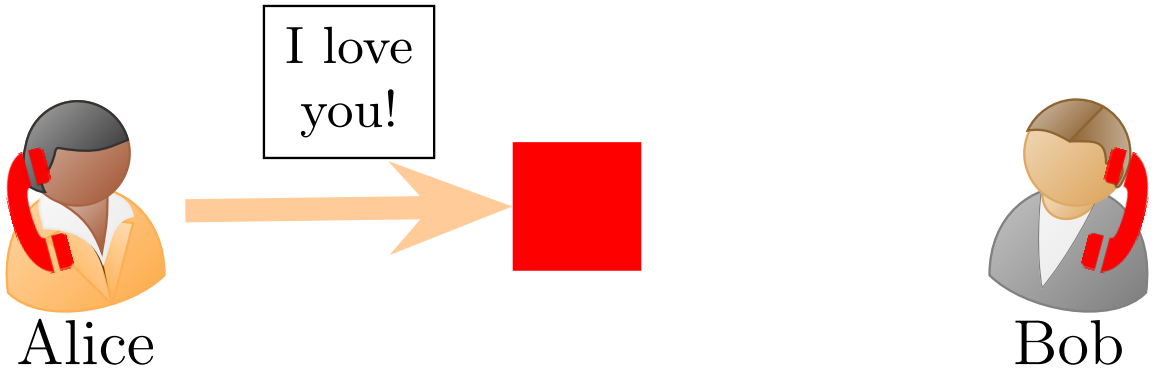

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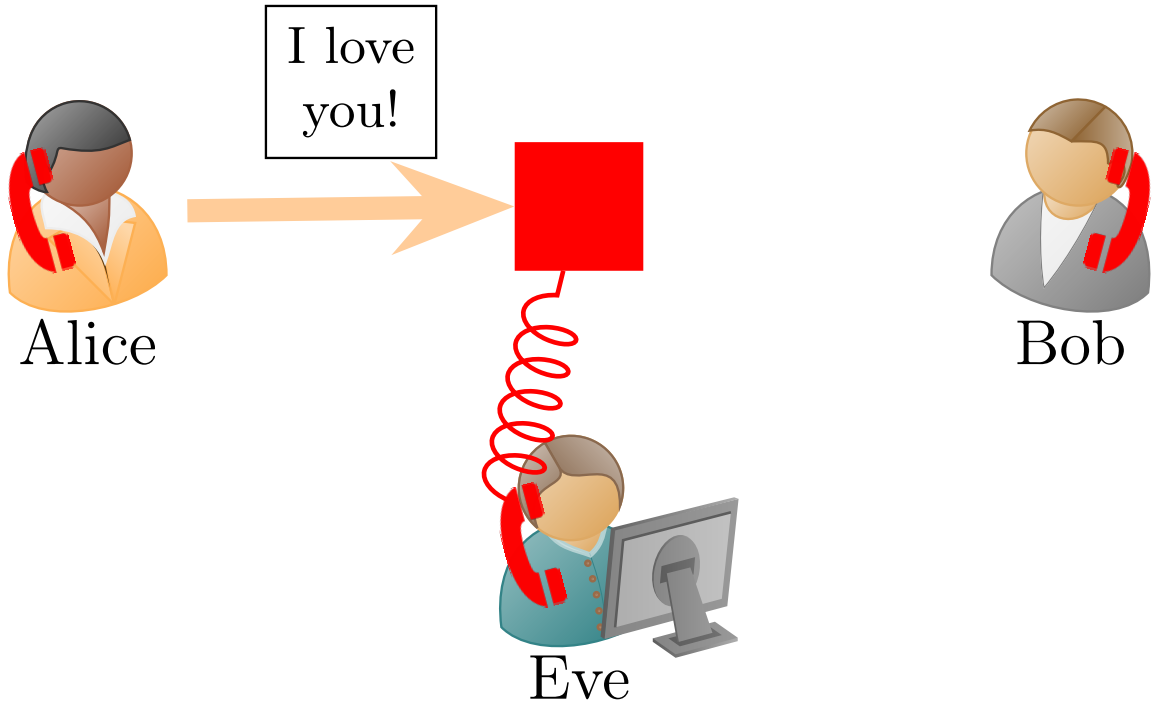

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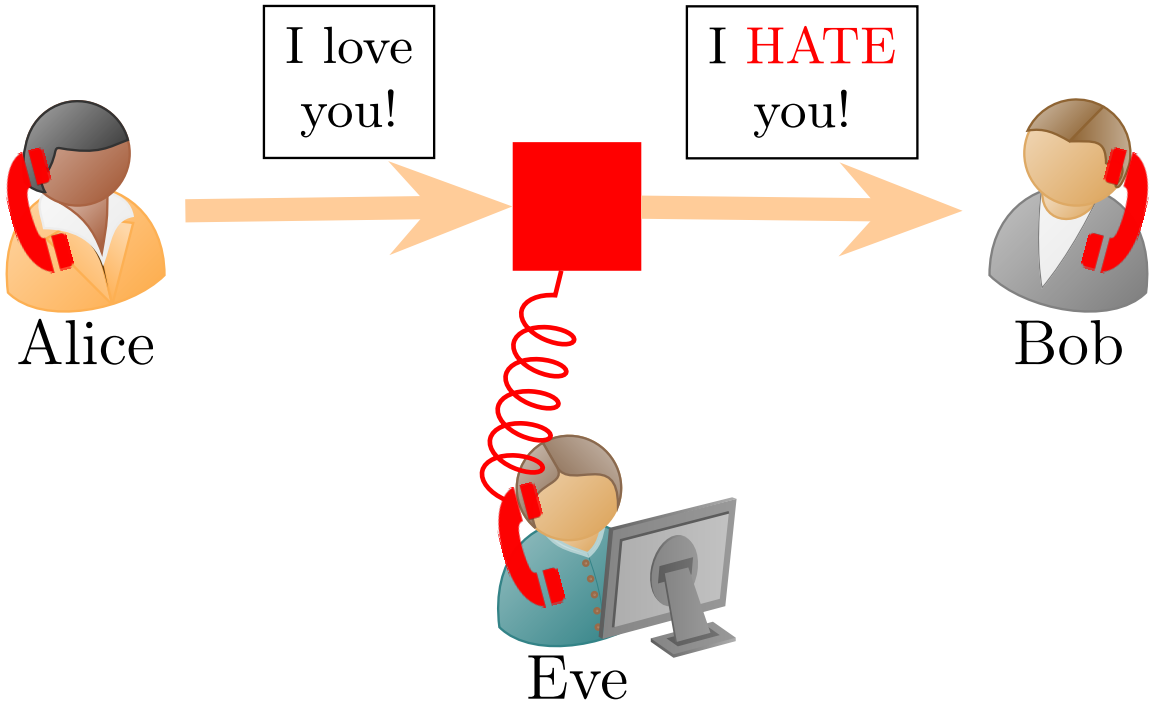

