each $1 \leq j \leq |V|$, from a normal distribution of zero mean and unit standard deviation (*i.e.*, $\Pr\left[a \leq r'_j \leq b\right] = \int_a^b (1/\sqrt{2\pi})\, \mathrm{e}^{-x^2/2}\, \mathrm{d}x$ for any $\emptyset \subset \left[a,b\right] \subset \mathbb{R}$), and then normalizing the coordinates so that the 2-norm of the resulting vector is 1, *i.e.*, setting $r_j = \dfrac{r'_j}{\sqrt{\sum_{i=1}^{n}(r'_i)^2}}$ for $j = 1, 2, \ldots, |V|$.

(b) For each $v \in V$, set $\ell_v = \begin{cases} 1, & \text{if } \overrightarrow{r} \cdot \overrightarrow{\ell_v} \geq 0 \\ -1, & \text{otherwise} \end{cases}$ where "$\cdot$" denotes the vector inner product.

Let ℓ_{opt} be an optimum node labeling function for SGN-CONST with $\left|F_{\ell_{\text{opt}}}\right|$ consistent edges. It can be shown that the above randomized algorithm provides a node labeling function ℓ_{approx} to SGN-CONST with $\left|F_{\ell_{\text{approx}}}\right|$ consistent edges such that

$$\mathbb{E}\left[\,\left|F_{\ell_{\text{approx}}}\right|\,\right] \geq \kappa \left|F_{\ell_{\text{opt}}}\right|$$

where $\kappa = \min\left\{ \min_{0\leq\theta\leq\pi} \dfrac{2\,\theta/\pi}{1-\cos\theta},\ \min_{0\leq\theta\leq\pi} \dfrac{2-(2\theta/\pi)}{1+\cos\theta} \right\}$. It can be shown using elementary calculus that $\kappa > 0.87856$; thus on an average the approximate solution retains *at least* 87.85% of the number of consistent edges in an optimal solution. The above randomized approach to compute the mappings P_v can be made deterministic (*i.e.*, can be "de-randomized") [70], but the derandomization procedure is complicated. Instead, one usually runs the randomized algorithm for computing the P_vs many times and accepts the best of these solutions; it can be shown that such an approach retains at least κ fraction of the optimal number of consistent edges with *very high probability* [84].

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