# Semi-supervised Learning

Acknowledgments: Tom Mitchell, Avrim Blum

June 23, 2016

# Semi-supervised Learning

- Supervised Learning models require labeled data
- Learning a reliable model usually requires plenty of labeled data
- Labeled Data: Expensive and Scarce
- Unlabeled Data: Abundant and Free/Cheap
  - ► E.g., webpage classification: easy to get unlabeled webpages
- Semi-supervised Learning: Devising ways of utilizing unlabeled data with labeled data to learn better models
  - Formally, given labeled training data  $\mathcal{D}^I = \{\mathbf{x}_i, y_i\}_{i=1}^L$ , and unlabeled data  $\mathcal{D}^u = \{\mathbf{x}_i\}_{i=L+1}^{L+U}$  (usually  $U \gg L$ ), the goal is to learn a classifier f better than using labeled data alone.

# Why/How Might Unlabeled Data Help?

- At first consideration, one may think that nothing can be gained by having access to unlabeled data.
- However, they provide information about the joint probability distribution over words.
- Example: university webpage classification
  - Supposed that using only labeled data, documents containing "homework" belong to the "course" category.
  - ▶ If we estimate the classification of many unlabeled documents, we may find that "lecture" occurs frequently in unlabeled documents that are believed to belong to the "course" category.
  - ► The co-occurrence of "homework" and "lecture" over the large set of unlabeled data allows to construct a more accurate classifier that considers both "homework" and "lecture" as indicators of positive examples.

## Using Expectation-Maximization for SSL

- Expectation-Maximization (EM) is a class of iterative algorithms for maximum likelihood or maximum a posteriori estimation in problems with incomplete data (Dempster, Laird, and Rubin, 1977)
- Unlabeled data are considered incomplete as they come without class labels.
- The EM algorithm:
  - ▶ First trains a classifier with only *labeled data* and uses the classifier to assign probabilistically-weighted class labels to each unlabeled example by calculating the expectation of the missing class labels.
  - ▶ It then trains a new classifier using all the documents and iterates.

### Incorporating Unlabeled Data with EM

#### Applying EM to Naive Bayes.

- ullet Inputs: Labeled data  $\mathcal{D}^I = \{\mathbf{x}_i, y_i\}_{i=1}^L$ , and unlabeled data  $\mathcal{D}^u = \{\mathbf{x}_i\}_{i=L+1}^{L+U}$
- Train an initial naive Bayes classifier,  $\hat{\theta}$ , using just  $\mathcal{D}'$ .
- Loop while classifier parameters improve, as measured by the change in the complete log probability of the labeled and unlabeled data and the prior:
  - ▶ (**E-step**) Use the current classifier,  $\hat{\theta}$ , to estimate component membership of each unlabeled example,  $P(c_j|d_i; \hat{\theta})$ .
  - ▶ (M-step) Re-estimate the classifier,  $\hat{\theta}$ , given the estimated component membership of each example,  $P(w_t|c_j;\hat{\theta})$  and  $P(c_i|\hat{\theta})$
- **Output:** A classifier  $\hat{\theta}$ , that takes an unlabeled document and predicts a class label.

# Incorporating Unlabeled Data with EM

E-step:

$$\begin{split} \mathbf{P}(y_{i} = c_{j} | d_{i}; \hat{\theta}) &= \frac{\mathbf{P}(c_{j} | \hat{\theta}) \mathbf{P}(d_{i} | c_{j}; \hat{\theta})}{\mathbf{P}(d_{i} | \hat{\theta})} \\ &= \frac{\mathbf{P}(c_{j} | \hat{\theta}) \prod_{k=1}^{|d_{i}|} \mathbf{P}(w_{d_{i,k}} | c_{j}; \hat{\theta})}{\sum_{r=1}^{|\mathcal{C}|} \mathbf{P}(c_{r} | \hat{\theta}) \prod_{k=1}^{|d_{i}|} \mathbf{P}(w_{d_{i,k}} | c_{r}; \hat{\theta})}. \end{split}$$

M-step:

$$\hat{\theta}_{w_t|c_j} \equiv P(w_t|c_j; \hat{\theta}) = \frac{1 + \sum_{i=1}^{|D|} N(w_t, d_i) P(y_i = c_j|d_i)}{|V| + \sum_{s=1}^{|V|} \sum_{i=1}^{|D|} N(w_s, d_i) P(y_i = c_j|d_i)},$$

$$\hat{\theta}_{c_j} \equiv P(c_j|\hat{\theta}) = \frac{1 + \sum_{i=1}^{|D|} P(y_i = c_j|d_i)}{|\mathcal{C}| + |\mathcal{D}|}.$$

# EM with Unlabeled Data Increases Accuracy

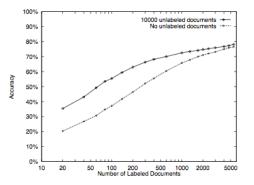


Figure 2. Classification accuracy on the 20 Newsgroups data set, both with and without 10,000 unlabeled documents. With small amounts of training data, using EM yields more accurate classifiers. With large amounts of labeled training data, accurate parameter estimates can be obtained without the use of unlabeled data, and the two methods begin to converge.