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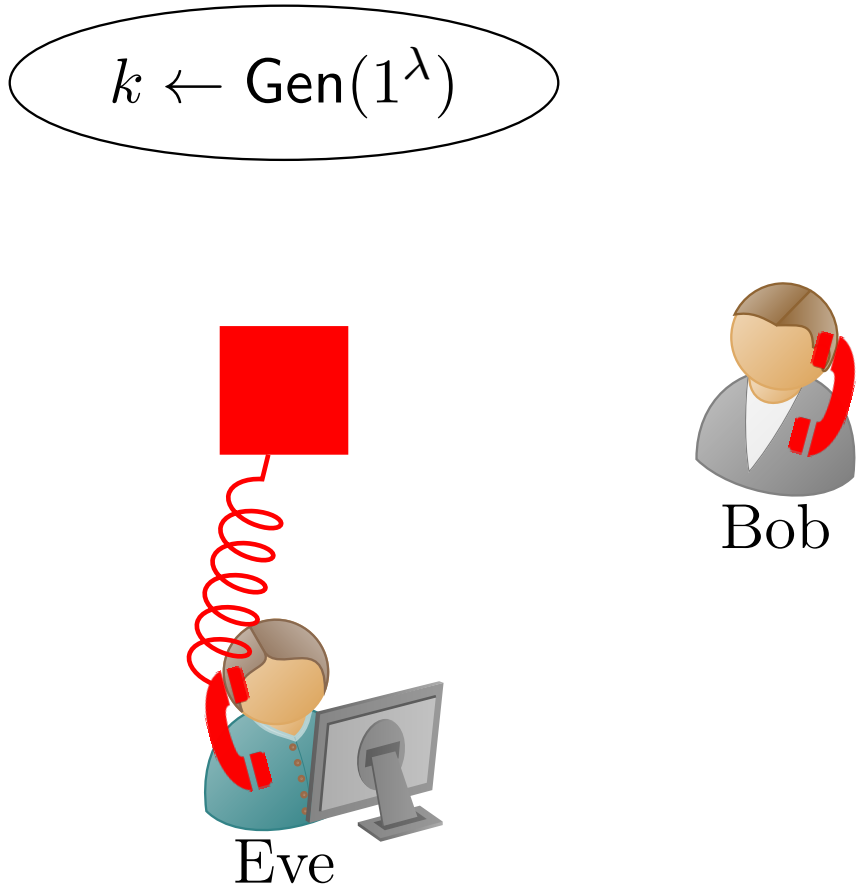

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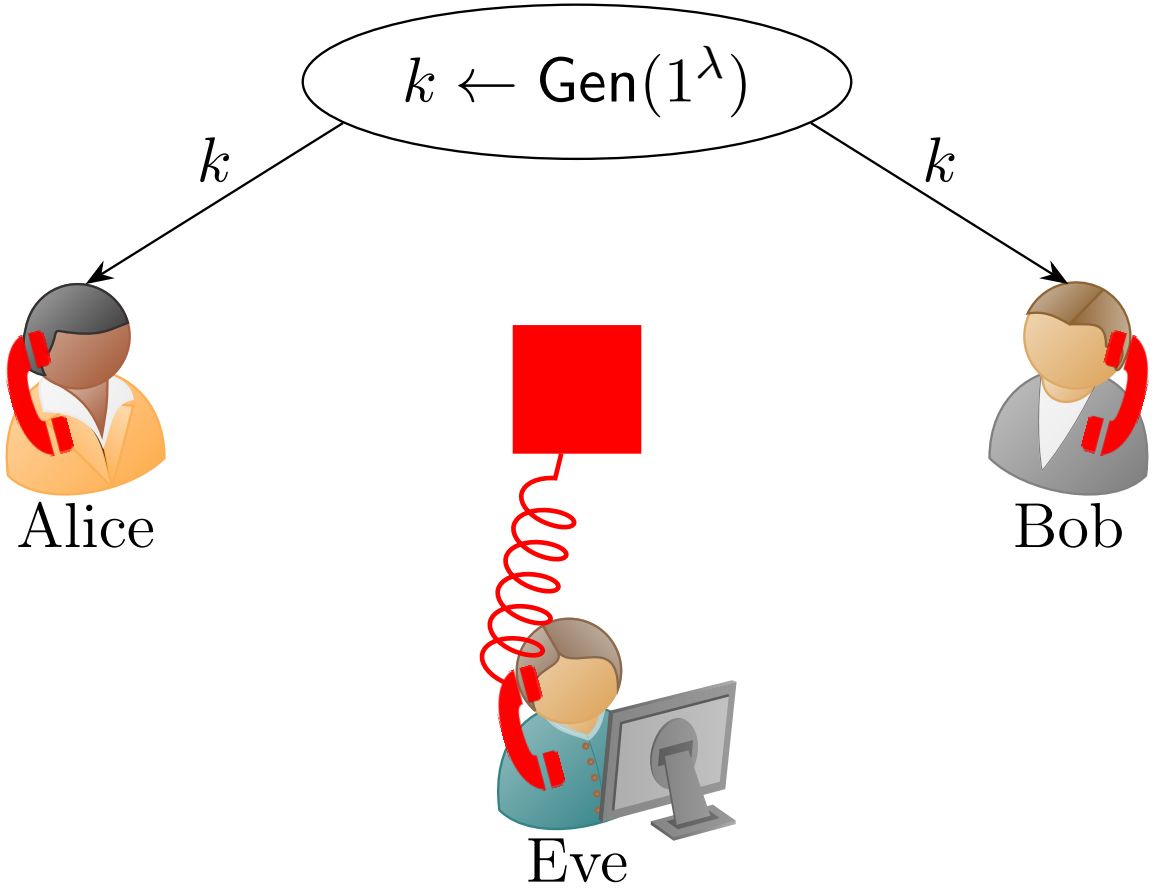

