Multilinear Factorization Machines for Multi-Task Multi-View Learning

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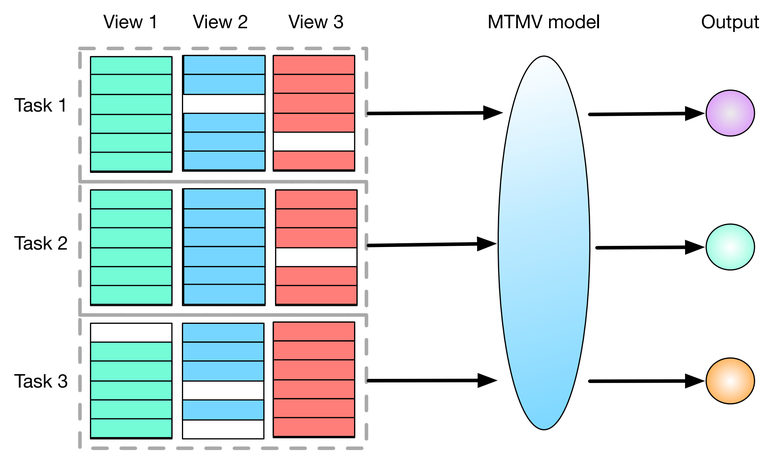
Paper Code

Multi-Task Multi-View Learning

Multi-Task Multi-View (MTMV) Learning: combine different views (e.g., images and texts) to learn multiple related tasks together.



- **Conventional approaches try to minimize the difference of the predictive functions among different views, while different views might not be consistent. How to exploit complementary information from multiple views?**
- **Some instances may not have certain views. How to explore the feature interactions between multiple views to estimate parameter under sparsity?**

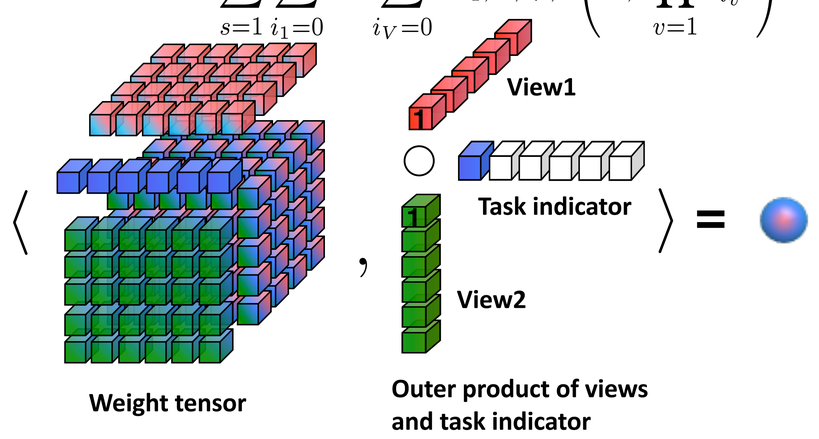
Multilinear Factorization Machines

We propose a multilinear structure for MTMV learning

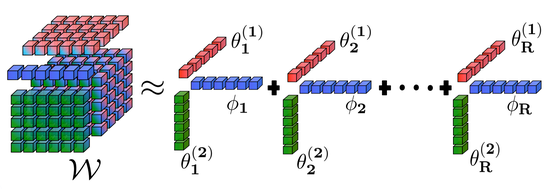
Sum up to full-order interaction by adding an extra feature with constant value 1 to the feature vector $\mathbf{z}^{(v)} = [1; \mathbf{x}^{(v)}] \in \mathbb{R}^{1+I_v}$

$$f_t(\{\mathbf{x}^{(v)}\}) = \left\langle \mathcal{W}, \mathbf{z}^{(1)} \circ \cdots \circ \mathbf{z}^{(V)} \circ \mathbf{e}_t \right\rangle = \langle \mathcal{W}, \mathcal{Z}_t \rangle$$

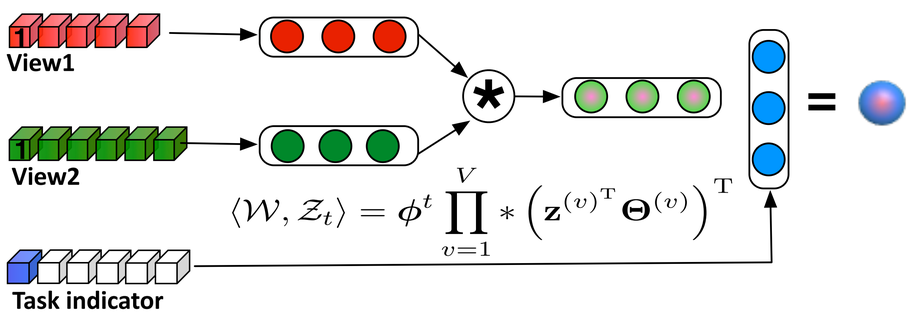
$$= \sum_{s=1}^{T} \sum_{i_1=0}^{I_1} \cdots \sum_{i_V=0}^{I_V} w_{i_1,\ldots,i_V,s} \left(e_{t,s} \prod_{v=1}^{V} z_{i_v}^{(v)} \right)$$