# EM with Unlabeled Data Increases Accuracy

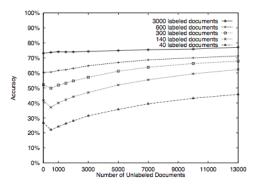


Figure 3. Classification accuracy while varying the number of unlabeled documents. The effect is shown on the 20 Newsgroups data set, with 5 different amounts of labeled documents, by varying the amount of unlabeled data on the horizontal axis. Having more unlabeled data helps. Note the dip in accuracy when a small amount of unlabeled data is added to a small amount of labeled data. We hypothesize that this is caused by extreme, almost 0 or 1, estimates of component membership,  $P(c_j|d_i,\hat{\theta})$ , for the unlabeled documents (as caused by naive Bayes' word independence assumption).

# The Evolution of Naive Bayes over two EM iteration on WebKB data using 2500 unlabeled documents

Table 3. Lists of the words most predictive of the course class in the WebKB data set, as they change over iterations of EM for a specific trial. By the second iteration of EM, many common course-related words appear. The symbol D indicates an arbitrary digit.

Iteration 0	Iteration 1	Iteration 2	
intelligence	DD		
DD	D	DD	
artificial	lecture	lecture	
understanding	cc	cc	
$DD_{W}$	$D^{\star}$	DD:DD	
dist	DD:DD	due	
identical	handout	$D^{\star}$	
rus	due	homework	
arrange	problem	assignment	
games	set	handout	
dartmouth	tay	set	
natural	DDam	hw	
cognitive	yurttas	exam	
logic	homework	problem	
proving	kfoury	DDam	
prolog	sec	postscript	
knowledge	postscript	solution	
human	exam	quiz	
representation	solution	chapter	
field	assaf	ascii	

#### EM Can Hurt Performance

When data do not fit the generative model assumption, i.e., mixture components that best explain the unlabeled data are not correlated with the class labels.

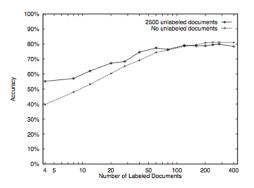


Figure 4. Classification accuracy on the WebKB data set, both with and without 2500 unlabeled documents. When there are small numbers of labeled documents, EM improves accuracy. When there are many labeled documents, however, EM degrades performance slightly—indicating a misfit between the data and the assumed generative model.

# Varying the Weight of the Unlabeled Data

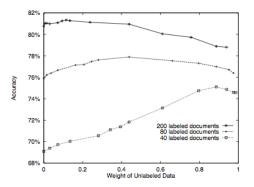
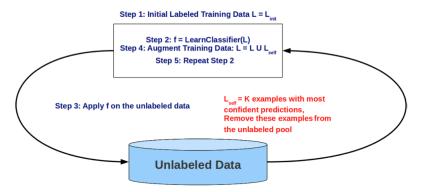


Figure 5. The effects of varying  $\lambda$ , the weighting factor on the unlabeled data in EM- $\lambda$ . These three curves from the WebKB data set correspond to three different amounts of labeled data. When there is less labeled data, accuracy is highest when more weight is given to the unlabeled data. When the amount of labeled data is large, accurate parameter estimates are attainable from the labeled data alone, and the unlabeled data should receive less weight. With moderate amounts of labeled data, accuracy is better in the middle than at either extreme. Note the magnified vertical scale.

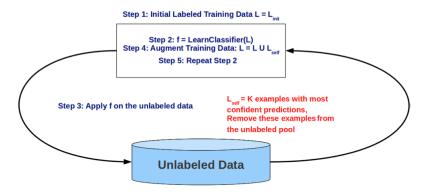
### The Self-Training Approach to SSL

- Given: Small amount of initial labeled training data and large amount of unlabeled data
- Idea: Train, predict, re-train using your own (best) predictions, repeat



## The Self-Training Approach to SSL

- Given: Small amount of initial labeled training data and large amount of unlabeled data
- Idea: Train, predict, re-train using your own (best) predictions, repeat



- Can be used with any supervised learner. Often works well in practice
- Caution: Prediction mistake can reinforce itself.