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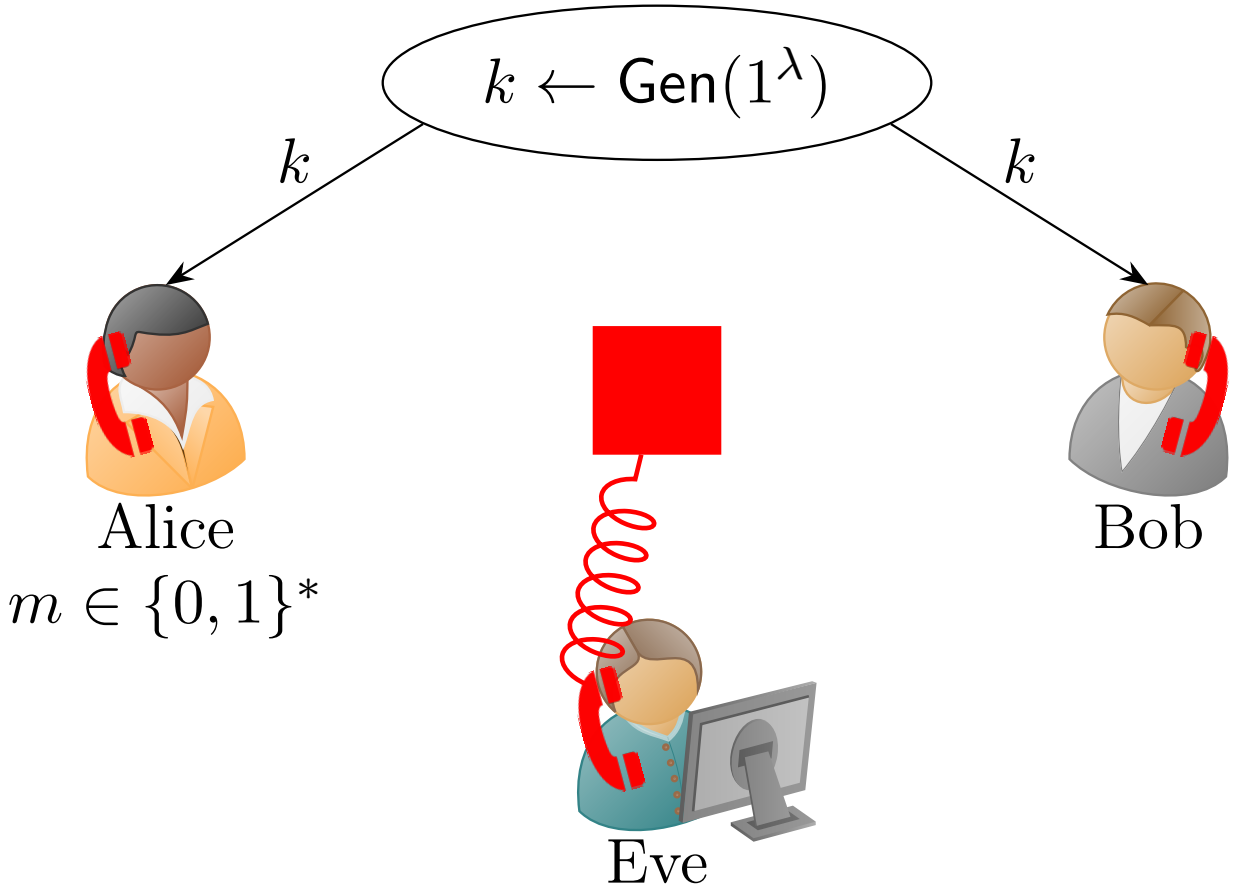

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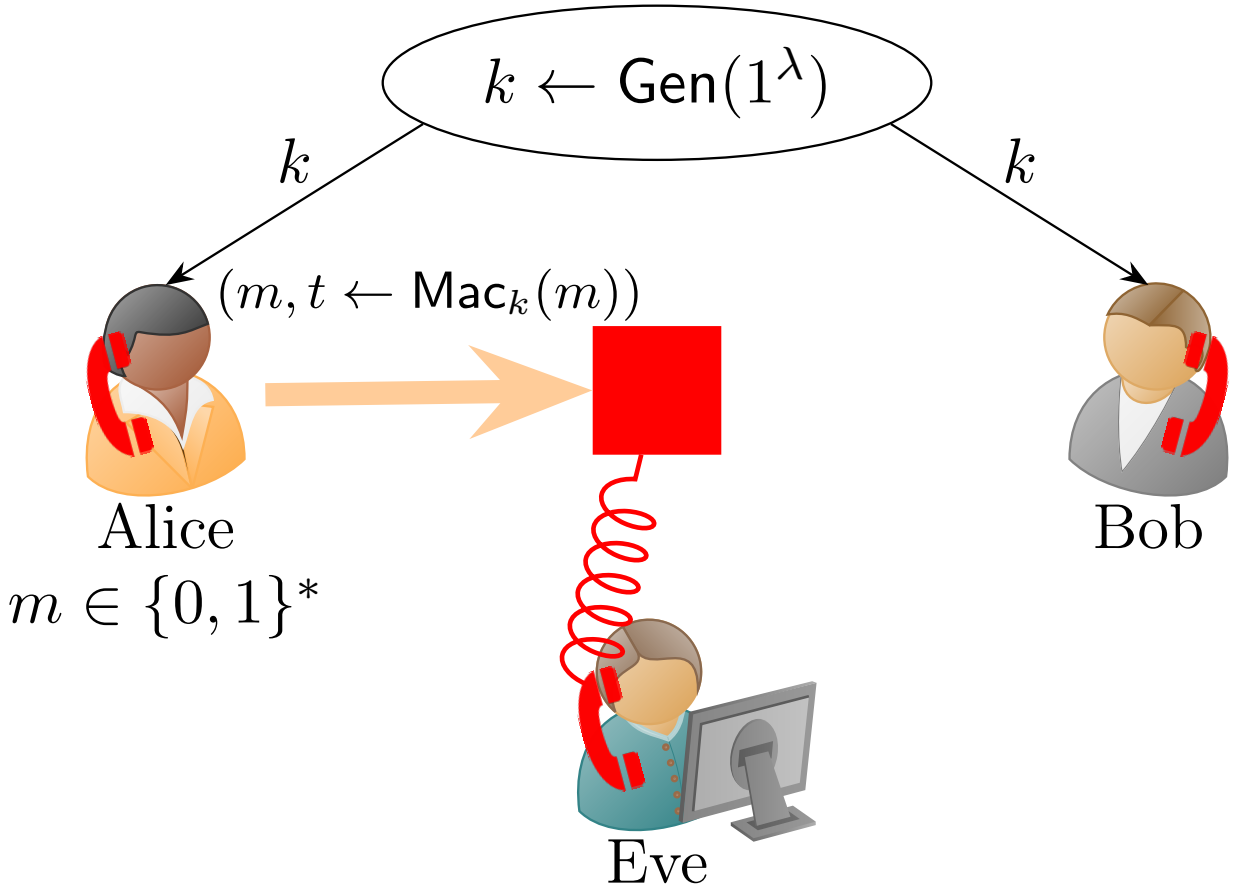

