CS 594 – Advanced Crypto (Spring 2026)

Alex Block

Lecture 5

February 2, 2026

Notes on the Random Oracle Model

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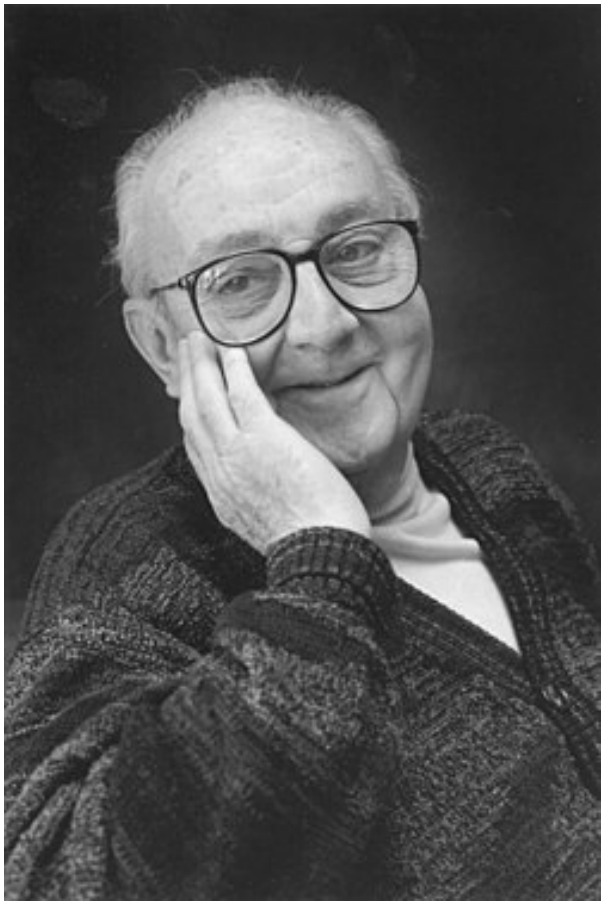

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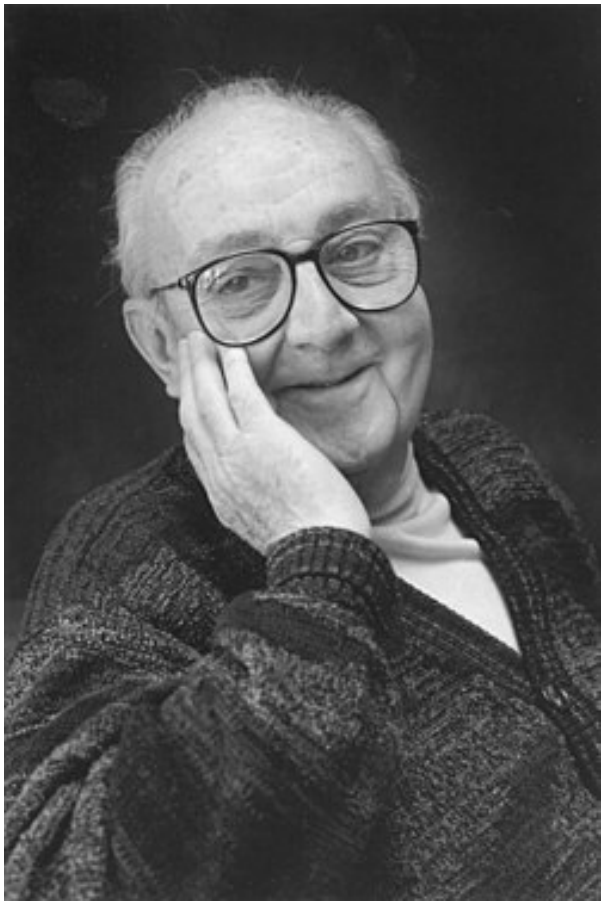

