CS 594 – Advanced Crypto (Spring 2026)

Alex Block

Lecture 4

January 28, 2026

Last Time: Random Oracle Model

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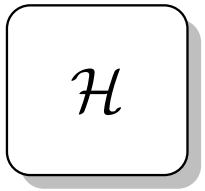
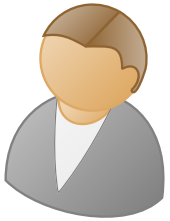


Last Time: Random Oracle Model

$\mathcal{H}$

Alice



Bob

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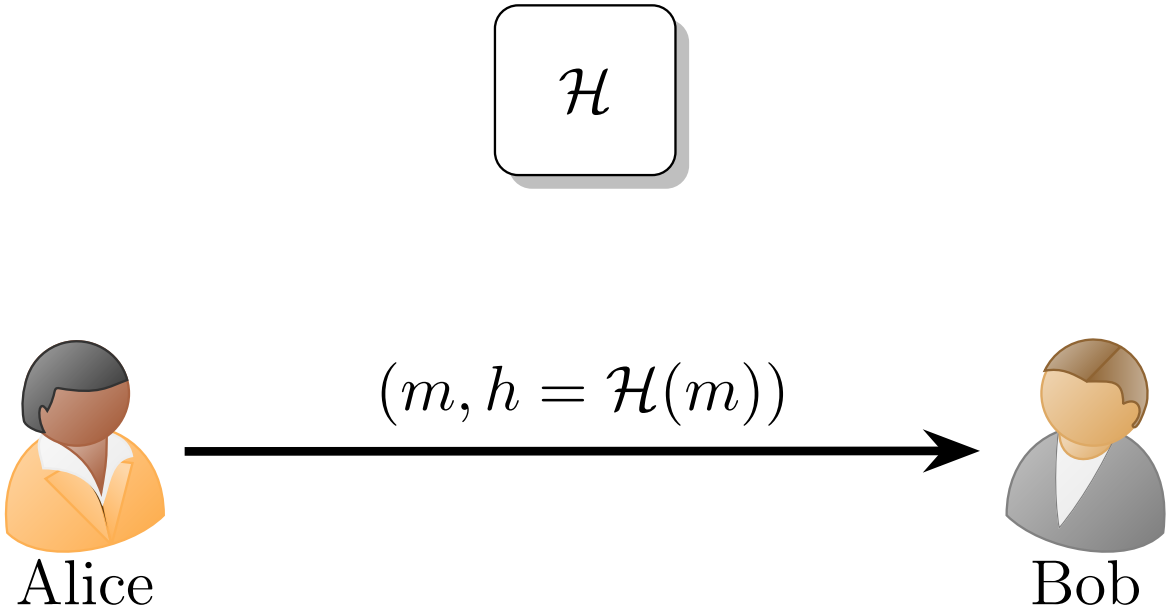

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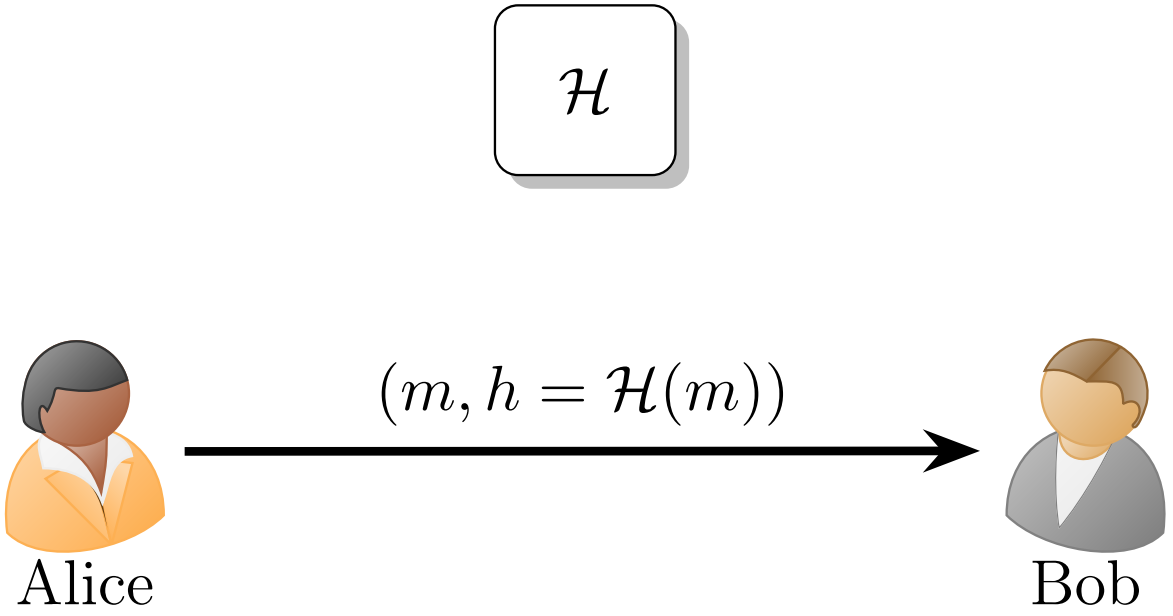

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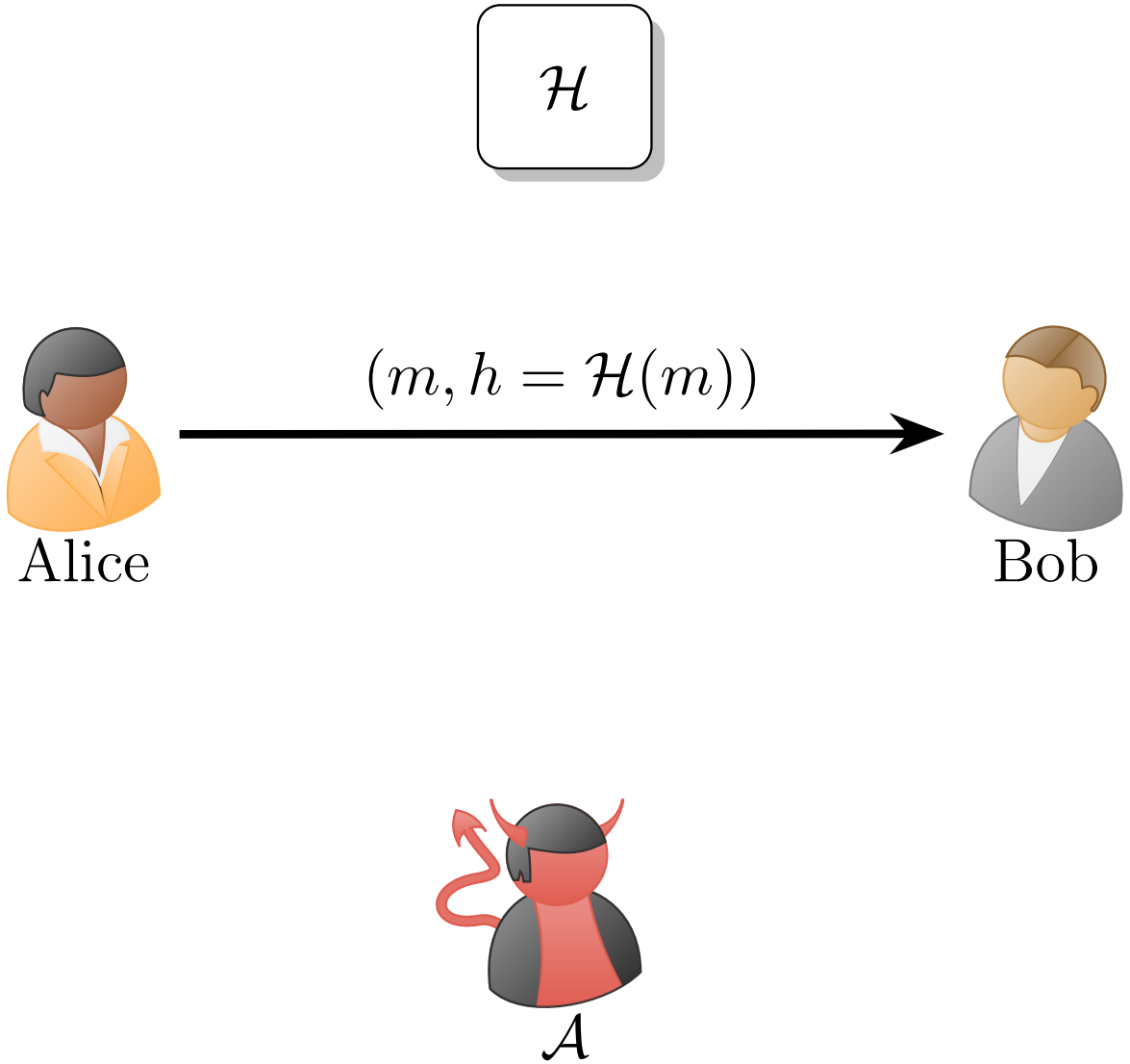

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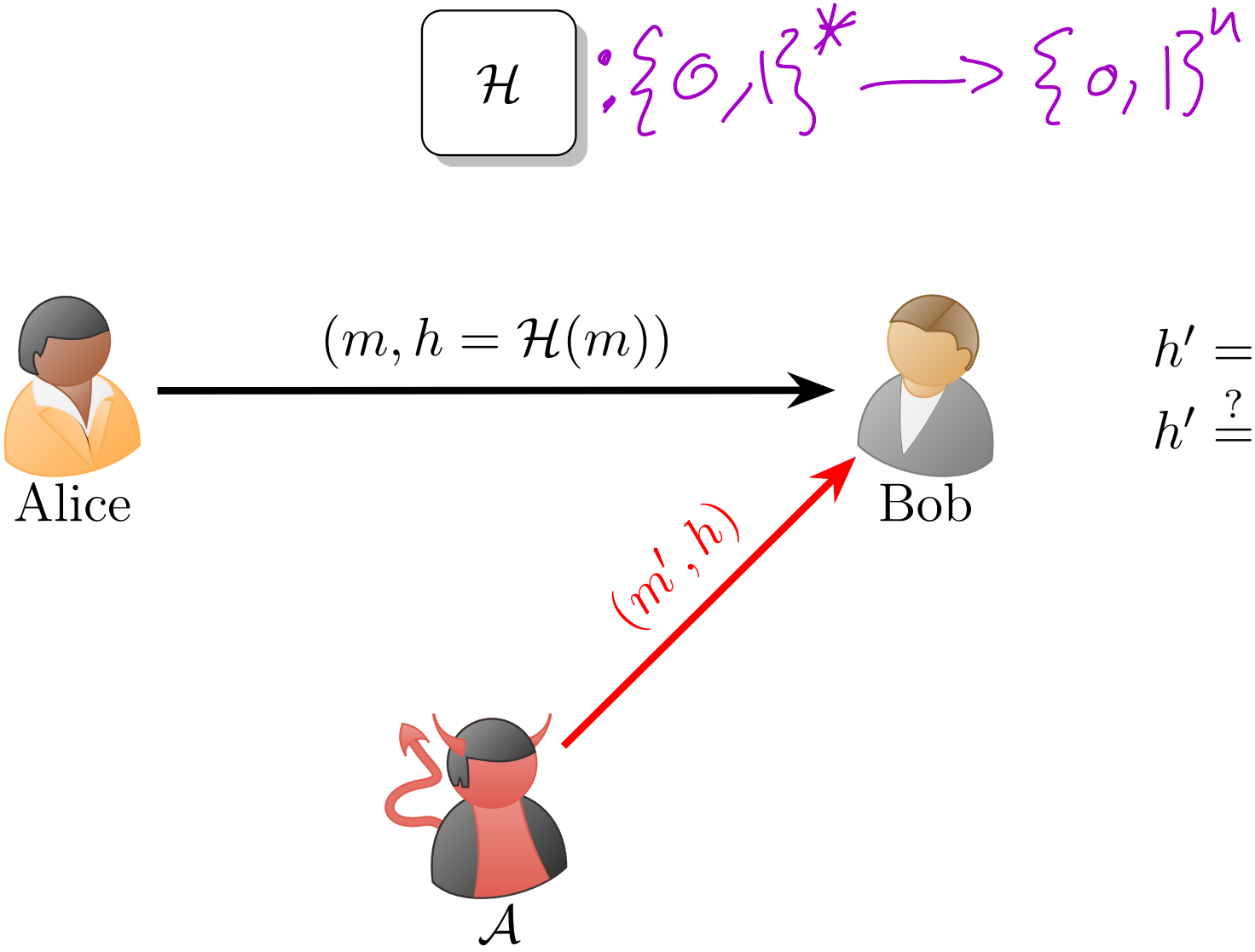

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$\underline{H(y)\text{:}}$
If $H[y]$ undefined
$H[y] \xleftarrow{\$} \{0,1\}^n$
return $H[y]$

Pseudorandom Functions

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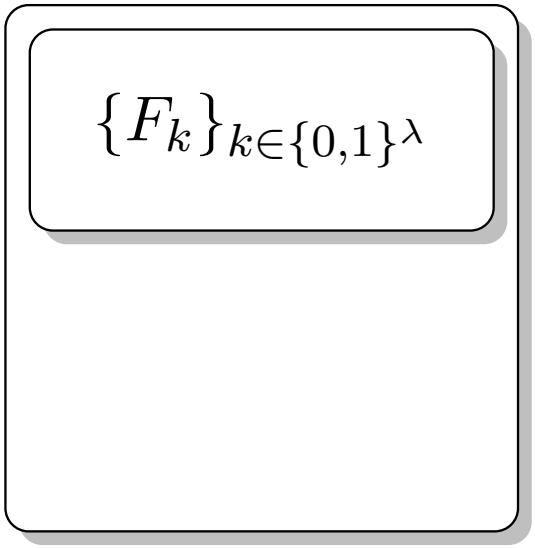

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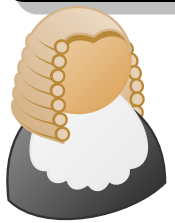

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- If D_0 and D_1 are computationally indistinguishable, we write $D_0 \approx D_1$.