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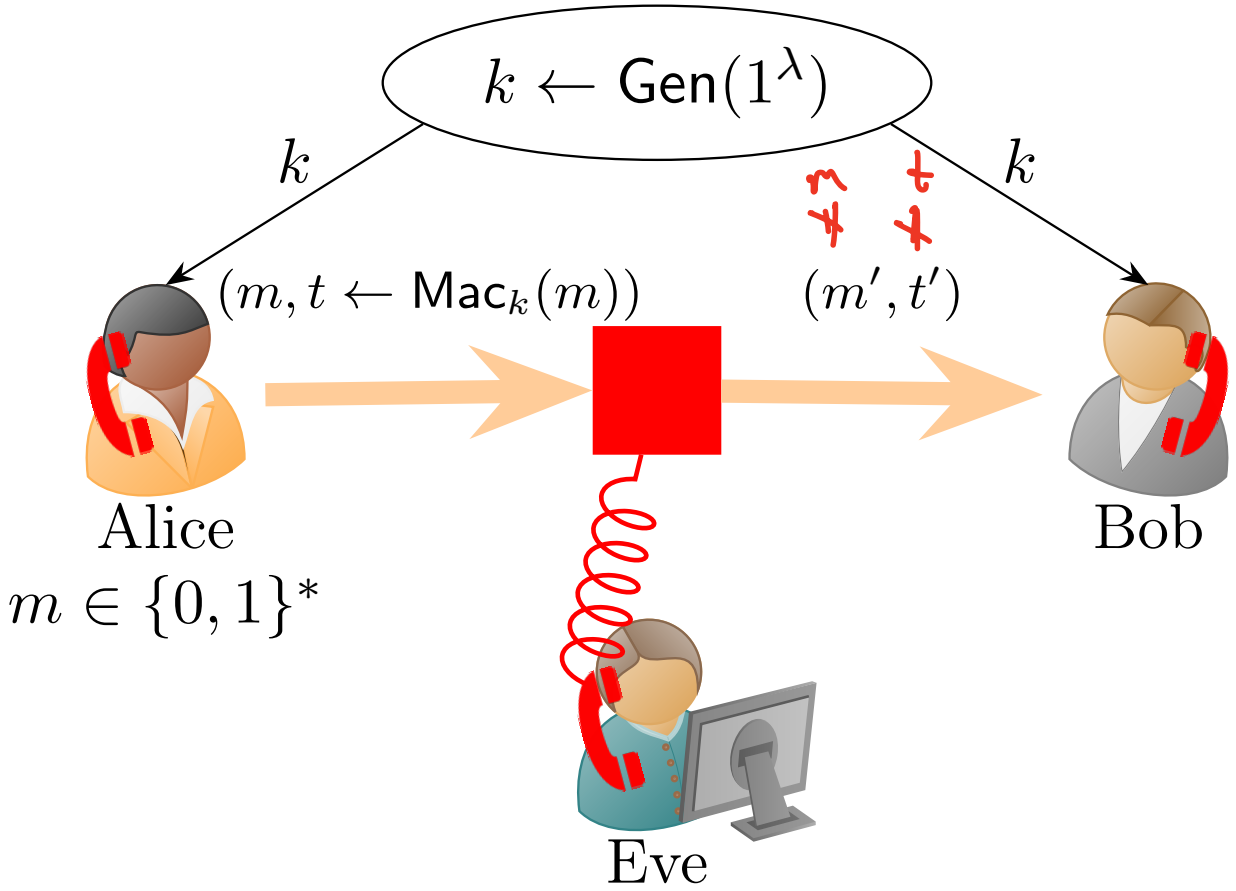

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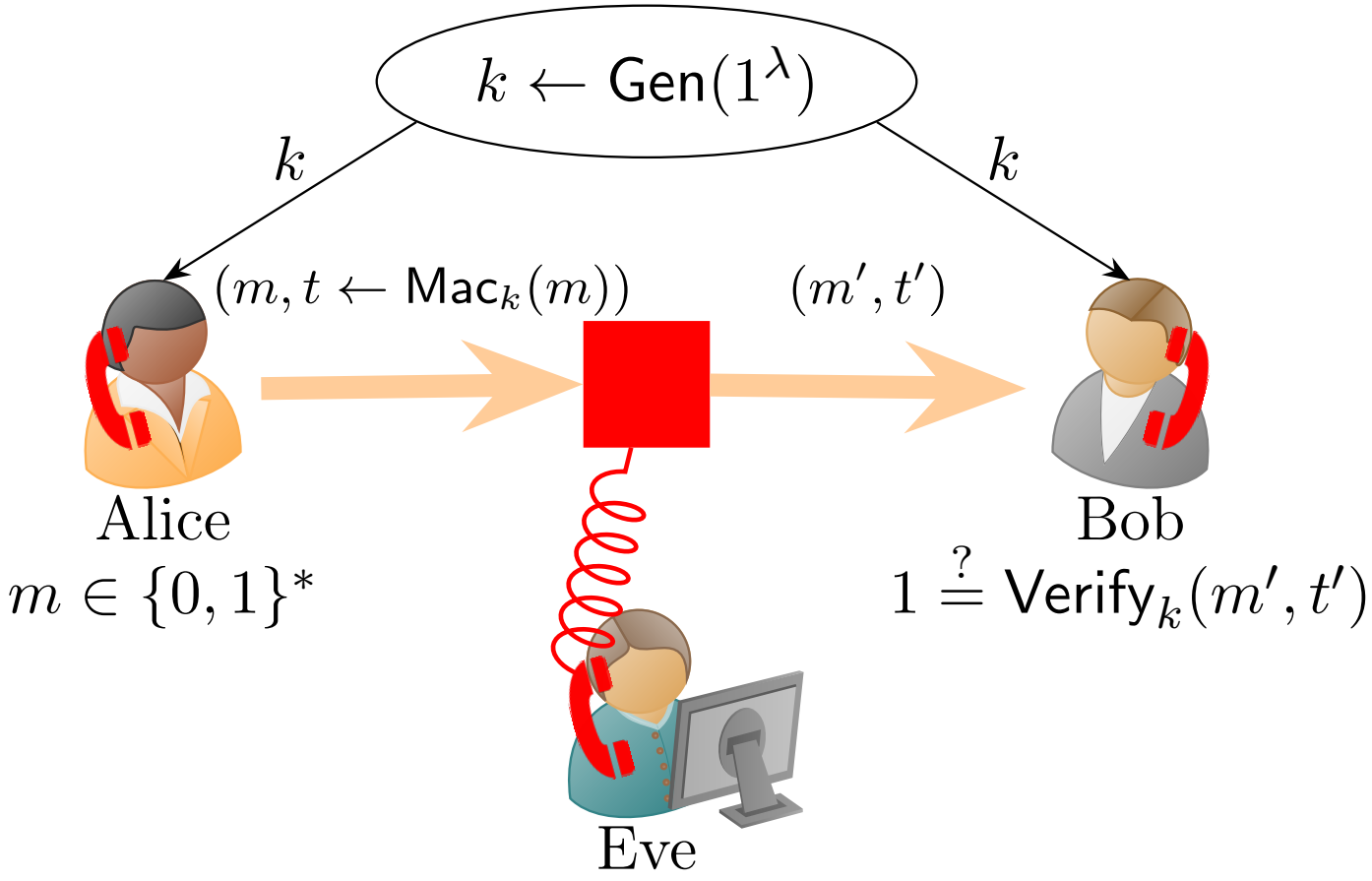