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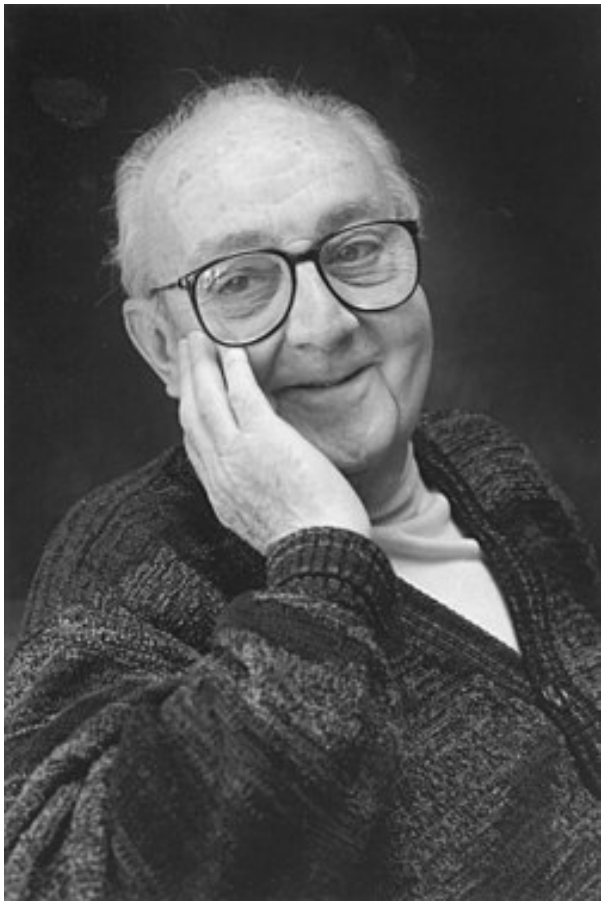

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  - Real-world hash functions **cannot** behave like a RO
  - Security proofs that model hash functions as ROs do not necessarily guarantee security in the real-world
    - In fact, there are examples of schemes which are secure in the RO model **but completely insecure** with any hash function

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  - Security proofs in the ROM tell us what security we can expect against attacks in the real world that do not depend on specific properties of the real-world hash function we are modeling as an RO.

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    - Do one-way functions really exist?
    - Does true randomness actually exist?

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A security proof in the random oracle model is *significantly* better than no proof at all!

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Collision Resistant Hash Functions

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- Note: we are only interested in hash functions with input domains larger than the output.

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If H_k is defined only for inputs of length $\ell'(\lambda) > \ell(\lambda)$, then we say that (Gen, H) is a *fixed-length hash function* (for inputs of length $\ell'(\cdot)$).

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1. ~~$k \leftarrow \mathsf{Gen}(1^\lambda)$~~
2. $x, x' \leftarrow \mathcal{A}(1^\lambda, k)$.
3. Output 1 if and only if $x \neq x'$ and $H_k(x) == H_k(x')$; else output 0.

No key?

H has collisions ✓ keys.

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$\underline{D_{0,k}(x, x') :}$
return $H_k(x) == H_k(x')$

$\underline{D_{1,k}(x, x') :}$
return $x == x'$

Adv:

$x = x'$

- $D_{0,k}(x,x) = D_{1,k}(x,x)$

$x \neq x'$

$D_{1,k}(x,x') = 0$

$\Downarrow$

$D_{0,k}(x,x') = 1$ w.p. $\mathrm{negl}(\lambda)$

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Random Oracles do not need keys!

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  - We believe this adversary *cannot be found efficiently*

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- We'll discuss this more in post-quantum crypto at the end of the course.

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- Useful for *lower bounds*
  - What are the *fastest* algorithms for breaking discrete-log given idealized access to group elements?
- So security in an idealized group model gives some confidence in the real-world security of group-based crypto.

Generic Group Model

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$(\mathbb{Z}_p, 1)$ example

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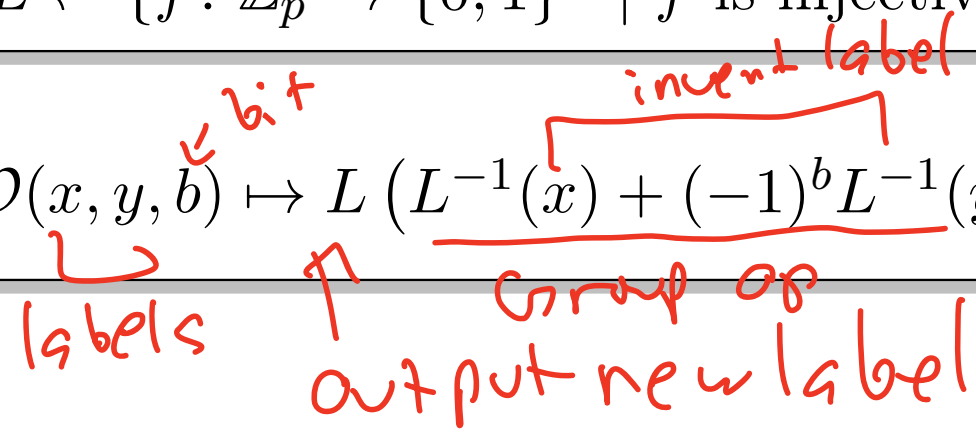