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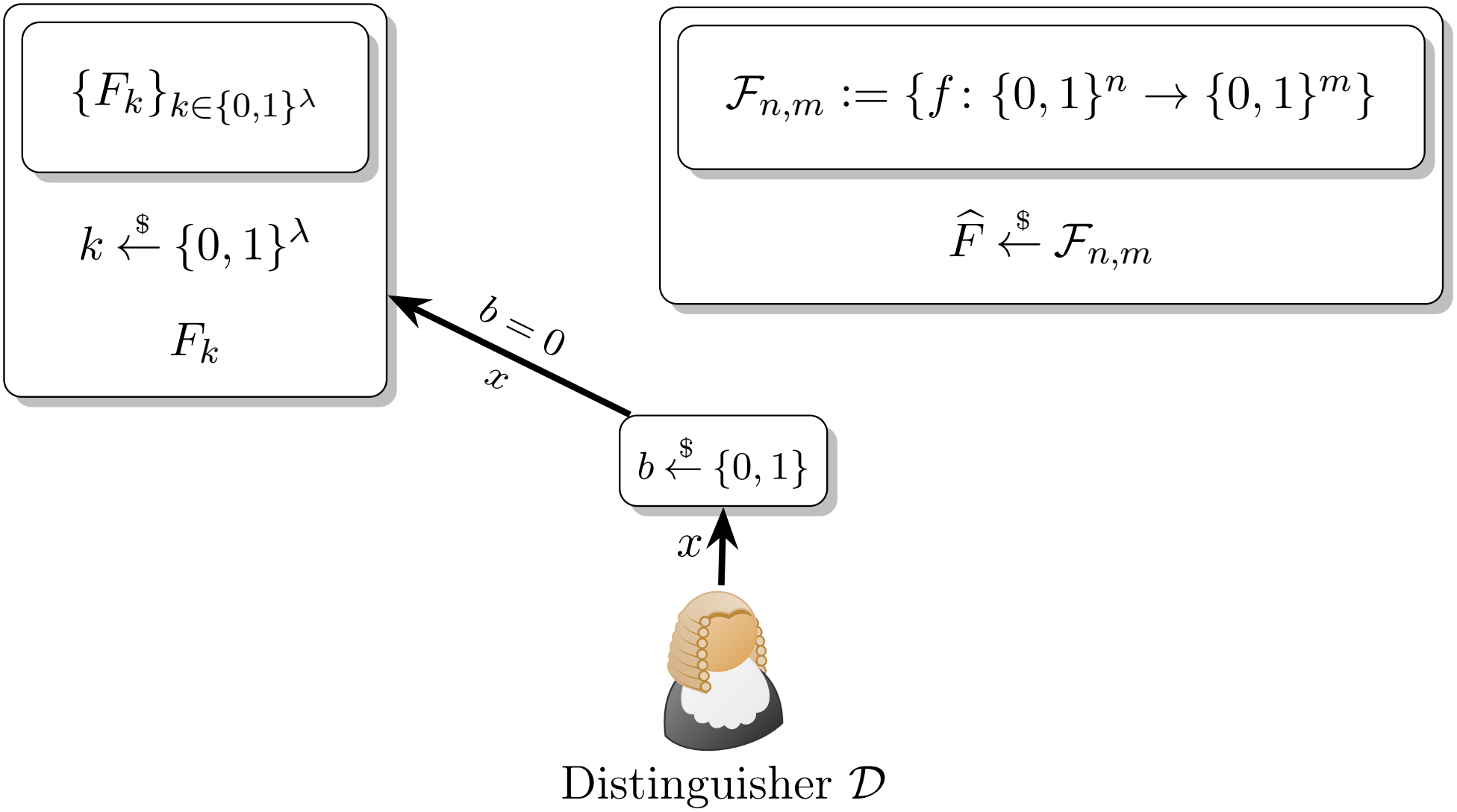

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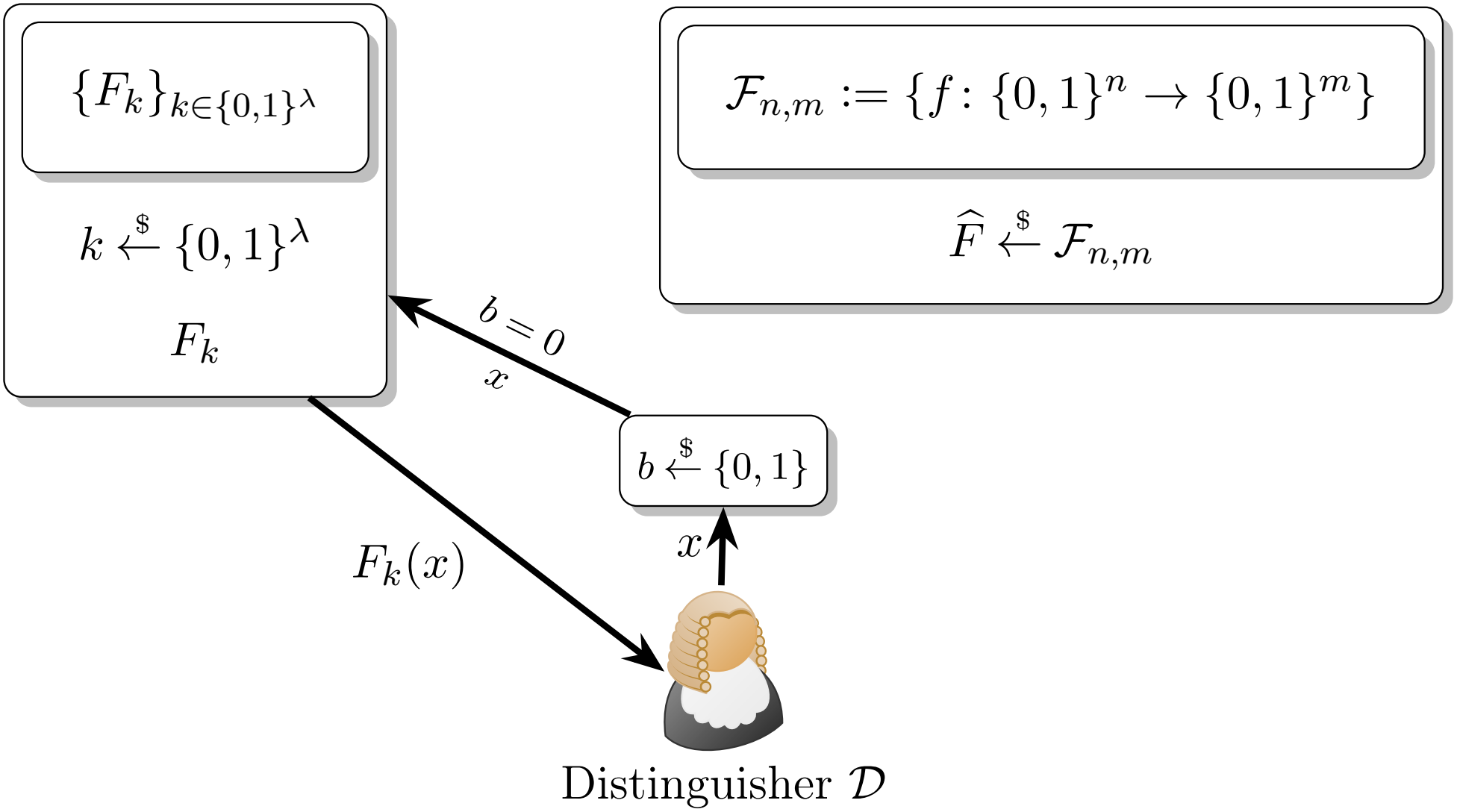

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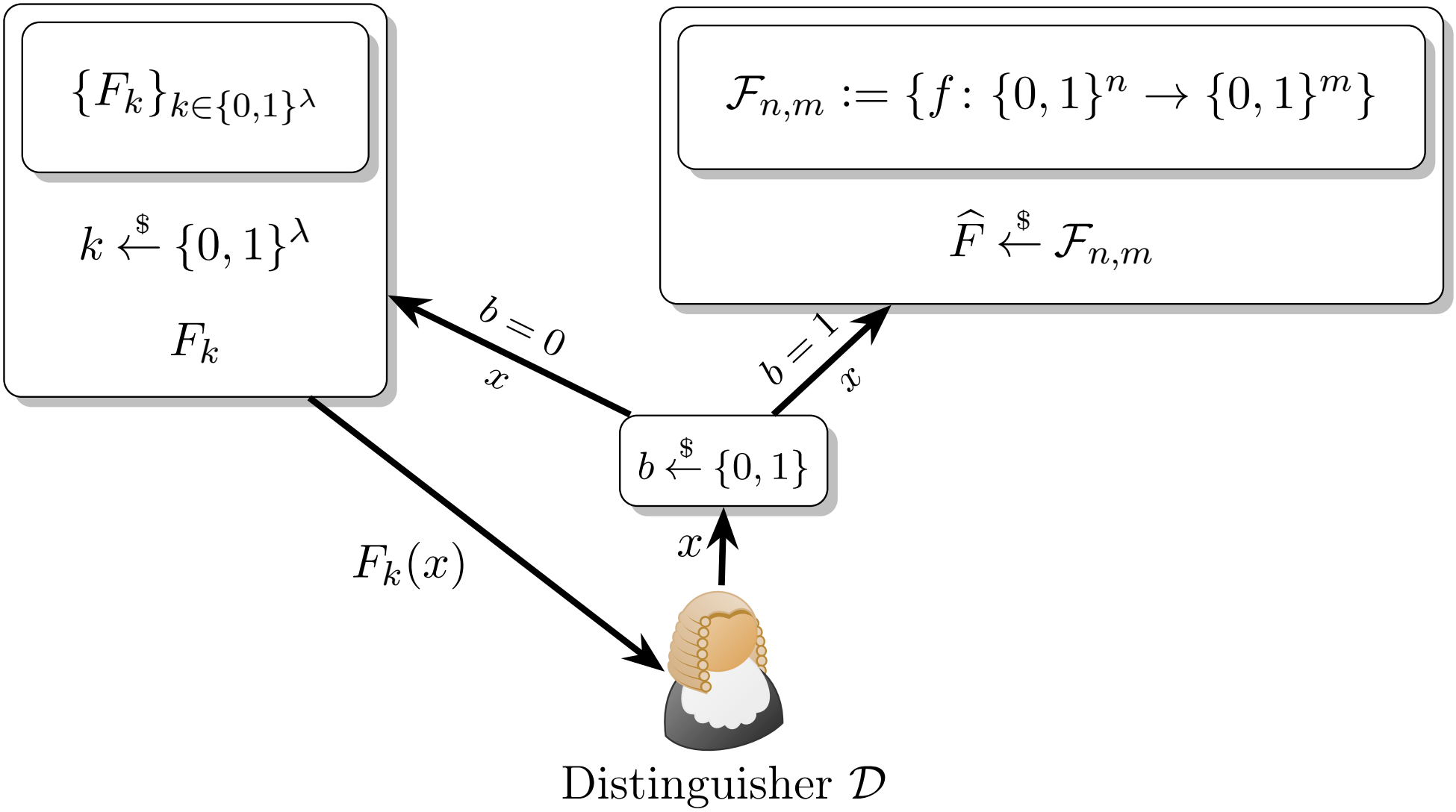

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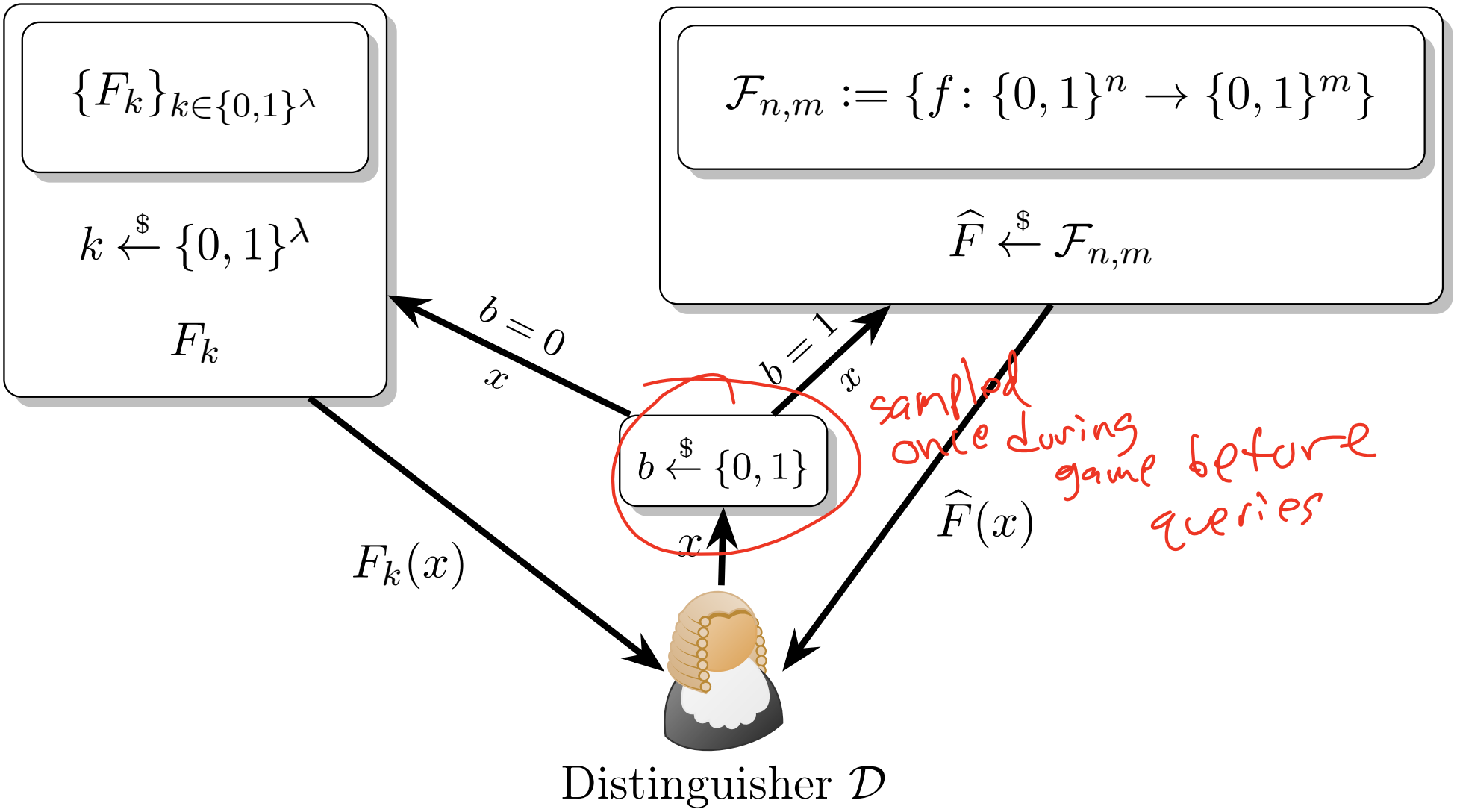