Eve does not have Key K

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- Also known as *existential unforgeability under adaptive chosen-message attacks*

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Mac-forge$_{\mathcal{A},\Pi}(\lambda)$ as a Security Game

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1^λ

$\mathcal{A}$

$\mathcal{C}$

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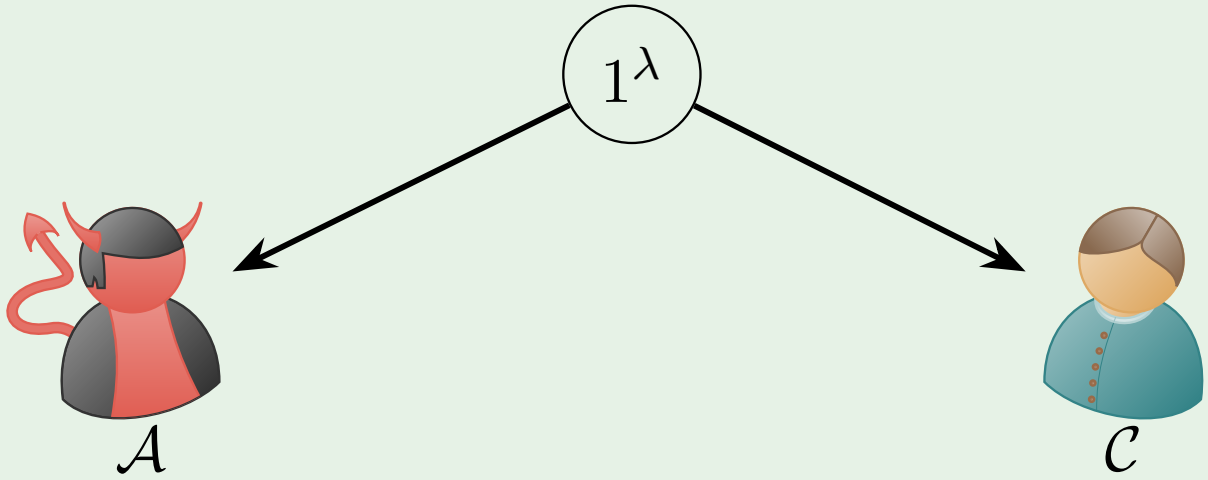

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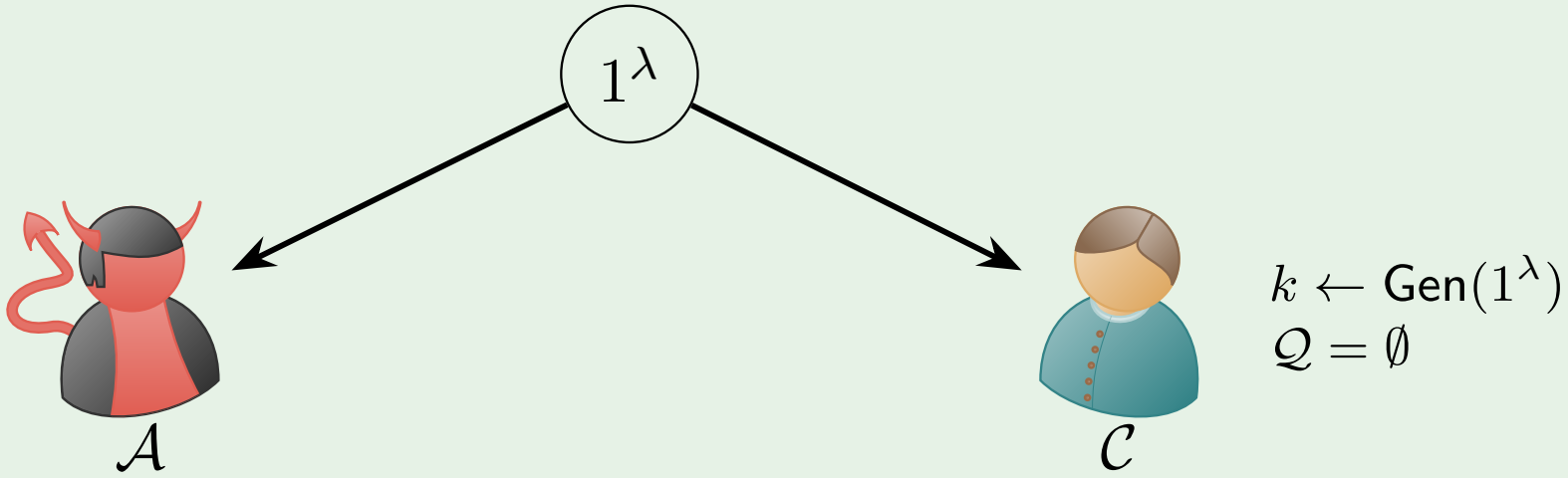

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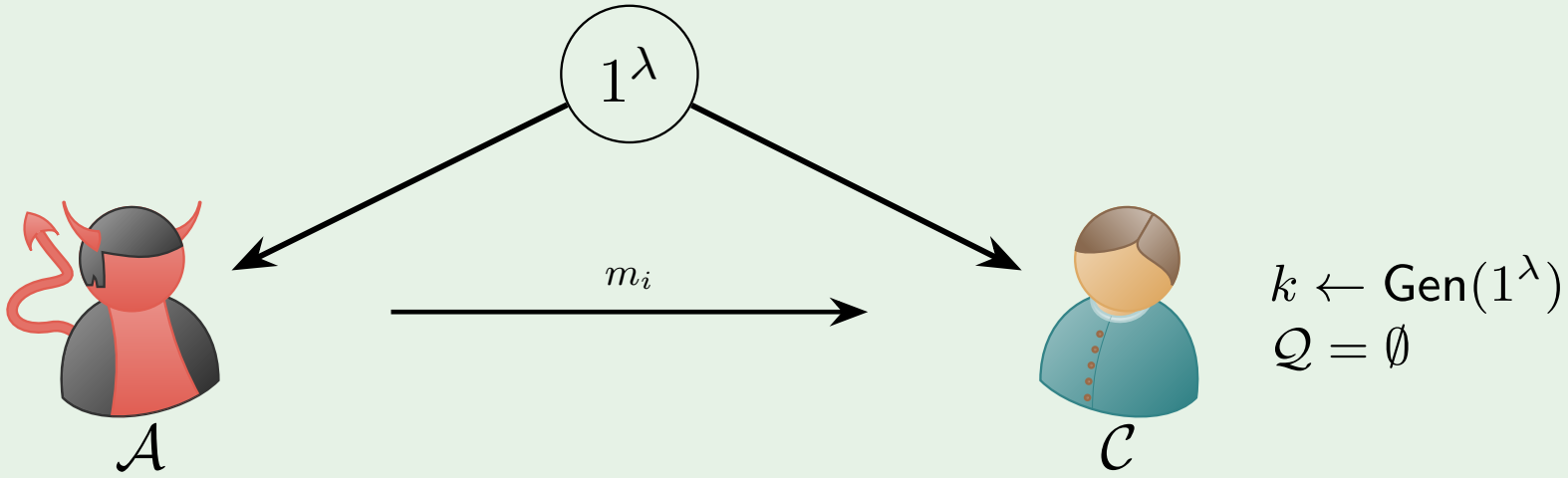