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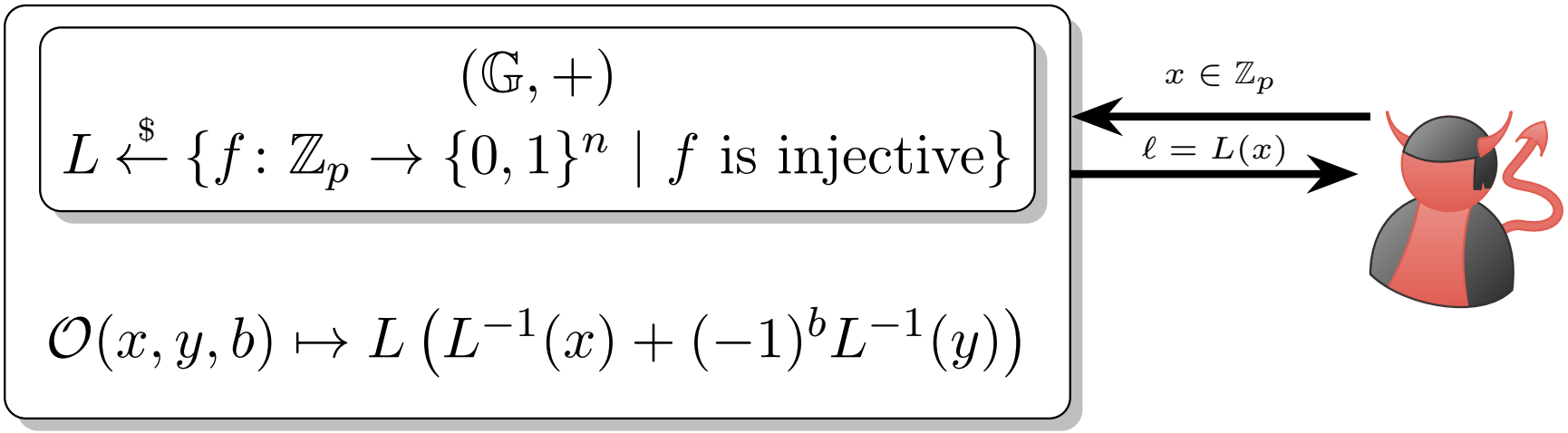

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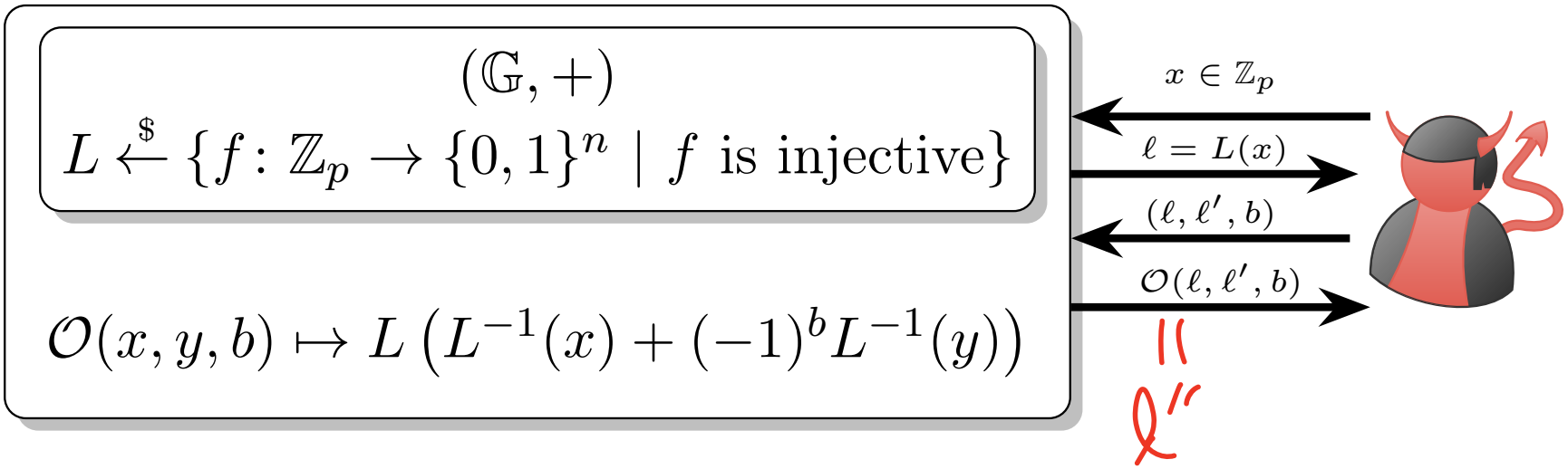
  2. There is an oracle $\mathcal{O}$ which computes the group operation on two elements.