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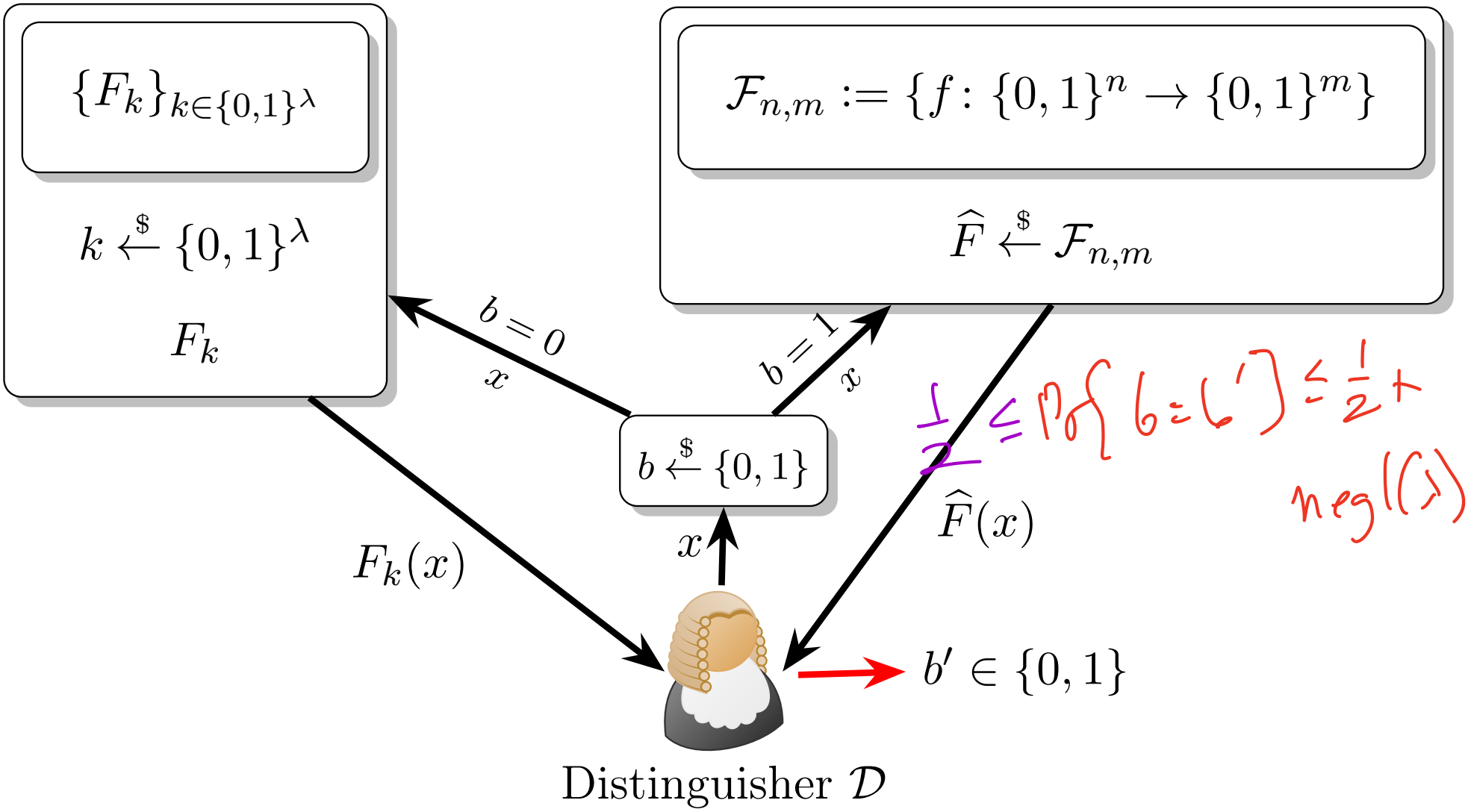

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  - In light of this, if $\mathcal{D}$ is given a function it can *read*, then it can output $b' = 0$ and win the game with noticeable probability.

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$$\{F_k^H : \{0,1\}^n \to \{0,1\}^m\}_{k \in \{0,1\}^\lambda} \qquad F_k^H(x) := H(k \| x)$$

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So a function family $\{F_k : \{0,1\}^n \to \{0,1\}^m\}_{k \in \{0,1\}^\lambda}$ is a secure PRF family if it is computationally indistinguishable from D_1.

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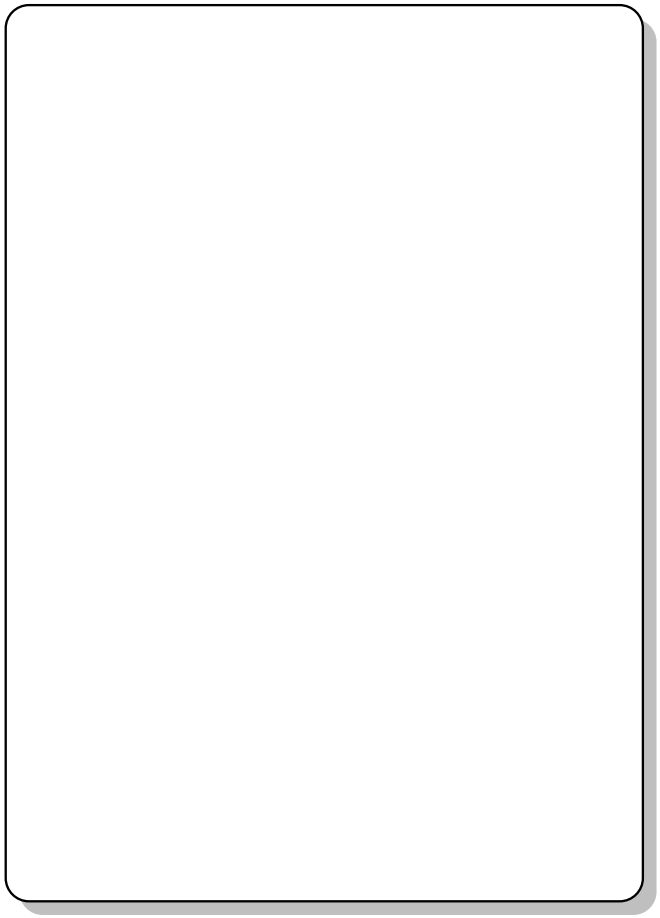

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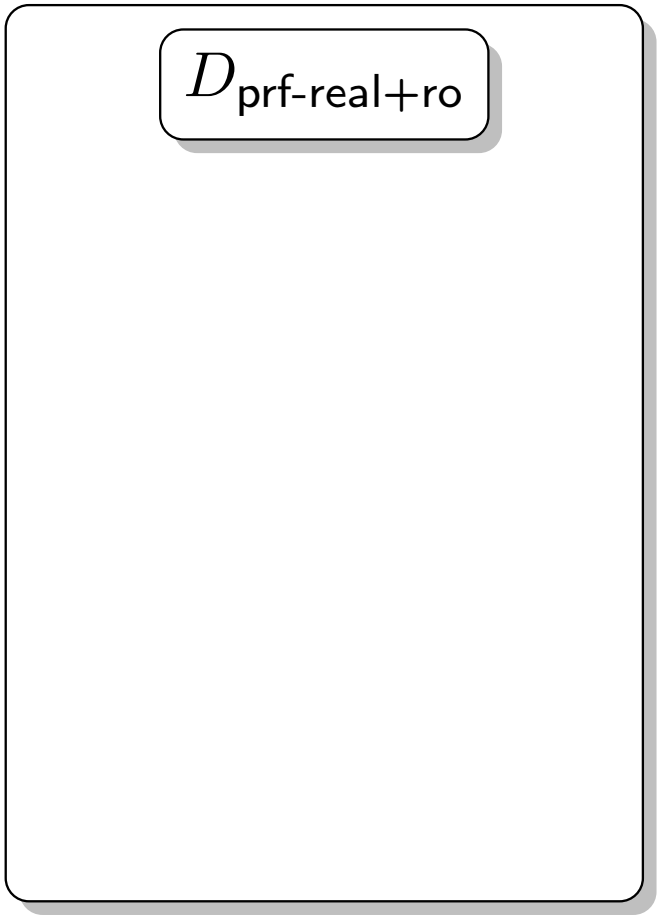

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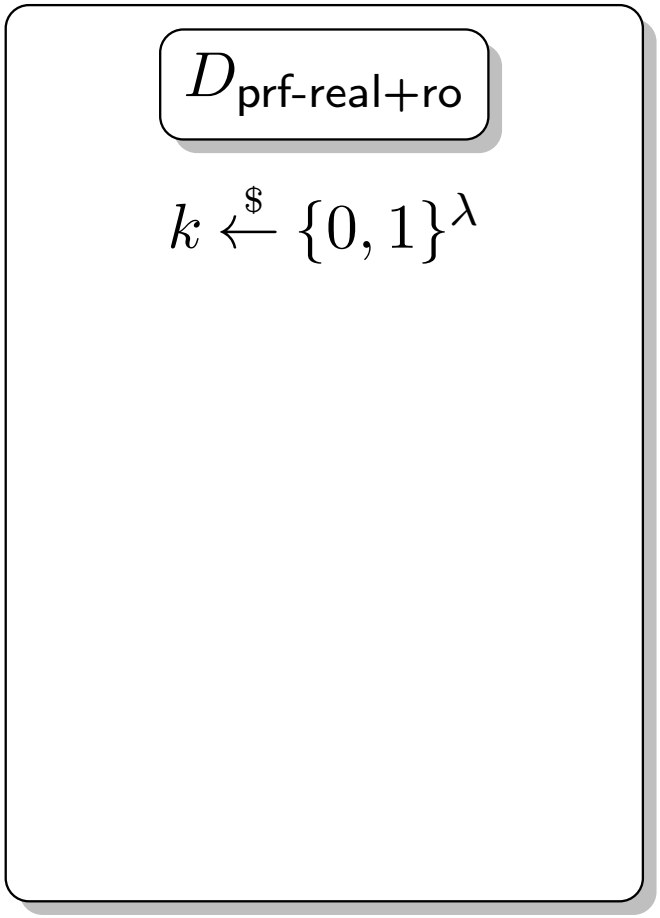

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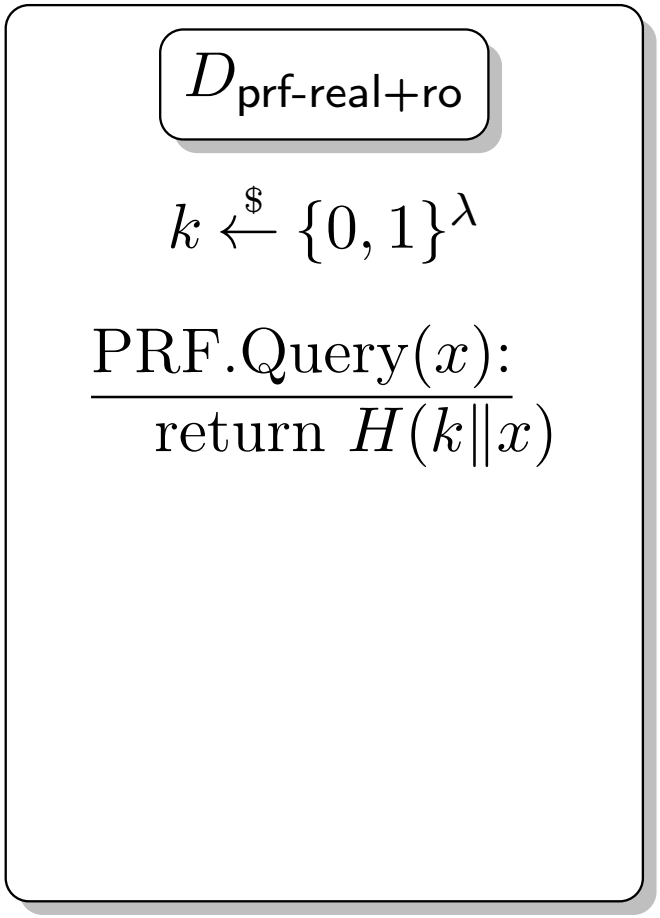

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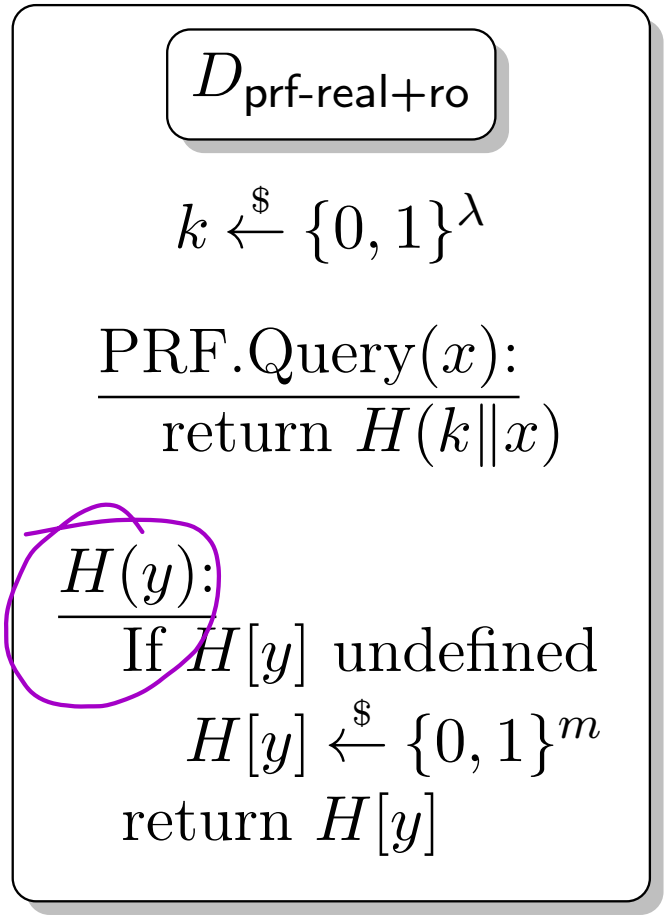