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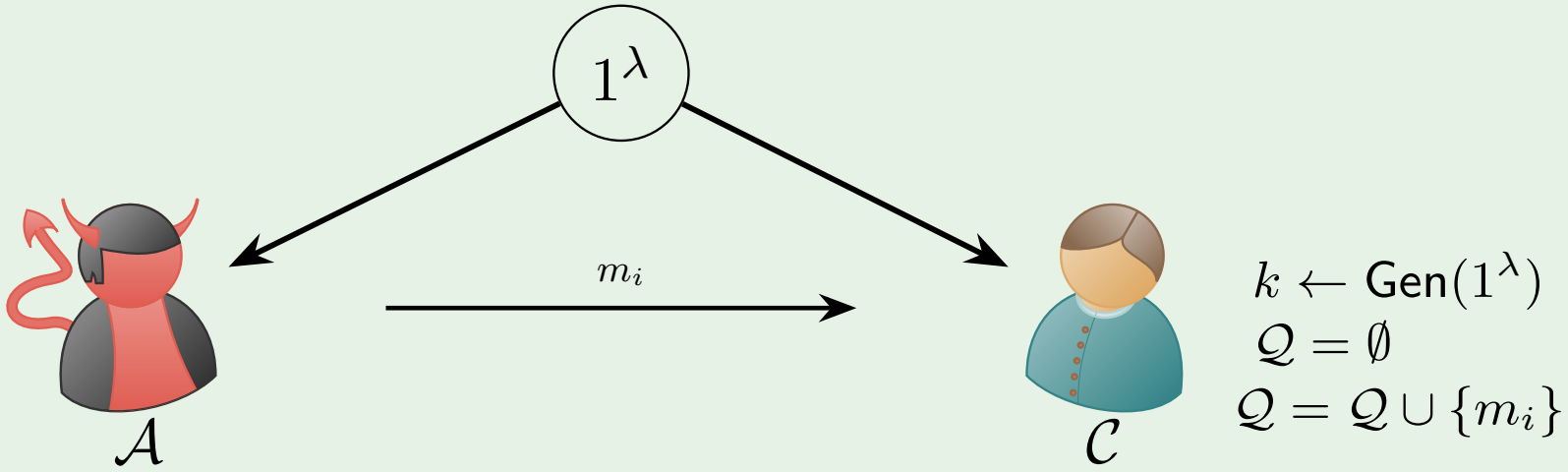

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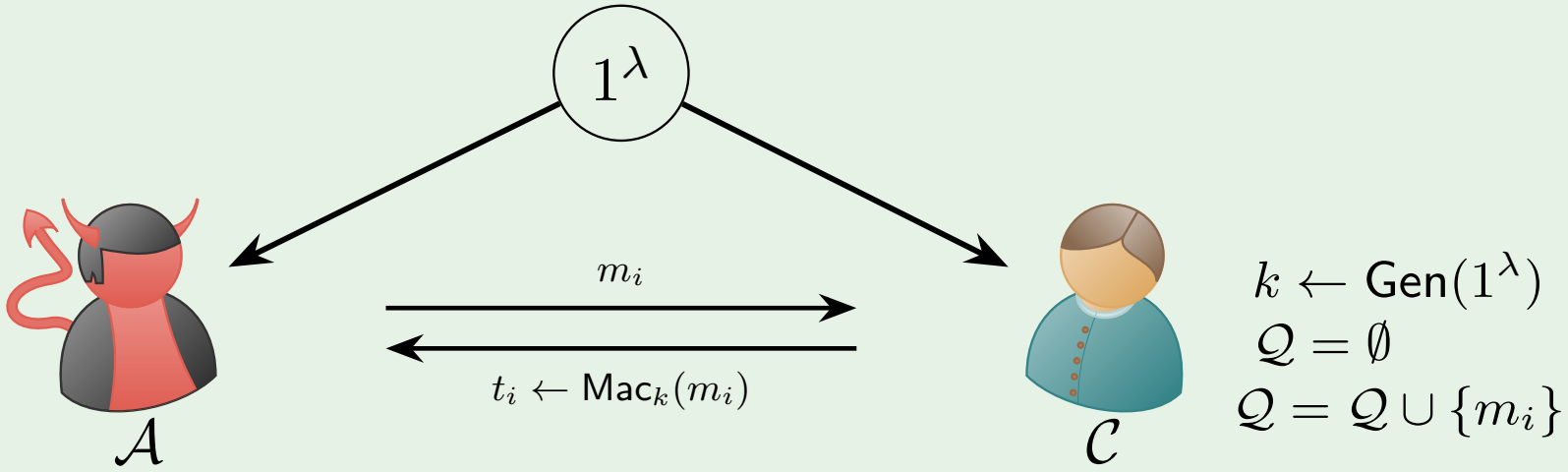