Applying tensor factorization on the weight tensor.



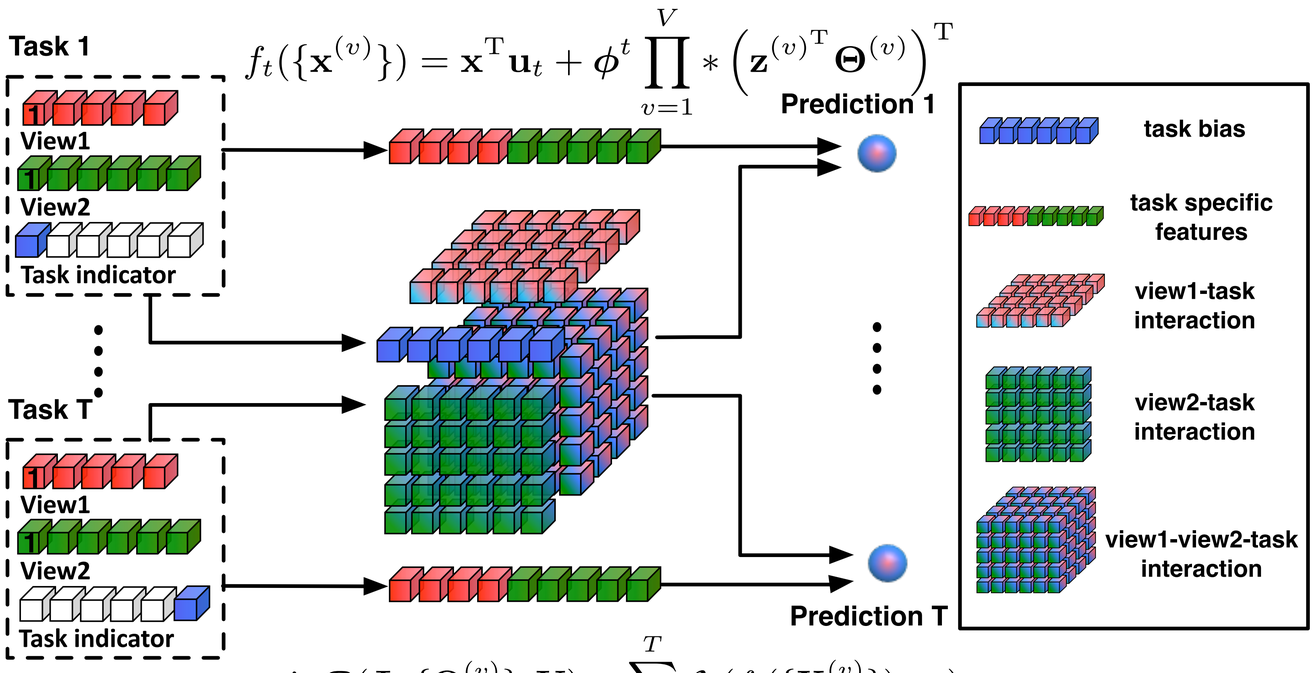
$$\mathcal{W} = \sum_{r=1}^{R} \phi_{\mathbf{r}} \circ \theta_{\mathbf{r}}^{(1)} \circ \theta_{\mathbf{r}}^{(2)}$$



$$\langle \mathcal{W}, \mathcal{Z}_t \rangle = \phi^t \prod_{v=1}^{V} * \left(\mathbf{z}^{(v)\mathrm{T}} \mathbf{\Theta}^{(v)} \right)^{\mathrm{T}}$$

It is too restrictive to constrain all tasks to share the common subspace. We design a predictive function to learn from both the task-specific feature map and the task-view shared multilinear structures.