Everyone has access to this

PRFs from Random Oracles

To prove the lemma, we show the following distributions are indistinguishable

$D_{\mathsf{prf\text{-}real+ro}}$

$k \xleftarrow{\$} \{0,1\}^\lambda$

PRF.Query(x):
return $H(k \| x)$

$H(y)$:
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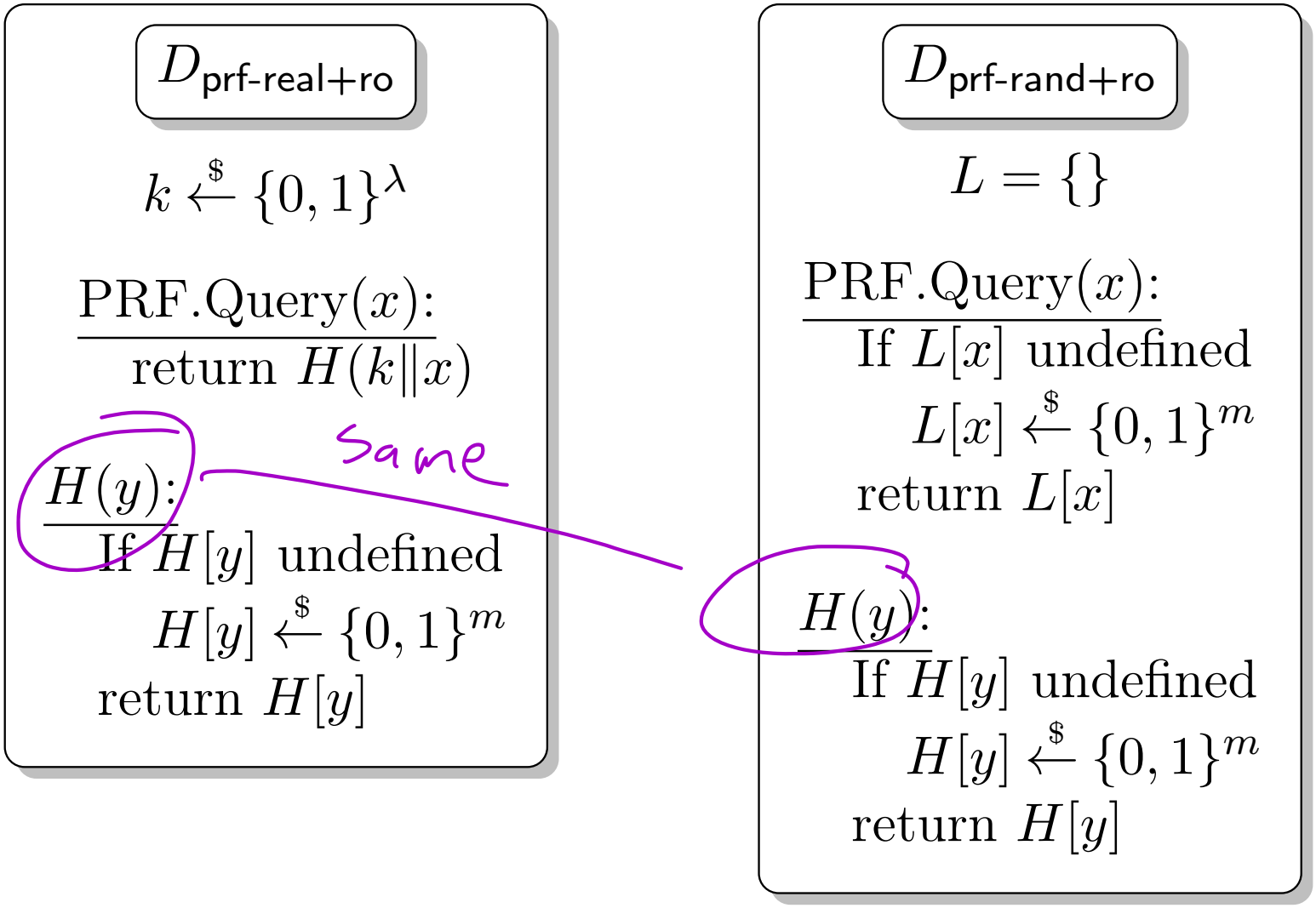

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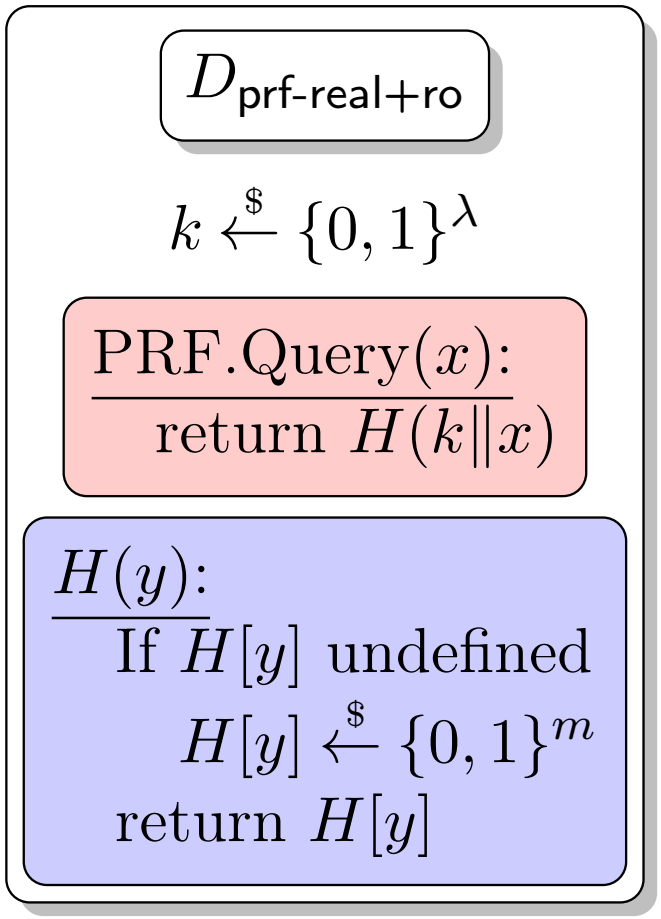

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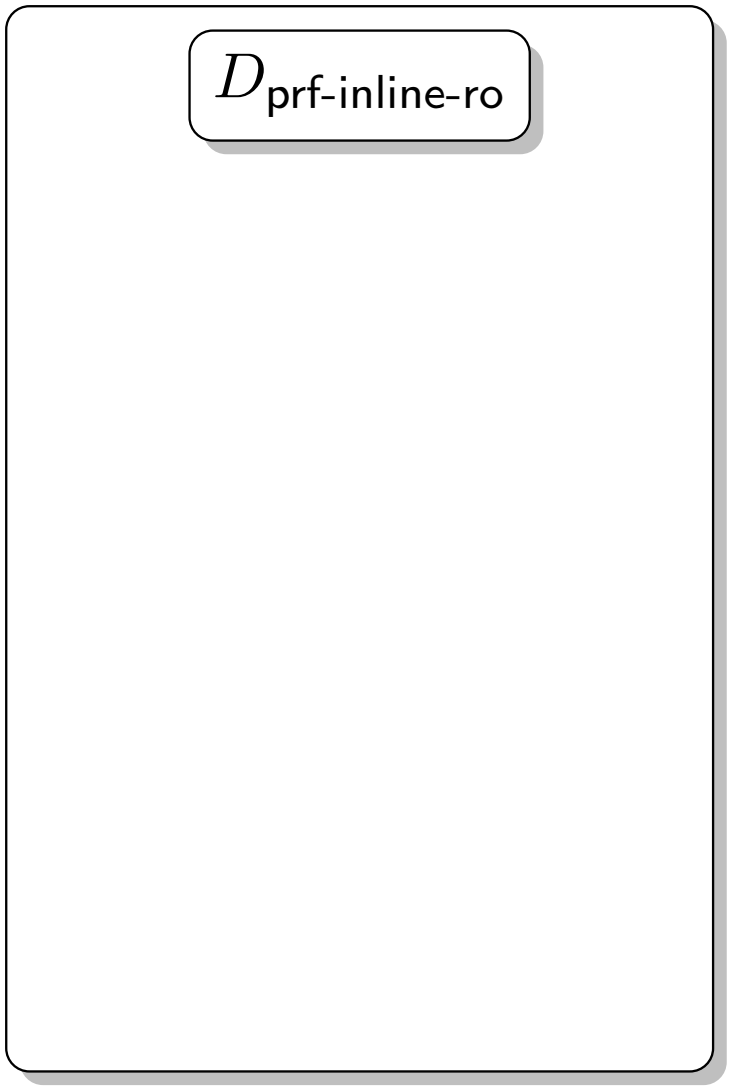

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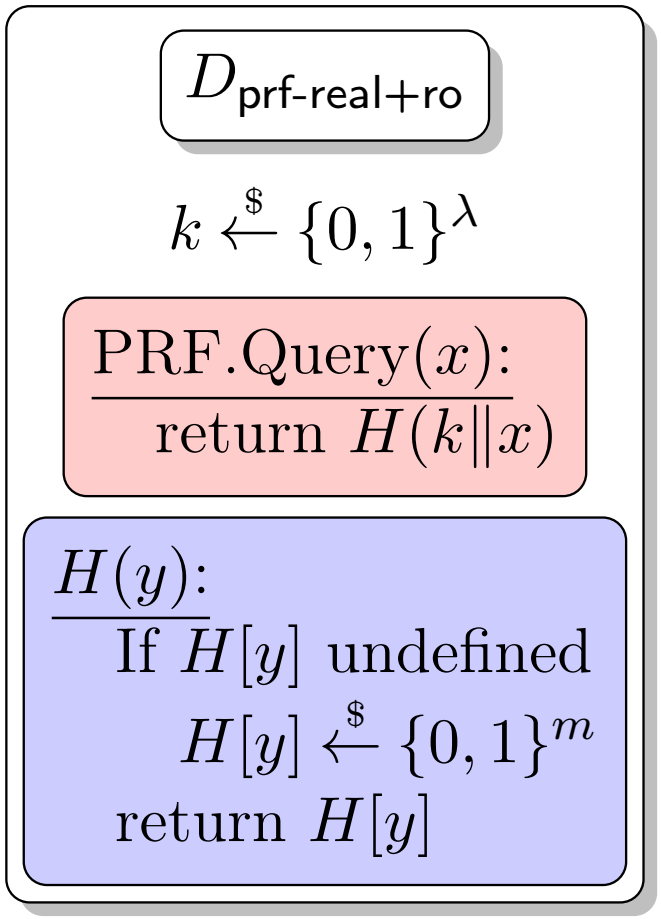

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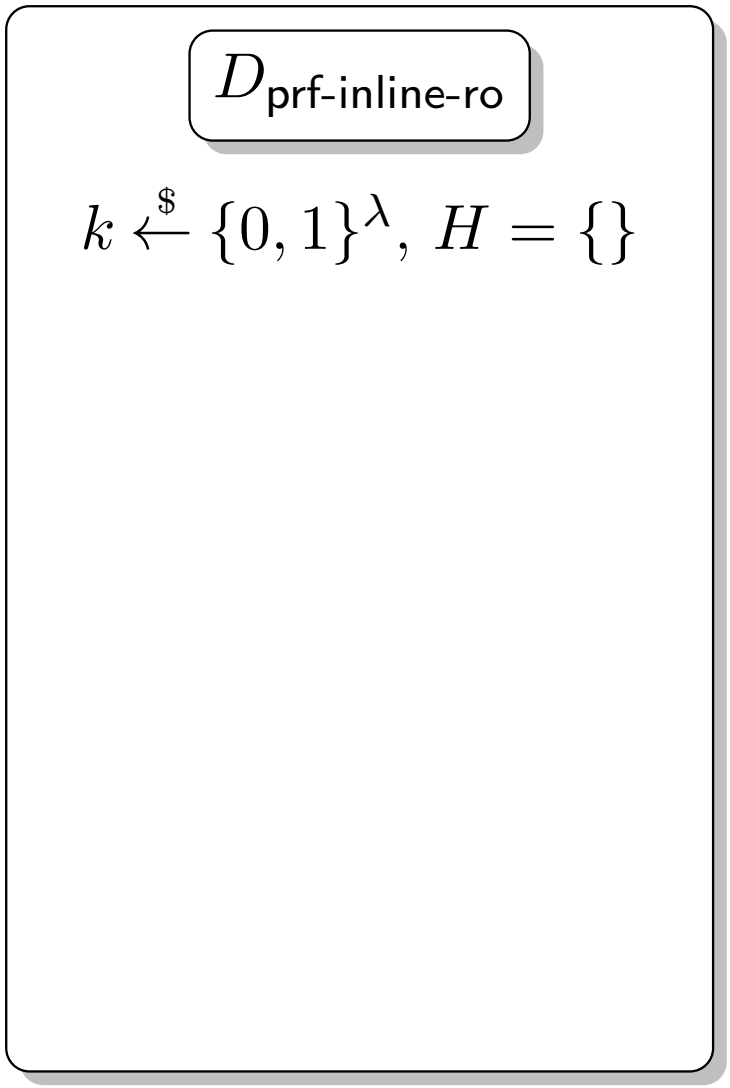