# Co-Training Approach to SSL

- Given: Labeled data  $\{\mathbf{x}_i, y_i\}_{i=1}^L$ , unlabeled data  $\{\mathbf{x}_i\}_{i=L+1}^{L+U}$
- Each example has 2 views:  $\mathbf{x} = [\mathbf{x}^{(1)}, \mathbf{x}^{(2)}]$
- How do we get different views?
  - Naturally available (different types of features for the same object)
    - ★ Webpages: view 1 from page text; view 2 from page URL
  - ... or by splitting the original features into two groups
- Assumption: Given sufficient data, each view is good enough to learn from
- Co-training: Utilize both views to learn better with fewer labeled examples
- Idea: Each view teaching (training) the other view
- Technical Condition: Views should be conditionally independent

### Redundantly Predictive Features

Assumption: Given sufficient data, either view is sufficient for learning

• There are  $f_1$  and  $f_2$  s.t.  $f(x) = f_1(x) = f_2(x) = y$  for all (x, y) pairs.

### Redundantly Predictive Features

Assumption: Given sufficient data, either view is sufficient for learning

• There are  $f_1$  and  $f_2$  s.t.  $f(x) = f_1(x) = f_2(x) = y$  for all (x, y) pairs.







#### Co-Training

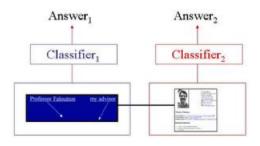
- Idea: Use small labeled sample to learn initial rules.
  - "my advisor" pointing to a page is a good indicator it is a faculty home page.
  - "I am teaching" on a page is a good indicator it is a faculty home page.



- Then look for unlabeled examples where one rule is confident and the other is not. Have it label the example for the other.
- Train 2 classifiers, one on each type of info. Use each to help train the other.
- Basic hope is that two views are consistent. Using agreement as proxy for labeled data.

### Co-Training

- Key idea: The classifiers  $C_1$  and  $C_2$  must:
  - Correctly classify labeled examples
  - Agree on classification of unlabeled.



# Co-Training Algorithm #1

- Given: Labeled data L, unlabeled data U
- Loop:
  - Train f<sub>1</sub> (hyperlink classifier) using L
  - ► Train f<sub>2</sub> (page classifier) using L
  - ▶ Allow  $f_1$  to label p positive, n negative examples from U
  - ▶ Allow  $f_2$  to label p positive, n negative examples from U
  - ▶ Add these self-labeled examples to *L*.

#### Co-Training Results on WebKB

Training Naive Bayes classifiers on 12 labeled examples, 1000 unlabeled.

	Page-based classifier	Hyperlink-based classifier	Combined classifier
Supervised training	12.9	12.4	11.1
Co-training	6.2	11.6	5.0

Table 2: Error rate in percent for classifying web pages as course home pages. The top row shows errors when training on only the labeled examples. Bottom row shows errors when co-training, using both labeled and unlabeled examples.

# Co-Training Results on WebKB

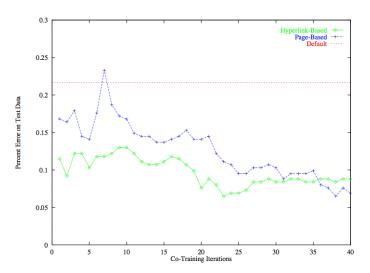


Figure 2: Error versus number of iterations for one run of co-training experiment.

# Co-Training Algorithm #2

- Given: Labeled data L, unlabeled data U
- Create two labeled datasets  $L_1$  and  $L_2$  from L using views 1 and 2
- Learn classifiers  $f_1$  from  $L_1$  and  $f_2$  from  $L_2$
- ullet Apply  $f_1$  and  $f_2$  on unlabeled data pool U to predict labels
  - Predictions are made only using their own set (view) of features
- Add k most confident predictions  $(\mathbf{x}, f_1(\mathbf{x}))$  of  $f_1$  to  $L_2$
- Add k most confident predictions  $(\mathbf{x}, f_2(\mathbf{x}))$  of  $f_2$  to  $L_1$
- Remove these examples from the unlabeled pool
- Re-train  $f_1$  using  $L_1$ ,  $f_2$  using  $L_2$
- Like self-training but two classifiers teaching each other
- Finally, use a voting or averaging to make predictions on the test data