$$f_t(\{\mathbf{x}^{(v)}\}) = \mathbf{x}^{\mathrm{T}} \mathbf{u}_t + \phi^t \prod_{v=1}^{V} * \left(\mathbf{z}^{(v)\mathrm{T}} \mathbf{\Theta}^{(v)} \right)^{\mathrm{T}}$$



$$\min \mathcal{R}(\mathbf{\Phi}, \{\mathbf{\Theta}^{(v)}\}, \mathbf{U}) = \sum_{t=1}^{T} \mathcal{L}_t(f_t(\{\mathbf{X}_t^{(v)}\}), \mathbf{y}_t) + \lambda \Omega_\lambda(\mathbf{\Phi}, \{\mathbf{\Theta}^{(v)}\}) + \gamma \Omega_\gamma(\mathbf{U})$$

Solved by alternating block coordinate descent w/ AdaGrad

- **Proposed MFMs can efficiently learn the task-specific feature map and the task-view shared multilinear structures from full-order interactions, without physically building the tensor.**
- **MFMs jointly factorize the interaction parameters for different orders to allow accurate parameter estimation under sparsity and renders the model with the capacity to avoid overfitting.**
- **The model complexity of MFMs is linear in the feature dimensionality, making it applicable to large-scale applications.**

Experiment Results

Datasets

Classification	#Feature	T	N_p	N_n
FOX	image(996), text(2,711)	4	178∼635	888∼1,345
DBLP	linkage(4,638), text(687)	6	635∼1,950	2,688∼3,985
Regression	**#Feature**	**T**	**N**	**Density**
MovieLens	users(943), movies(1,599), tags(1,065)	10	758∼39,895	6.3%
Amazon	users(1,805,364), items(192,978), text(83,143)	5	349,038∼1,015,189	0.001%

T: number of tasks
N: total number of instances
Np (Nn): number of positive (negative) instances in each task
Density: the density of the user-item matrix

Average results of 10 times of random sampling: n% labeled instances as training set (n=10,20, and 30), 10% as validation set, 40% as testing set

Comparison Methods

- **rMTFL:** robust multi-task feature learning algorithm
- **IteM²:** transductive MTMV classification algorithm
- **CSL-MTMV:** state-of-the-art inductive MTMV learning algorithm
- **Factorization Machine (FM)**: state-of-the-art factorization model
- **Tensor Factorization (TF):** factorize highest-order weight tensor
- **Multilinear Tensor Factorization (MFM):** proposed method
  - **MFM-T:** only using tensor part (**U** is fixed as a zero matrix)
  - **MFM-F:** using F-norm regularizers for all parameters
  - **MFM-F-S:** using $\ell_{2,1}$-norm on **U** for joint feature selection
    F-norm on the rest parameters

FOX

Training Ratio	Measure	rMTFL	FM	TF	IteM²	CSL-MTMV	MFM-T	MFM-F	MFM-F-S
10%	ACC	0.8816±0.011	0.7883±0.011	0.8460±0.035	0.4052±0.076	0.8986±0.011	0.9259±0.019	**0.9343±0.012**	**0.9364±0.011**
	F1	0.6911±0.035	0.2930±0.046	0.6362±0.044	0.3598±0.030	0.7335±0.029	0.7799±0.053	**0.8076±0.038**	**0.8119±0.027**
	AUC	0.9109±0.013	0.7764±0.018	0.8681±0.038	0.5326±0.036	0.9342±0.011	0.9678±0.015	**0.9763±0.008**	**0.9777±0.009**
20%	ACC	0.9039±0.013	0.8087±0.011	0.8546±0.025	0.5091±0.078	0.9264±0.005	0.9551±0.005	**0.9569±0.010**	**0.9612±0.005**
	F1	0.7654±0.026	0.3764±0.050	0.6632±0.051	0.3306±0.068	0.8004±0.012	0.8721±0.012	**0.8769±0.027**	**0.8882±0.014**
	AUC	0.9353±0.016	0.8260±0.012	0.8751±0.029	0.4954±0.043	0.9705±0.003	0.9883±0.003	**0.9885±0.006**	**0.9922±0.002**
30%	ACC	0.9314±0.005	0.8255±0.007	0.8767±0.082	0.4289±0.134	0.9390±0.004	0.9641±0.007	**0.9709±0.003**	**0.9697±0.004**
	F1	0.8051±0.015	0.4448±0.026	0.7302±0.132	0.3314±0.056	0.8341±0.012	0.9000±0.018	**0.9185±0.010**	**0.9149±0.010**
	AUC	0.9709±0.005	0.8393±0.012	0.9010±0.091	0.5365±0.039	0.9812±0.003	0.9916±0.003	**0.9949±0.001**	**0.9949±0.001**