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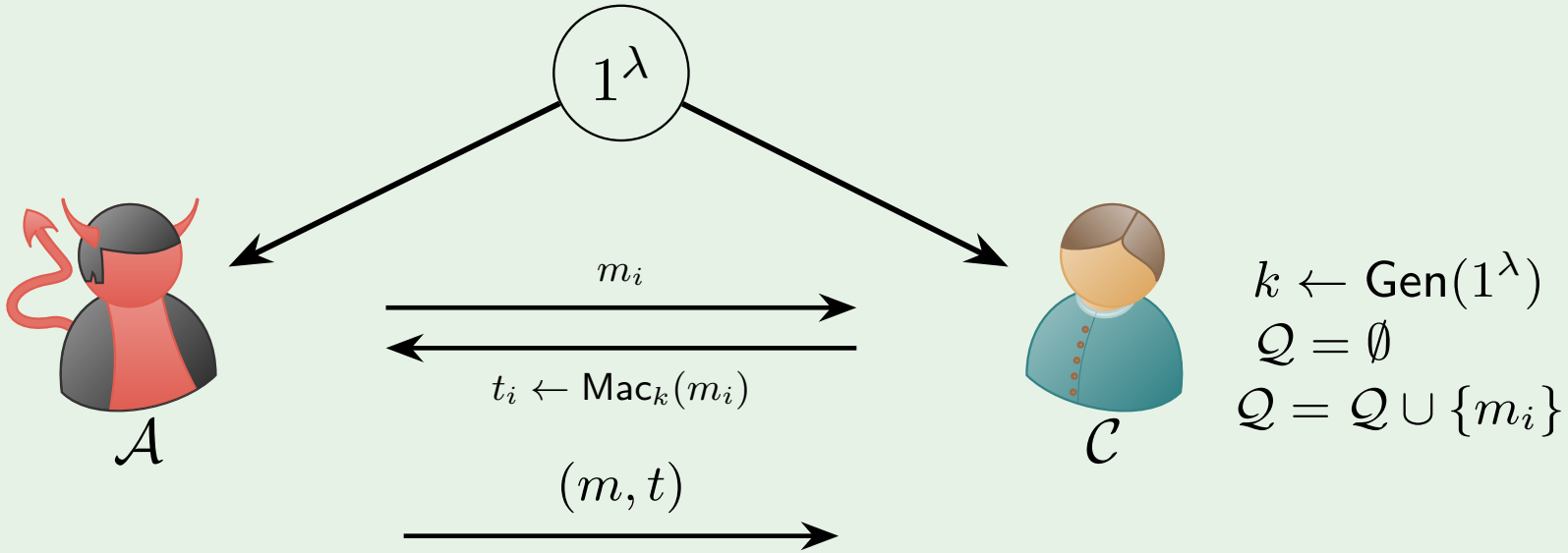

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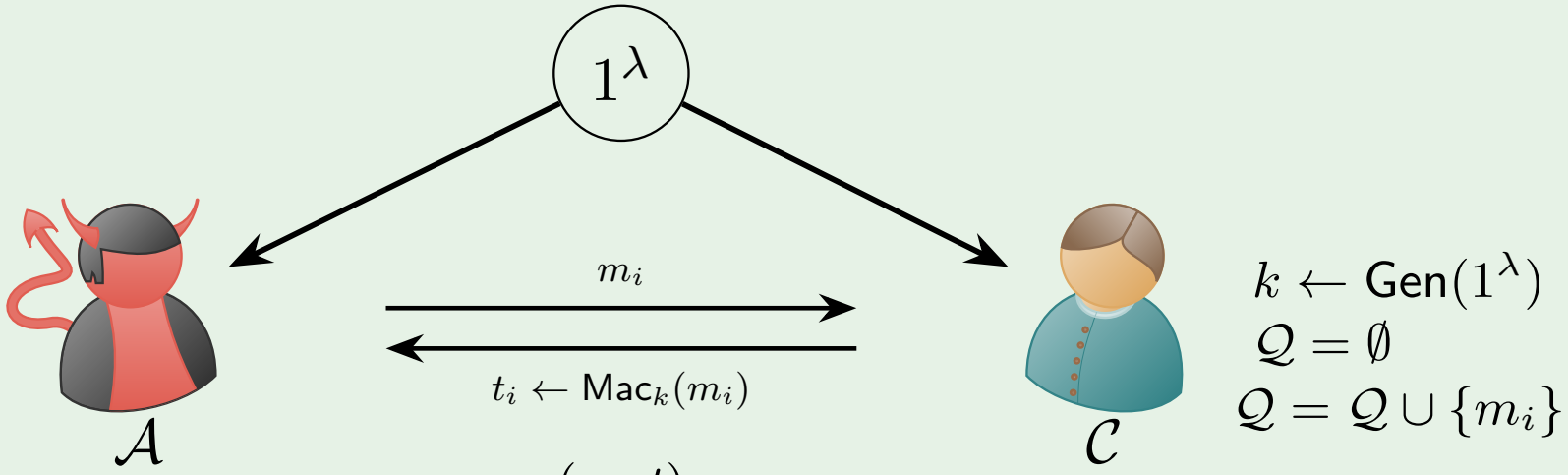

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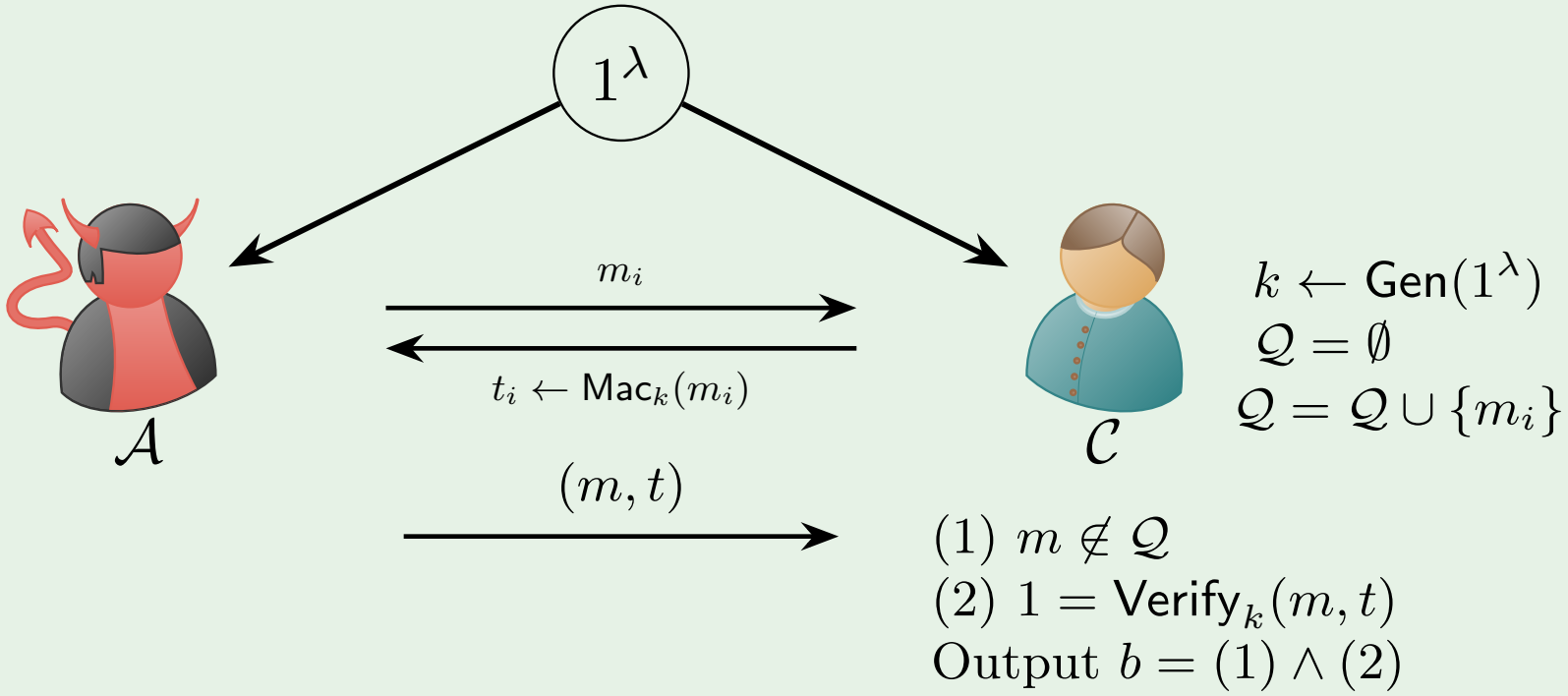