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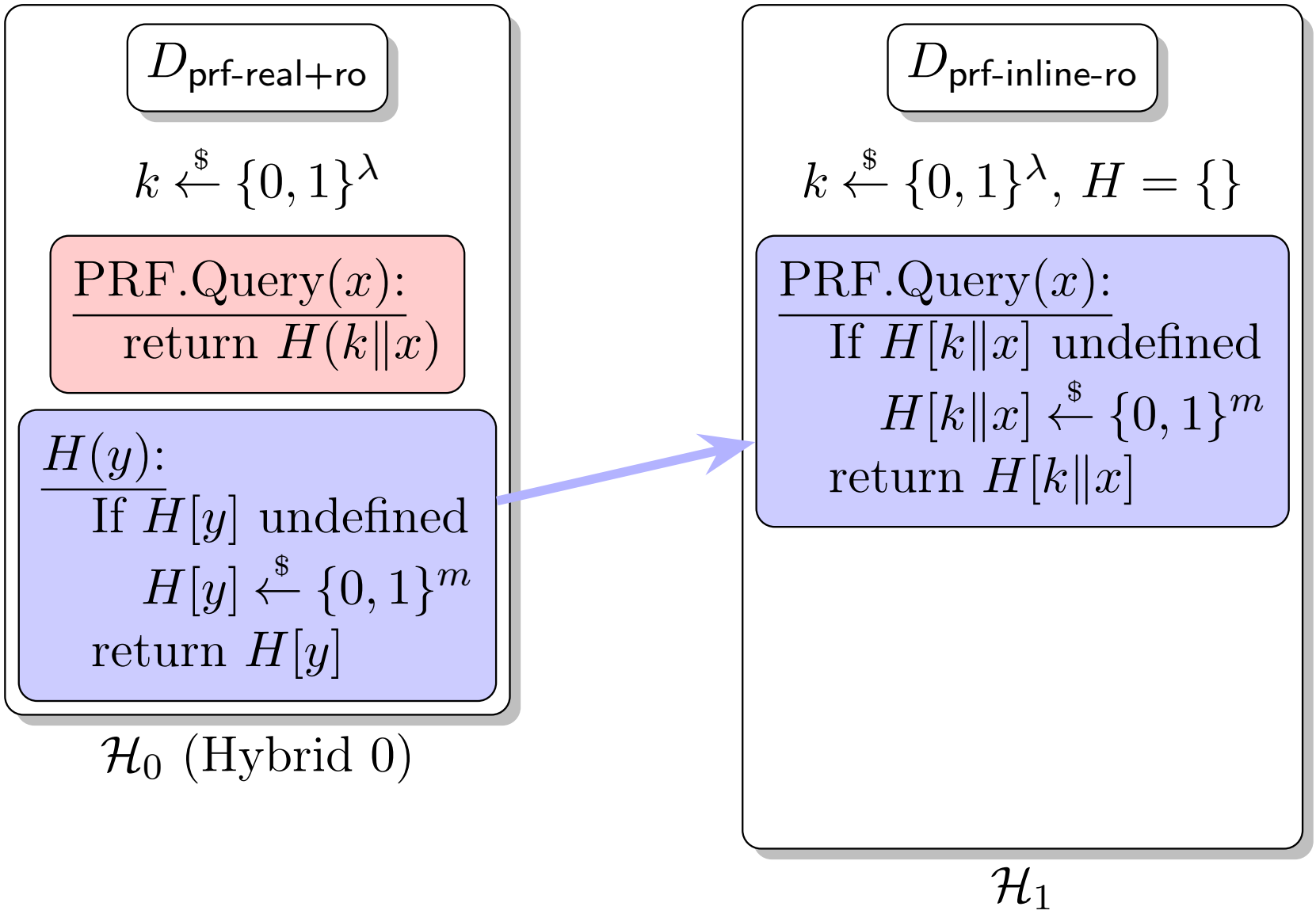

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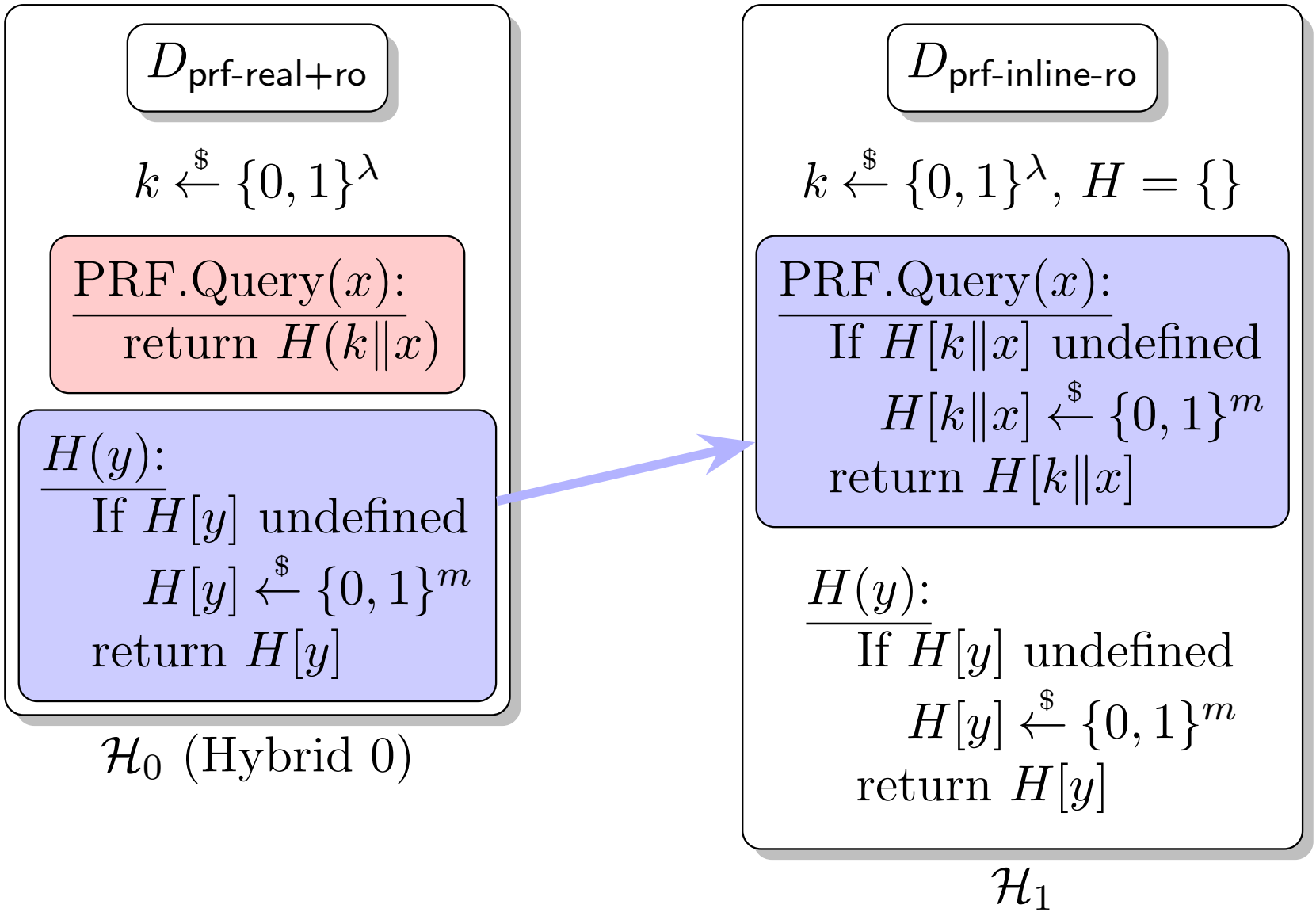

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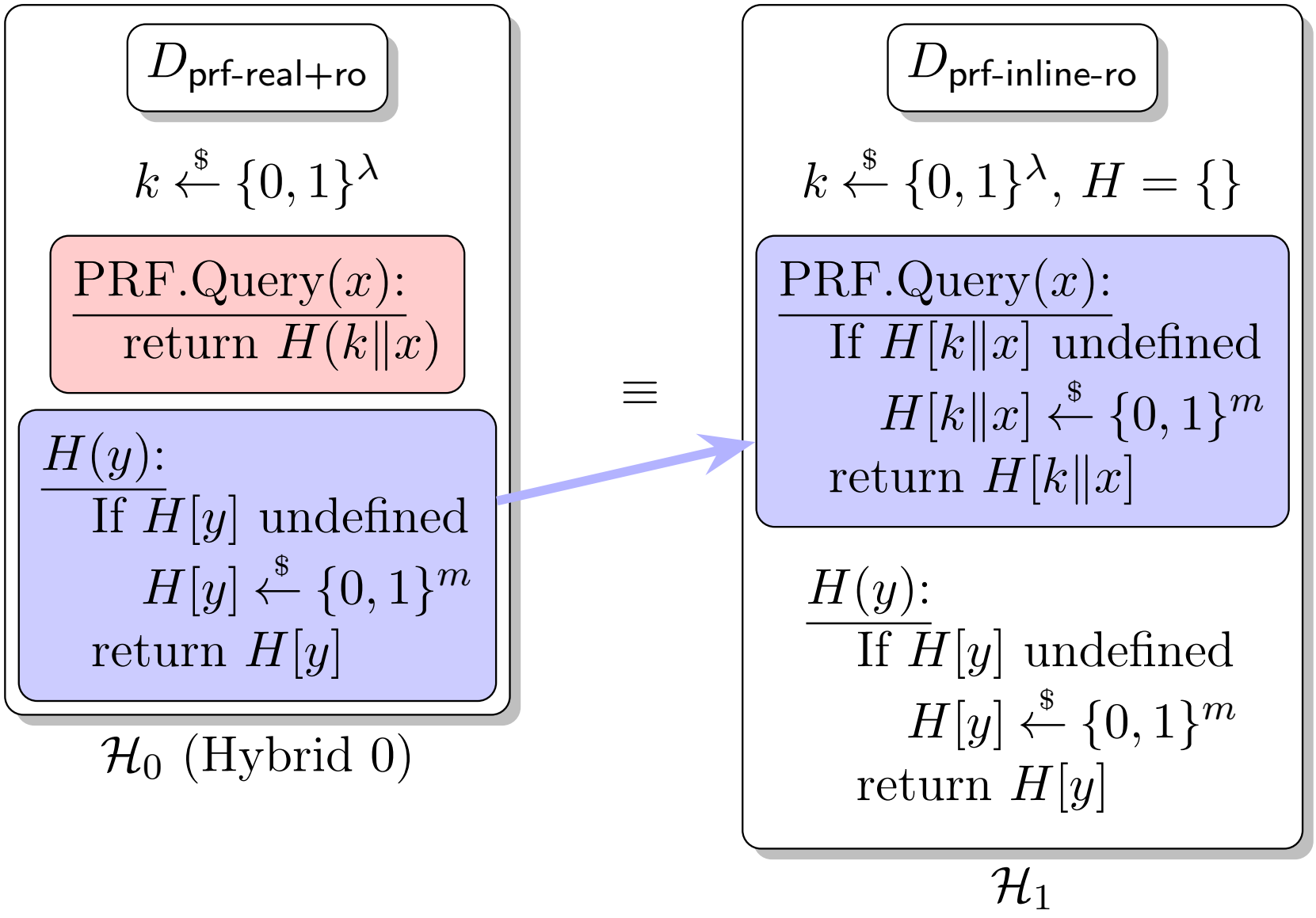

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Claim 2

$\mathcal{H}_0$ and $\mathcal{H}_1$ are identical distributions.

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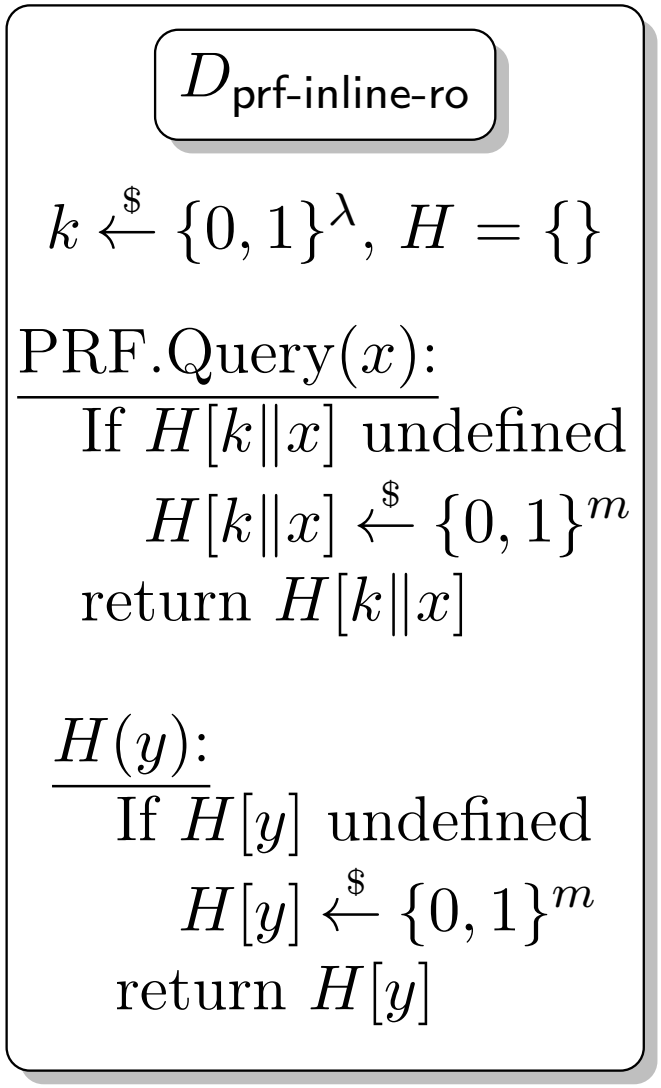
Follows immediately from the construction of our function F_k. □

- Intuition: All we've done is replaced "return $H(k\|x)$" with how the Random Oracle H is defined!