DBLP

Training Ratio	Measure	rMTFL	FM	TF	IteM²	CSL-MTMV	MFM-T	MFM-F	MFM-F-S
10%	ACC	0.8057±0.004	0.7264±0.004	0.7471±0.011	0.6223±0.004	0.7290±0.005	0.8008±0.004	**0.8058±0.004**	**0.8062±0.005**
	F1	0.5395±0.015	0.0732±0.019	0.5606±0.011	0.3176±0.007	0.4402±0.004	0.5278±0.018	**0.5469±0.014**	**0.5471±0.015**
	AUC	0.7888±0.007	0.6264±0.023	0.7723±0.009	0.5310±0.007	0.6890±0.006	0.8039±0.010	**0.8113±0.010**	**0.8120±0.009**
20%	ACC	0.8319±0.004	0.7628±0.007	0.7878±0.007	0.6309±0.003	0.7760±0.002	0.8346±0.004	**0.8374±0.004**	**0.8371±0.004**
	F1	0.6447±0.008	0.2680±0.038	0.6247±0.014	0.3494±0.006	0.5295±0.007	0.6274±0.013	**0.6499±0.012**	**0.6508±0.012**
	AUC	0.8374±0.005	0.7548±0.022	0.8200±0.010	0.5550±0.006	0.7655±0.005	0.8531±0.006	**0.8658±0.005**	**0.8632±0.005**
30%	ACC	0.8412±0.004	0.7978±0.005	0.8191±0.008	0.6256±0.003	0.8037±0.003	0.8501±0.004	**0.8527±0.004**	**0.8535±0.004**
	F1	0.6796±0.010	0.4312±0.021	0.6670±0.021	0.3569±0.009	0.5869±0.007	0.6800±0.013	**0.6891±0.012**	**0.6892±0.009**
	AUC	0.8590±0.005	0.8351±0.010	0.8498±0.009	0.5563±0.006	0.8083±0.006	0.8757±0.005	**0.8866±0.006**	**0.8866±0.006**

MovieLens

Training Ratio	Measure	rMTFL	FM	TF	CSL-MTMV	MFM-T	MFM-F	MFM-F-S
10%	RMSE	1.1861±0.008	1.0251±0.003	1.5679±0.099	1.05013±0.005	1.0078±0.005	**1.0069±0.005**	**0.9976±0.004**
	MAE	0.8516±0.004	0.8422±0.004	1.2497±0.088	0.8516±0.004	0.8142±0.005	**0.8082±0.005**	**0.8022±0.004**
20%	RMSE	1.0631±0.005	0.9898±0.003	1.2519±0.069	1.0214±0.004	**0.9877±0.003**	0.9977±0.003	**0.9857±0.003**
	MAE	0.8539±0.005	0.7997±0.004	0.9801±0.053	0.8294±0.004	**0.7987±0.003**	0.8023±0.003	**0.7927±0.004**
30%	RMSE	0.9917±0.003	**0.9765±0.003**	1.2066±0.061	1.0082±0.003	0.9795±0.003	0.9887±0.004	**0.9785±0.003**
	MAE	0.8159±0.003	**0.7815±0.003**	0.9380±0.045	0.8189±0.003	0.7885±0.002	0.7823±0.004	**0.7789±0.004**

Amazon

Training Ratio	Measure	FM	TF	MFM-T	MFM-F	MFM-F-S
10%	RMSE	0.9834±0.001	3.6044±0.003	**0.9775±0.001**	0.9857±0.001	**0.9825±0.002**
	MAE	0.7420±0.001	3.4574±0.005	0.7249±0.001	**0.7158±0.002**	**0.7129±0.001**
20%	RMSE	0.9814±0.001	3.5611±0.018	**0.9764±0.001**	0.9845±0.001	**0.9775±0.001**
	MAE	0.7343±0.002	3.3965±0.030	0.7255±0.001	**0.7112±0.001**	**0.7086±0.001**
30%	RMSE	0.9782±0.002	3.4962±0.018	**0.9705±0.002**	0.9841±0.001	**0.9733±0.001**
	MAE	0.7257±0.002	3.2945±0.034	**0.7001±0.001**	0.7115±0.001	**0.7078±0.001**

Due to the memory overhead, rMTFL and CSL-MTMV are not compared.

- **Due to view inconsistency, previous MTMV methods such as IteM2, and CSL-MTMV perform worse than rMTFL.**
- **MFMs consistently outperform compared methods, and improve 6~10% over best compared methods.**
- **MFM-F-S performs the best, and task-specific linear feature map is more important for classification than for regression.**

Acknowledgments

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