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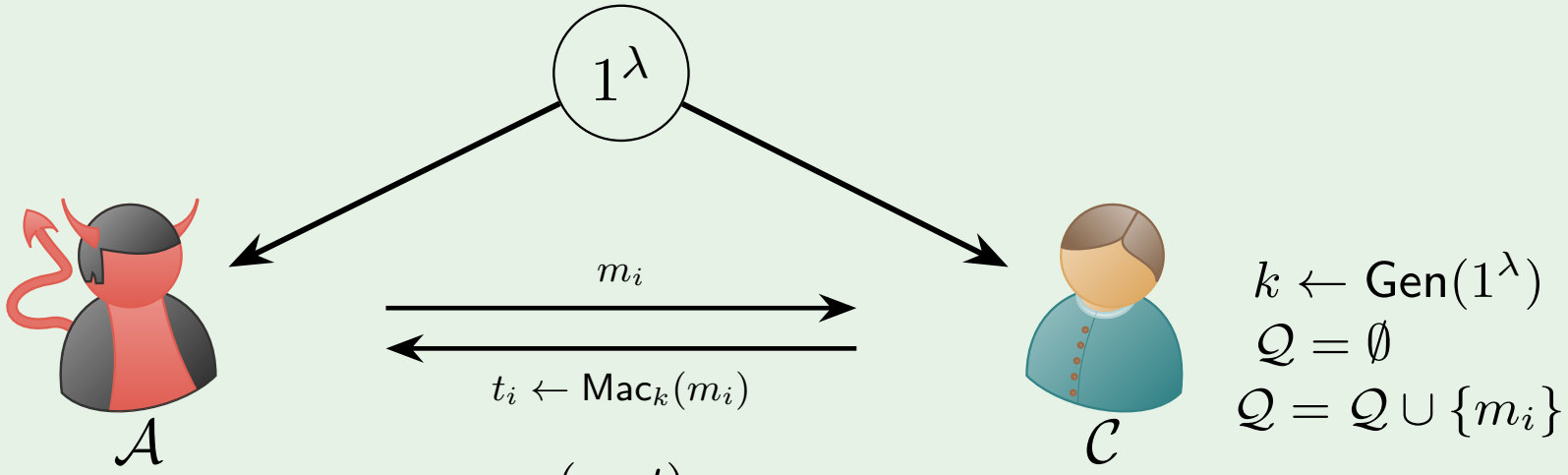

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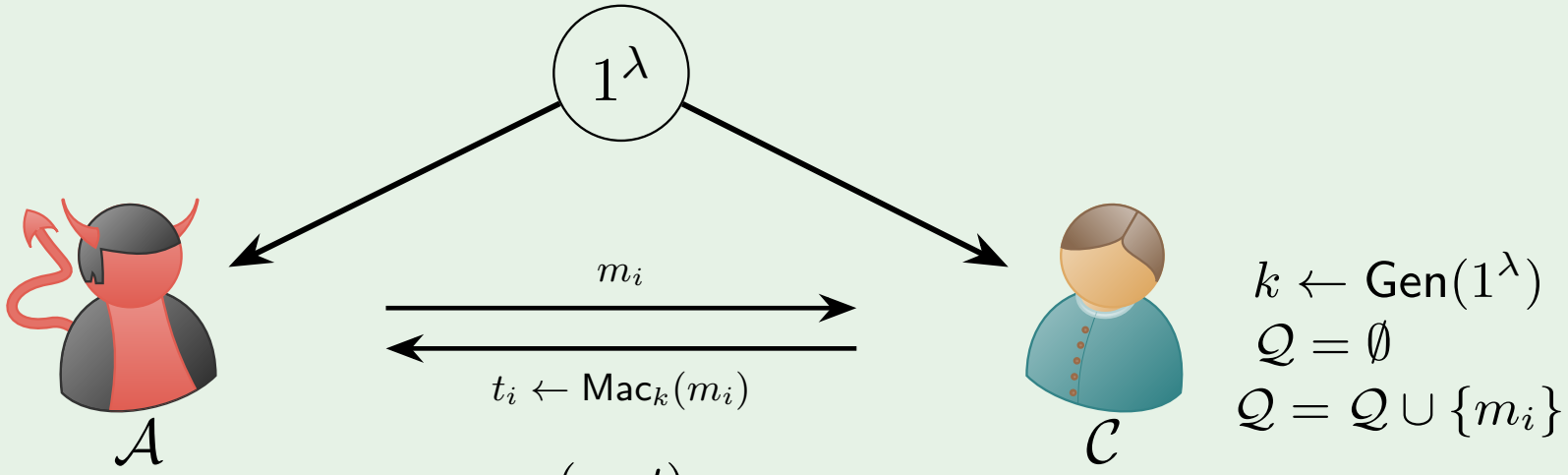

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- How likely is it for $\mathcal{A}$ to win the game?

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  - Caveat 2: hardware continuously improves over time

Concrete Security of MACs

Definition 2 ((t, q, ε)-secure MAC)

A MAC $\Pi = (\mathsf{Gen}, \mathsf{Mac}, \mathsf{Verify})$ is (t, q, ε)-secure if for every $\lambda \in \mathbb{Z}^+$ and every adversary $\mathcal{A}$ running in time at most $t(\lambda)$ and making at most $q(\lambda)$ oracle queries/challenges, it holds that

$$\Pr[\mathsf{Mac\text{-}forge}_{\mathcal{A},\Pi}(\lambda) = 1] \leqslant \varepsilon(\lambda).$$

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  - Don't have to worry about hardware improvements.

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- We just need to choose the security parameter λ large enough for the adversary to basically have no chance of winning!