Proof via Hybrid Argument



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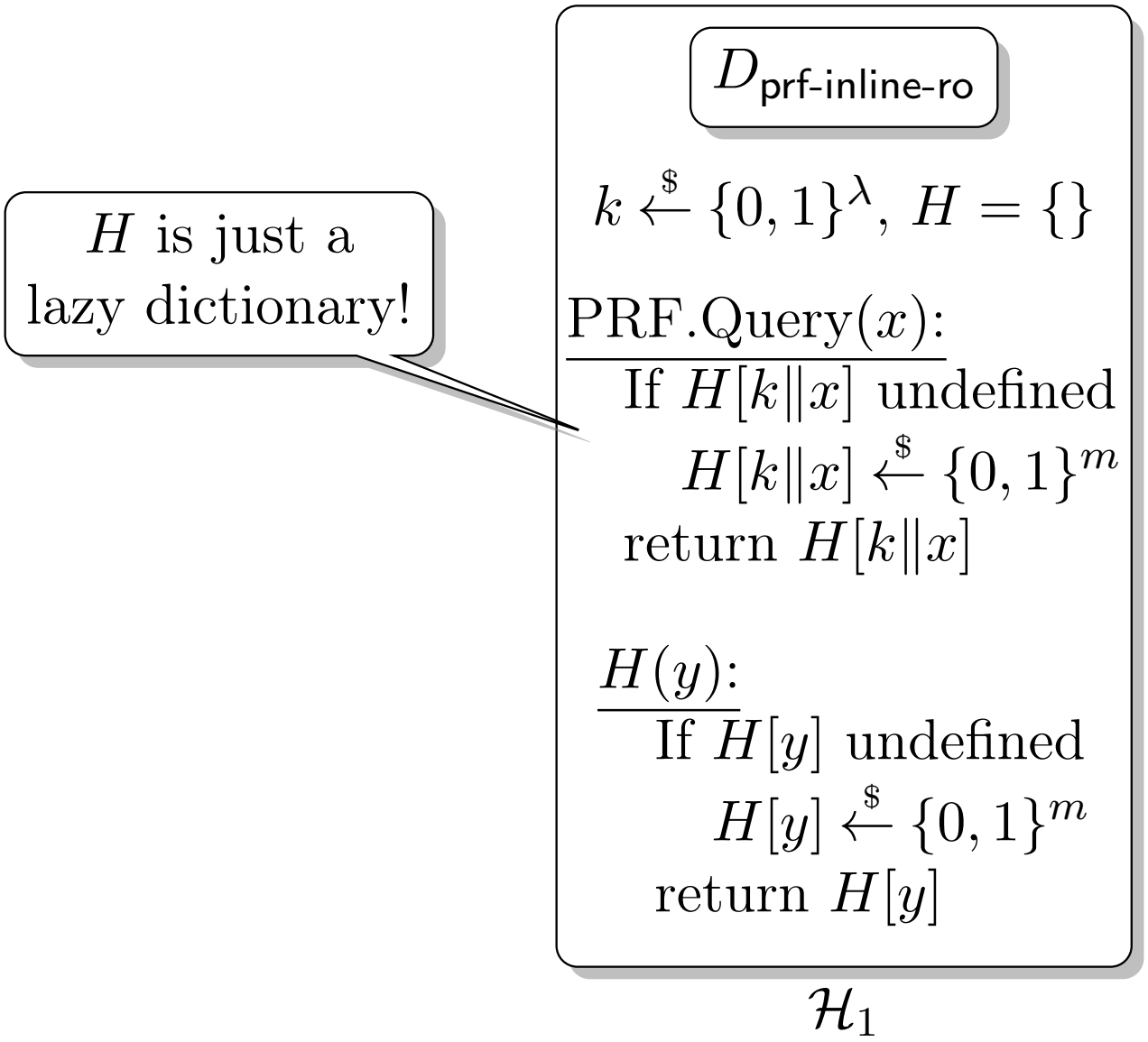

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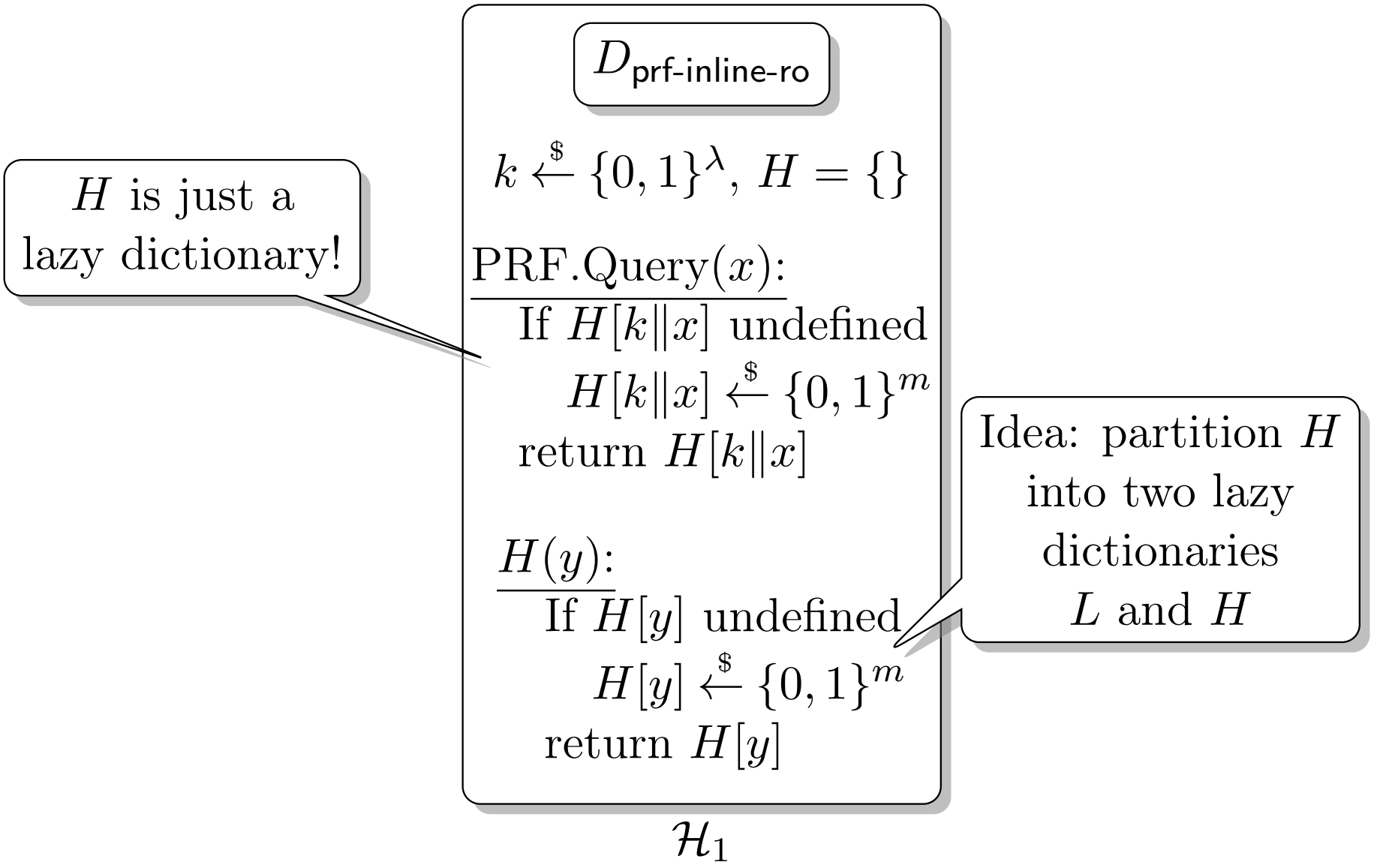

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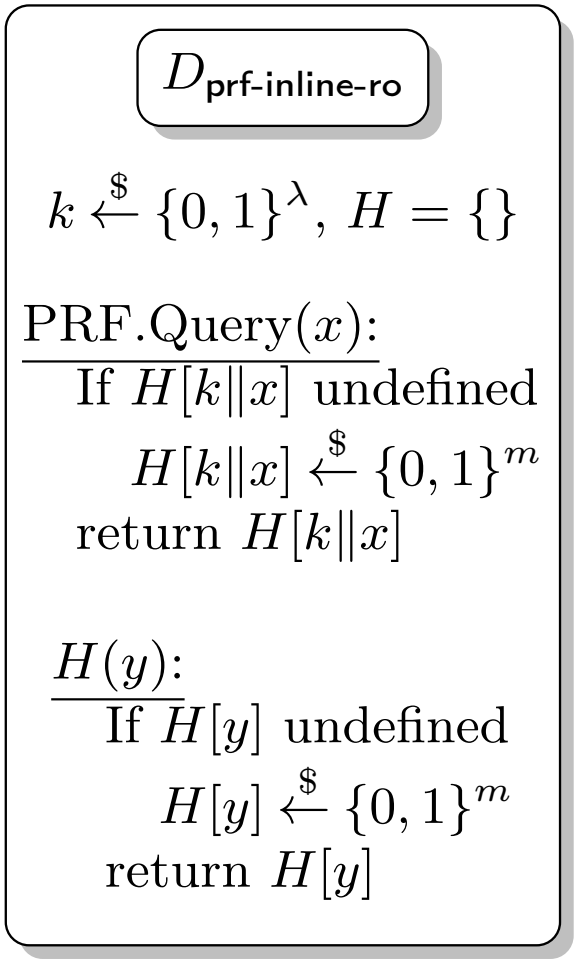

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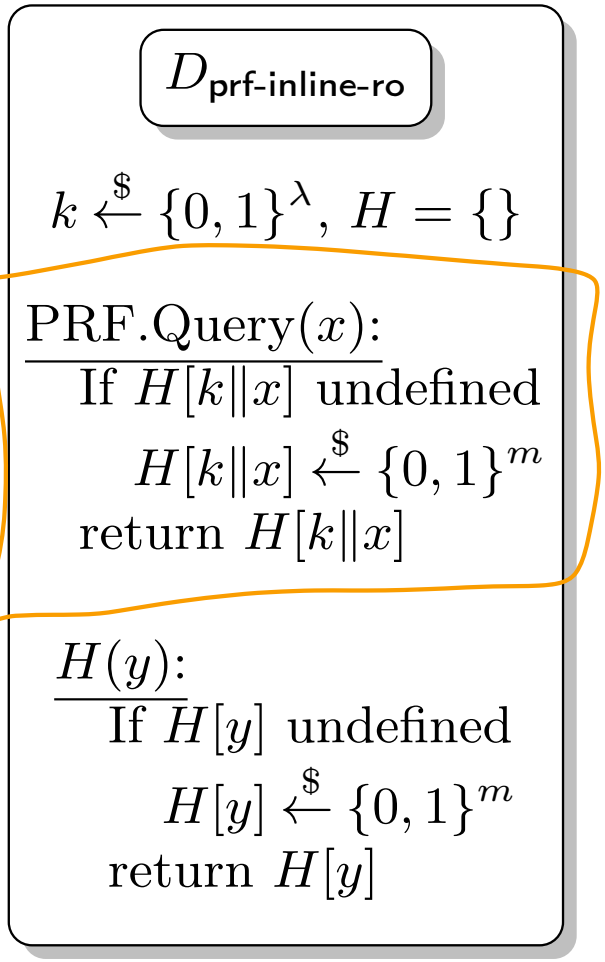

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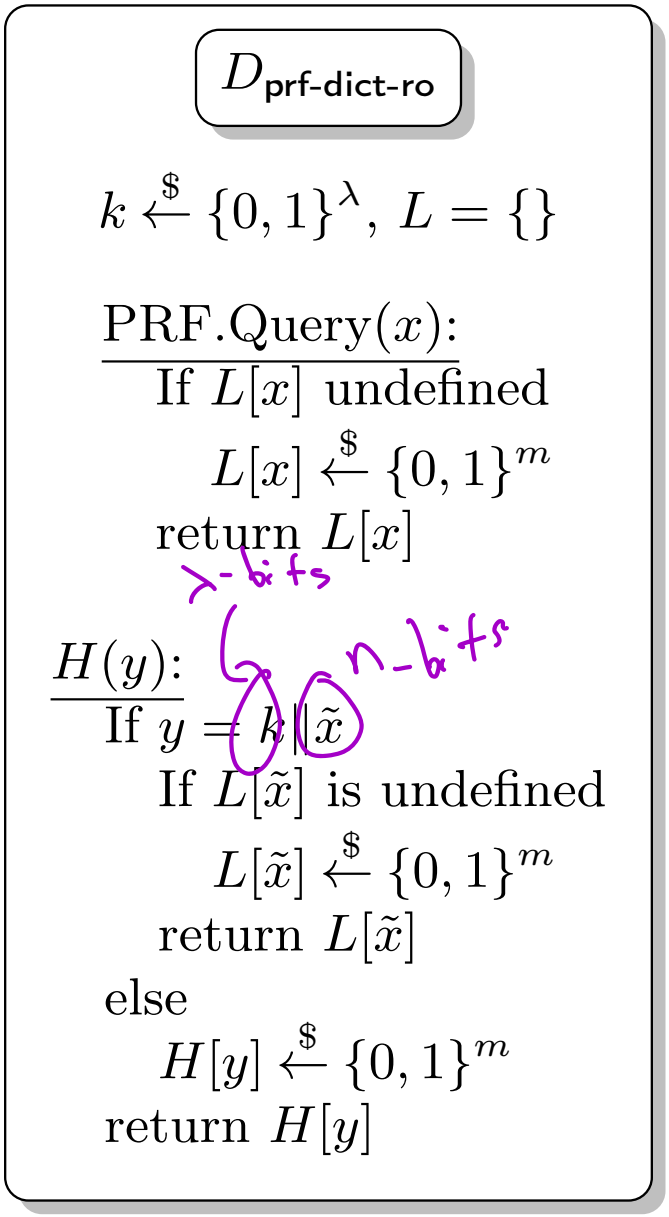

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Claim 3

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m-bits m-bits

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- In $\mathcal{H}_2$, this statement is identical!
- So $\mathcal{H}_1$ and $\mathcal{H}_2$ are identical.

