SET THEORETIC PROOFS & IDENTITIES

Useful Definitions

For $A$, $B$ subsets of universal set $U$:

$x \in A \cap B \iff x \in A \land x \in B$

$x \in A \cup B \iff x \in A \lor x \in B$

$x \in A - B \iff x \in A \land x \notin B$

$x \in A^C \iff x \notin A$

$(x, y) \in A \times B \iff x \in A \land y \in B$

Subset Proofs: $A \subseteq B$

To Prove: $A \subseteq B$

Proof:
Suppose $x \in A$. [$x$ is a particular but arbitrarily chosen element of $A$]

\[\vdash \text{deductions based on definition of } A\]

Then $x \in B$.
Therefore $A \subseteq B$.  \[\square\]

Equality Proofs: $A = B$

To Prove: $A = B$

Proof:

Subproof that $A \subseteq B$:
Suppose $x \in A$. [$x$ is a particular but arbitrarily chosen element of $A$]

\[\vdash \text{deductions based on definition of } A\]

Then $x \in B$.
Therefore, $A \subseteq B$.

Subproof that $B \subseteq A$:
Suppose $x \in B$. [$x$ is a particular but arbitrarily chosen element of $B$]

\[\vdash \text{deductions based on definition of } B\]

Then $x \in A$.
Therefore, $B \subseteq A$.

Thus $A \subseteq B \land B \subseteq A$. Therefore, $A = B$.  \[\square\]
Set Identities from 6.2 of Epp

Anything in capital letters is a set (with nothing special assumed about it) that is a subset of a universal set \( U \).

1. **Commutative Laws:**
   
   \[ A \cup B = B \cup A \]
   \[ A \cap B = B \cap A \]

2. **Associative Laws:**
   
   \[ (A \cup B) \cup C = A \cup (B \cup C) \]
   \[ (A \cap B) \cap C = A \cap (B \cap C) \]

3. **Distributive Laws:**
   
   \[ (A \cup B) \cap C = (A \cap C) \cup (B \cap C) \]
   \[ (A \cap B) \cup C = (A \cup C) \cap (B \cup C) \]

4. **Identity Laws:**
   
   \[ A \cup \emptyset = A \]
   \[ A \cap U = A \]

5. **Complement Laws:**
   
   \[ A \cup A^C = U \]
   \[ A \cap A^C = \emptyset \]

6. **Double Complement Law:**
   
   \[ (A^C)^C = A \]

7. **Idempotent Laws:**
   
   \[ A \cup A = A \]
   \[ A \cap A = A \]

8. **Universal Bound Laws:**
   
   \[ A \cup U = U \]
   \[ A \cap \emptyset = \emptyset \]

9. **De Morgan’s Laws:**
   
   \[ (A \cup B)^C = A^C \cap B^C \]
   \[ (A \cap B)^C = A^C \cup B^C \]

10. **Complements of \( U \) and \( \emptyset \):**
    
    \[ U^C = \emptyset \]
    \[ \emptyset^C = U \]

11. **Set Difference Law:**
    
    \[ A - B = A \cap B^C \]