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1. (Bilinear map) For all $u, u' \in \mathbb{G}_1$ and $v, v' \in \mathbb{G}_2$, we have $e(u + u', v) = e(u, v) + e(u', v)$ and $e(u, v + v') = e(u, v) + e(u, v')$;

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Theorem 1 (Informal; Zhandry & Zhang, ASIACRYPT'23)

The GBM is strictly stronger than the GGM, which is strictly stronger than the ROM.

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AGM Requirement

Non-group elements given as input to an oracle must not depend on group elements.

Adv.

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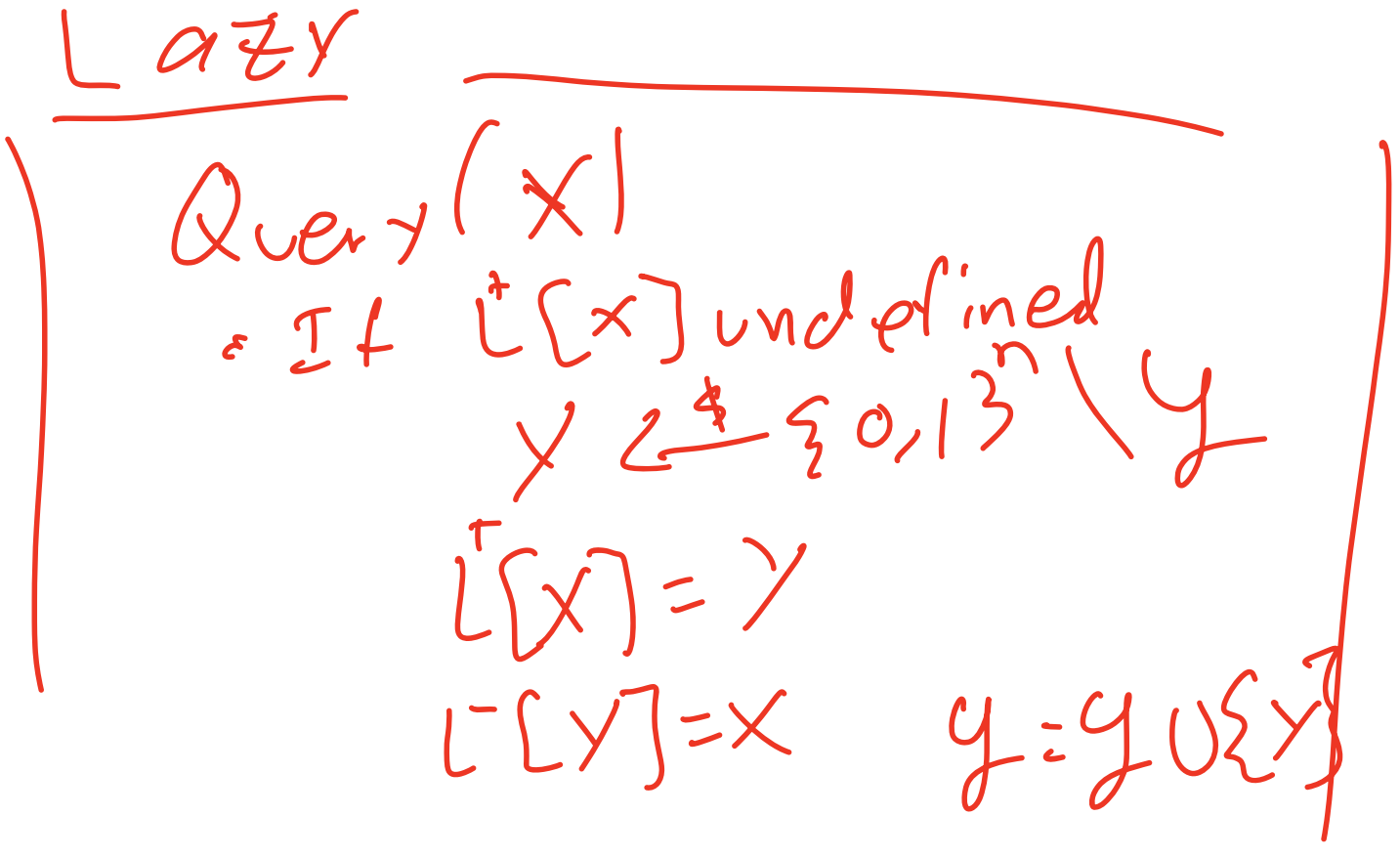
- *Ideal Permutation Model*: ROM but you have oracle access to a random permutation *and its inverse*.



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- Ideal Cipher Model: F_k and F_k^{-1} are indistinguishable from random *for every key k.*

Next Time: Secret Sharing