□

Proof via Hybrid Argument

$D_{\text{prf-dict-ro}}$

$k \xleftarrow{\$} \{0,1\}^\lambda$, $L = \{\}$

PRF.Query(x):
  If $L[x]$ undefined
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  return $L[x]$

$H(y)$:
  If $y = k \| \tilde{x}$
    If $L[\tilde{x}]$ is undefined
      $L[\tilde{x}] \xleftarrow{\$} \{0,1\}^m$
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$\mathcal{H}_2$

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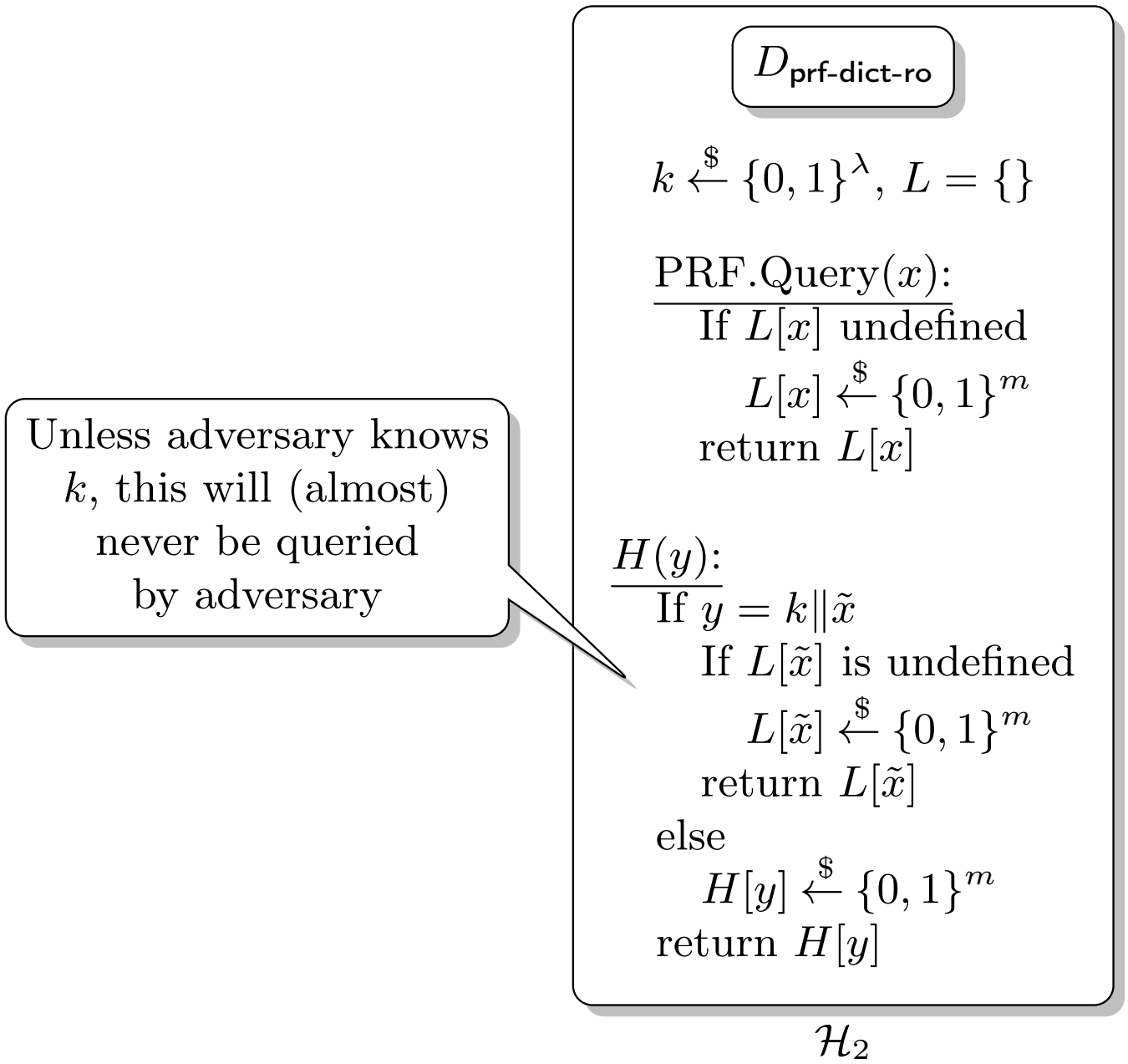

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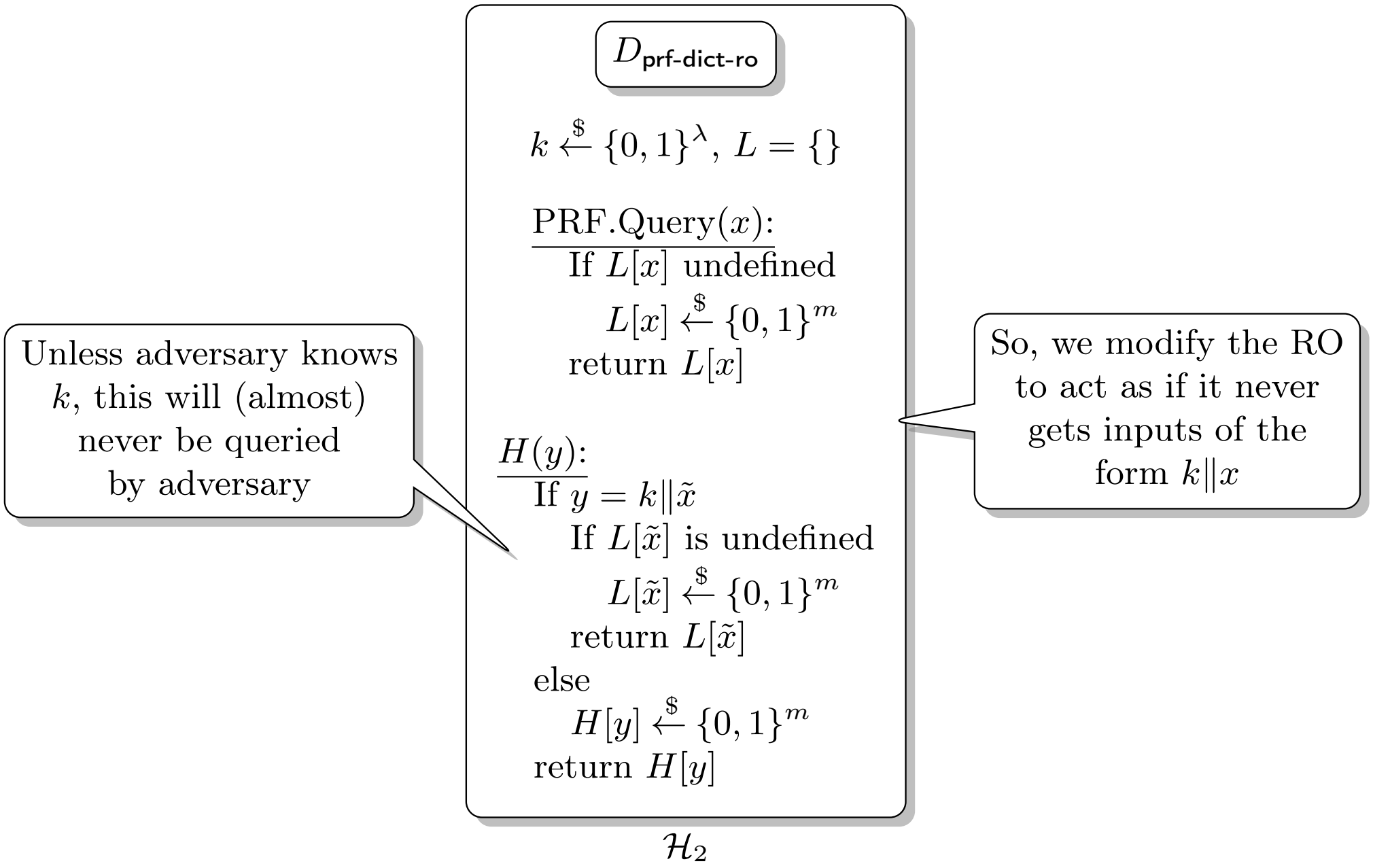

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$\mathcal{H}_2$

Unless adversary knows k, this will (almost) never be queried by adversary

So, we modify the RO to act as if it never gets inputs of the form $k\|x$

However, we will track if RO is queried on an input $k\|x$

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$\mathcal{H}_2$

$D_{\text{prf-dict-ro-bad}}$

$\mathcal{H}_3$

Proof via Hybrid Argument

$D_{\text{prf-dict-ro}}$

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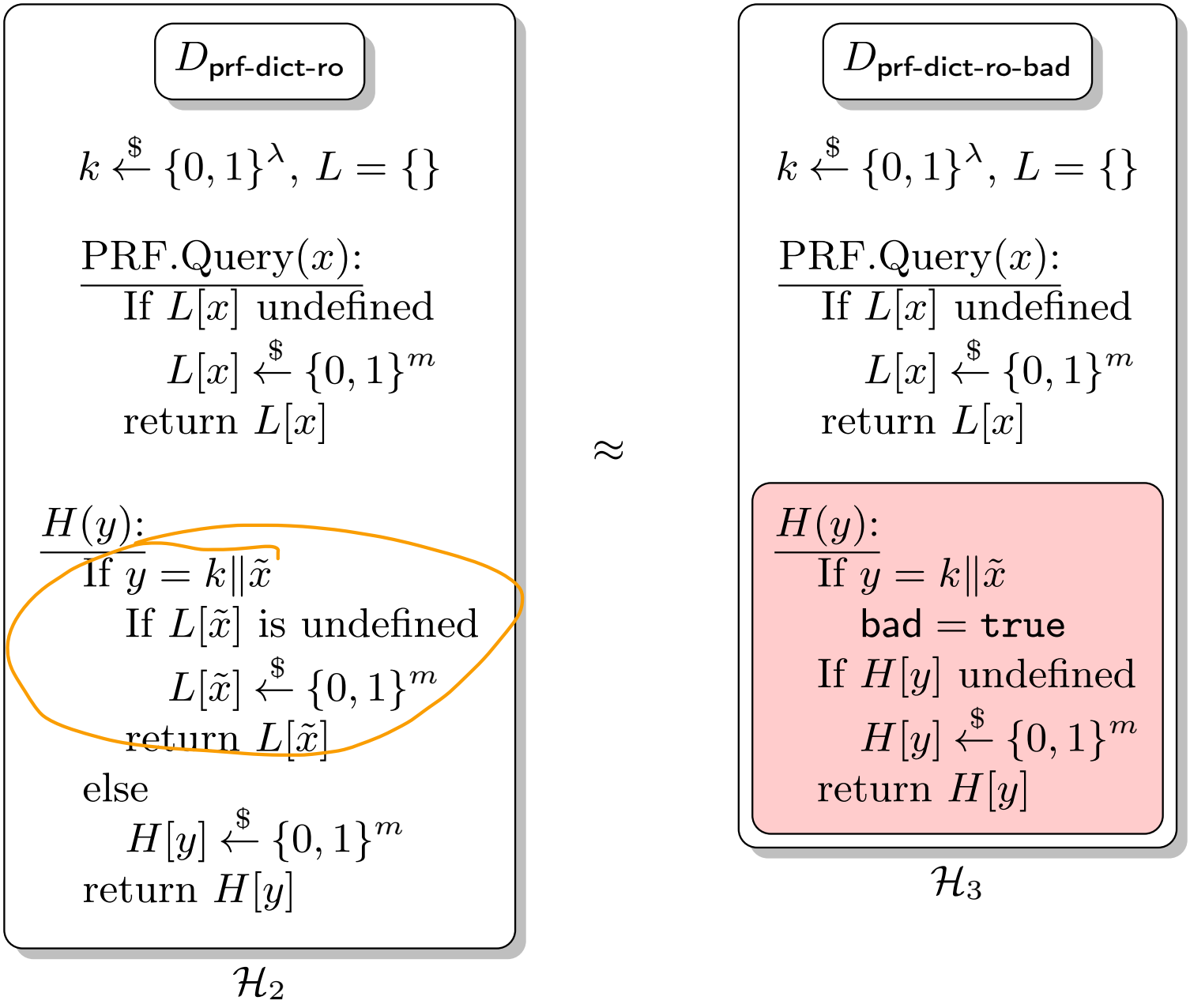
$H(y)$:
  If $y = k \| \tilde{x}$
    bad = `true`
  If $H[y]$ undefined
    $H[y] \xleftarrow{\$} \{0,1\}^m$
  return $H[y]$

$\mathcal{H}_3$

Proof via Hybrid Argument



Proof via Hybrid Argument

Claim 4

$\mathcal{H}_2$ and $\mathcal{H}_3$ are computationally indistinguishable.

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- An adversary can distinguish between $\mathcal{H}_2$ and $\mathcal{H}_3$ if and only if they query H at $y = k \| \widetilde{x}$.

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Proof via Hybrid Argument

Claim 4

$\mathcal{H}_2$ and $\mathcal{H}_3$ are computationally indistinguishable.

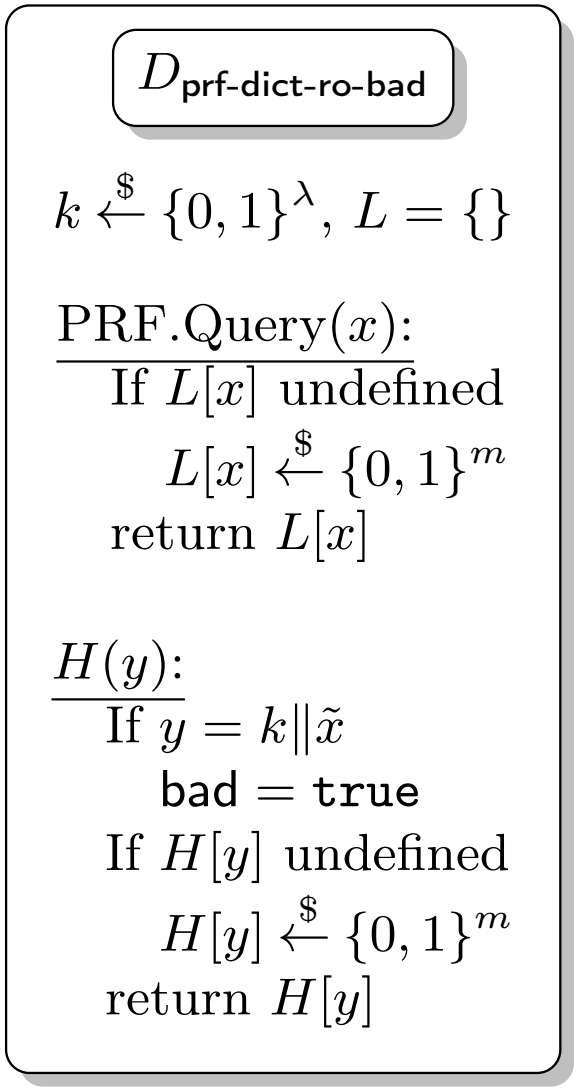
Proof.

- An adversary can distinguish between $\mathcal{H}_2$ and $\mathcal{H}_3$ if and only if they query H at $y = k \| \widetilde{x}$.
- Since $k \xleftarrow{\$} \{0,1\}^\lambda$ and k is hidden from any adversary, the probability and adversary can guess k is $2^{-\lambda}$, which is negligible.

$\leq 2^{-\lambda}(1 - 2^{-m}) \leq 2^{-\lambda}$

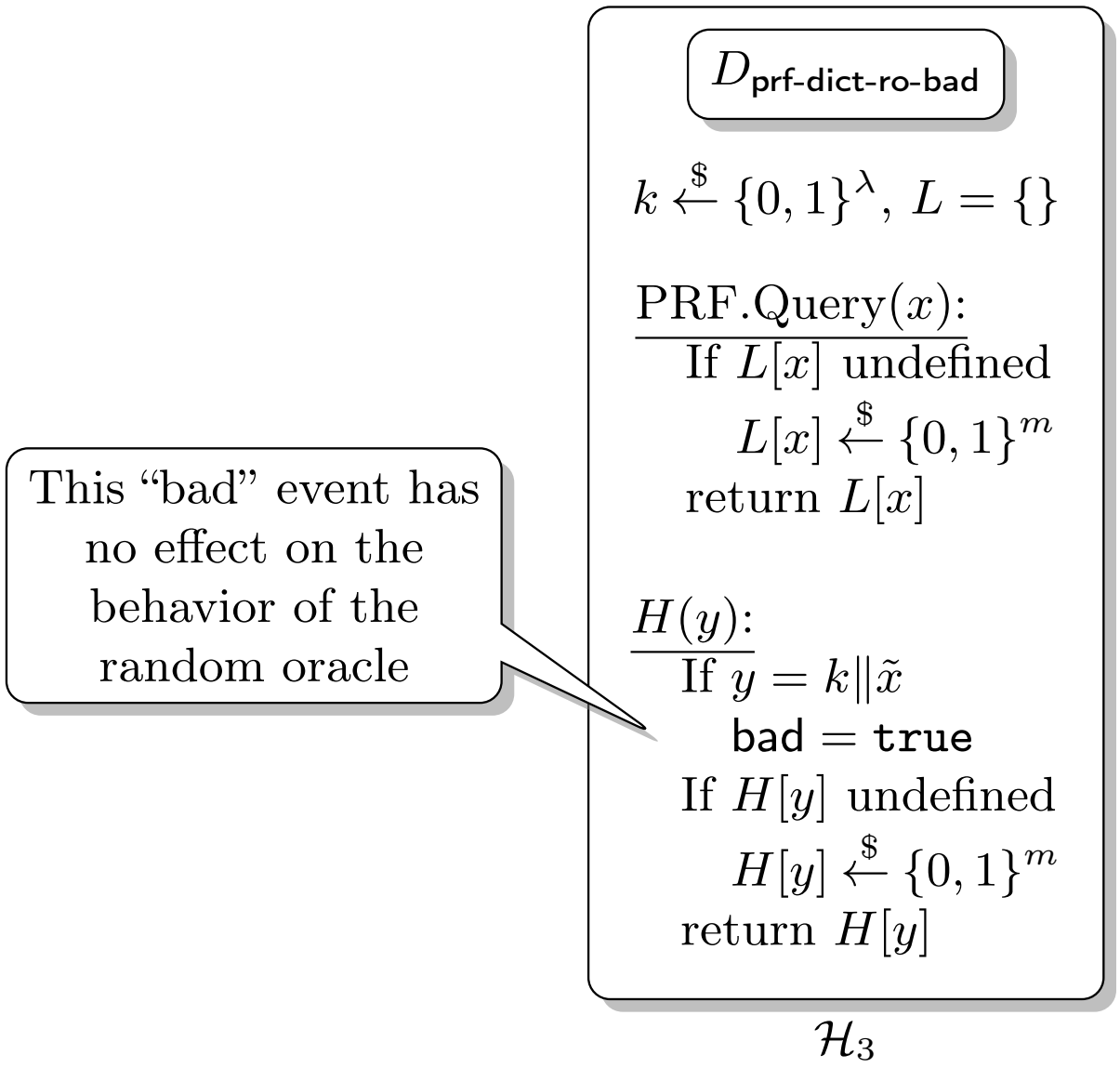
□

Proof via Hybrid Argument



$\mathcal{H}_3$

Proof via Hybrid Argument



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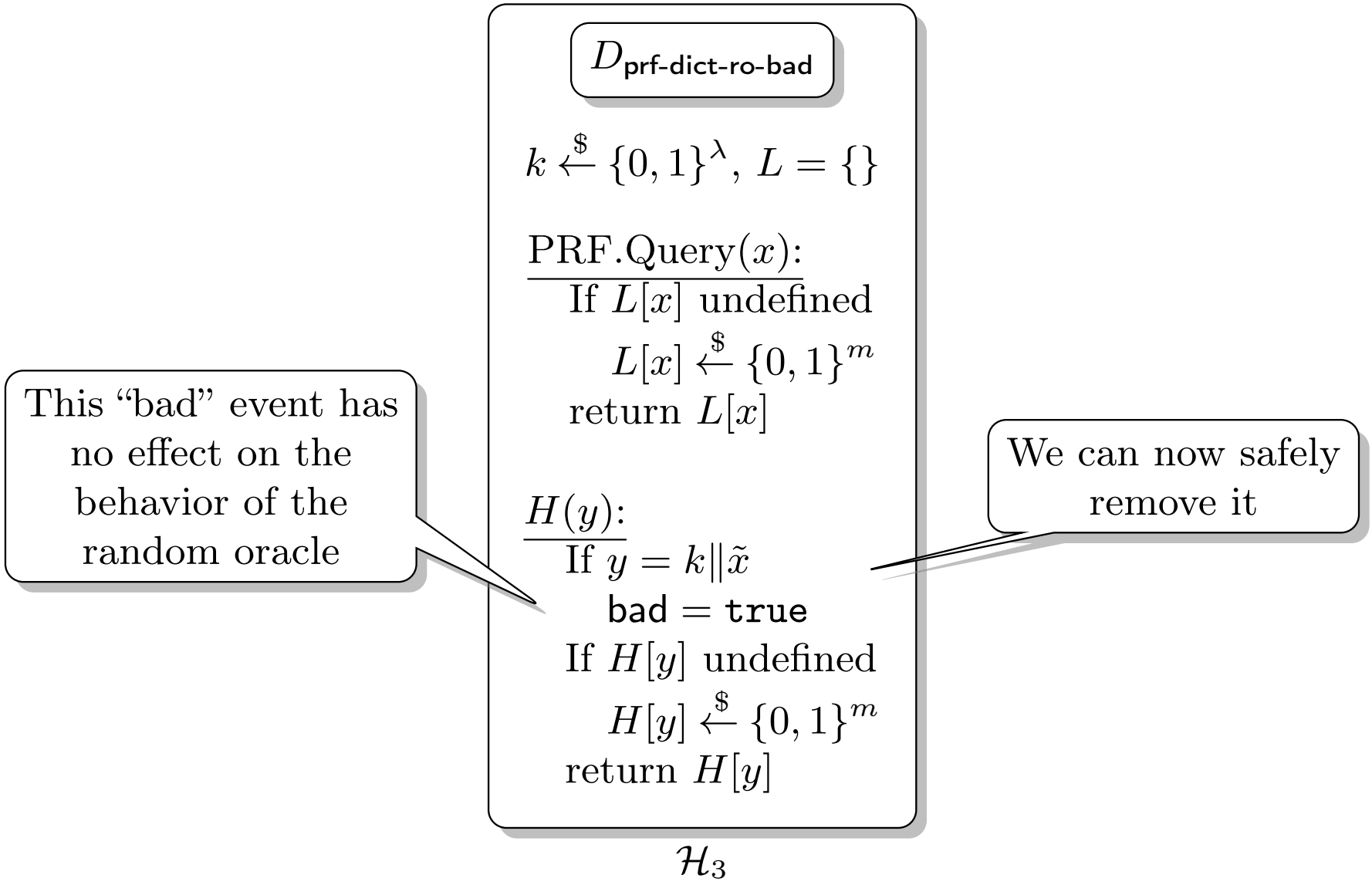

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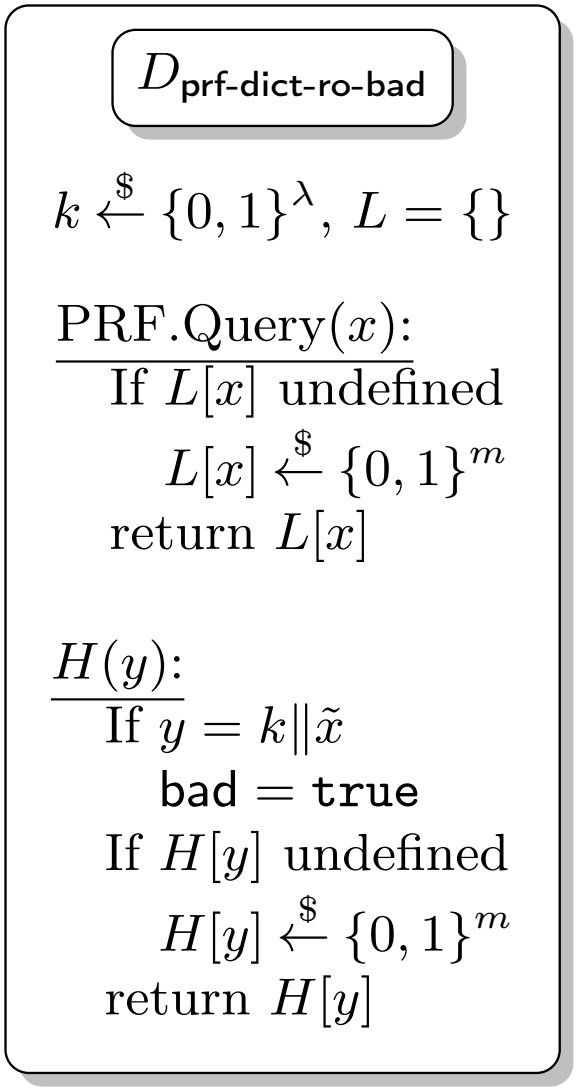

$\mathcal{H}_3$

Proof via Hybrid Argument

$D_{\text{prf-dict-ro-bad}}$

$k \xleftarrow{\$} \{0,1\}^\lambda$, $L = \{\}$

PRF.Query(x):
  If $L[x]$ undefined
    $L[x] \xleftarrow{\$} \{0,1\}^m$
  return $L[x]$

$H(y)$:
  If $y = k \| \tilde{x}$
    bad = true
  If $H[y]$ undefined
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$\mathcal{H}_3$

$D_{\text{prf-rand+ro}}$

$\mathcal{H}_4$

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$\equiv$

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Proof via Hybrid Argument

Claim 5

$\mathcal{H}_3$ and $\mathcal{H}_4$ are identical distributions.

Proof via Hybrid Argument

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$\mathcal{H}_3$ and $\mathcal{H}_4$ are identical distributions.

Proof.

The "bad" event does not change the behavior of the random oracle, so it has no effect on any adversary trying to distinguish between these two hybrids. □

Proof via Hybrid Argument (Conclusion)

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- We have proved that the function family $\{F_k^H : \{0,1\}^n \to \{0,1\}^m\}_{k \in \{0,1\}^\lambda}$ such that $F_k^H(x) := H(k \| x)$ is a secure PRF family.

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PRF Family

$$\mathcal{H}_0 \equiv \mathcal{H}_1 \equiv \mathcal{H}_2 \approx \mathcal{H}_3 \equiv \mathcal{H}_4.$$

PRF sec. Def.

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- $\mathcal{H}_0$ is the distribution corresponding to the PRF family.

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- We have proved that the function family $\{F_k^H : \{0,1\}^n \to \{0,1\}^m\}_{k \in \{0,1\}^\lambda}$ such that $F_k^H(x) := H(k \| x)$ is a secure PRF family.
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$$\mathcal{H}_0 \not\equiv \mathcal{H}_1 \not\equiv \mathcal{H}_2 \approx \mathcal{H}_3 \not\equiv \mathcal{H}_4.$$

$$\approx \quad \approx \qquad \approx$$

- $\mathcal{H}_0$ is the distribution corresponding to the PRF family.
- $\mathcal{H}_4$ is the distribution corresponding to random functions $\mathcal{F}_{n,m}$.
- To get from $\mathcal{H}_0$ to $\mathcal{H}_4$, we make a new hybrid $\mathcal{H}_i$ for $i \in [4]$ which only *slightly* changes $\mathcal{H}_{i-1}$.

Proof via Hybrid Argument (Conclusion)

- We have proved that the function family $\{F_k^H : \{0,1\}^n \to \{0,1\}^m\}_{k \in \{0,1\}^\lambda}$ such that $F_k^H(x) := H(k \| x)$ is a secure PRF family.
- We did so via a hybrid argument, which showed that:

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- $\mathcal{H}_0$ is the distribution corresponding to the PRF family.
- $\mathcal{H}_4$ is the distribution corresponding to random functions $\mathcal{F}_{n,m}$.
- To get from $\mathcal{H}_0$ to $\mathcal{H}_4$, we make a new hybrid $\mathcal{H}_i$ for $i \in [4]$ which only *slightly* changes $\mathcal{H}_{i-1}$.
- Then we argue that $\mathcal{H}_{i-1}$ and $\mathcal{H}_i$ are *close*.

Next Time

- More discussion on the Random Oracle Model.
- Overview of other Idealized Models.