CS 476 – Programming Language Design

William Mansky

Questions

Nobody has responded yet.

Hang tight! Responses are coming in.

From Typed Lambda Calculus to OCaml

- User-friendly syntax
- Basic types, tuples, records
- Inductive datatypes and pattern-matching
- Local declarations
- References
- ➢ Type inference
- Generics/polymorphism

Type Inference

- Typed lambda calculus has type annotations on arguments:

$$\lambda x\!:\text{int}.\,\lambda y\!:\text{int}.\,x \qquad\qquad \lambda x\!:\text{int}\to\text{int}.\,\lambda y\!:\text{int}.\,x\ y$$

- But OCaml doesn't always need them:

```
let f x y = x + y
let g f x = f x + f 3
```

- How does it figure out the types?

Type Inference

- Our typing rules have the form:

$$\frac{\Gamma \vdash l_1 : \tau_1 \rightarrow \tau_2 \quad \Gamma \vdash l_2 : \tau_1}{\Gamma \vdash l_1\ l_2 : \tau_2} \qquad \frac{(n \text{ is a number literal})}{\Gamma \vdash n : \text{int}}$$

- First-pass inference algorithm: mark literals with their types, then pass types down through the proof tree
  - Like the `type_of` function for imperative/OO expressions

Direct Type Inference

$$\frac{}{\Gamma \vdash \texttt{if 2+3=5 then 1 else 2} : ?}$$

Direct Type Inference

$$\frac{\Gamma \vdash \texttt{2+3=5} : ? \quad \Gamma \vdash \texttt{1} : ? \quad \Gamma \vdash \texttt{2} : ?}{\Gamma \vdash \texttt{if 2+3=5 then 1 else 2} : ?}$$

Direct Type Inference

$$\dfrac{\dfrac{\Gamma \vdash \texttt{2+3} : ? \quad \Gamma \vdash \texttt{5} : ?}{\Gamma \vdash \texttt{2+3=5} : ?} \quad \Gamma \vdash \texttt{1} : ? \quad \Gamma \vdash \texttt{2} : ?}{\Gamma \vdash \texttt{if 2+3=5 then 1 else 2} : ?}$$

Direct Type Inference

$$\dfrac{\dfrac{\Gamma \vdash \texttt{2+3} : ? \qquad \Gamma \vdash \texttt{5} : \text{int}}{\Gamma \vdash \texttt{2+3=5} : ?} \qquad \Gamma \vdash \texttt{1} : \text{int} \qquad \Gamma \vdash \texttt{2} : \text{int}}{\Gamma \vdash \texttt{if 2+3=5 then 1 else 2} : ?}$$

Direct Type Inference

$$\dfrac{\dfrac{\Gamma \vdash \texttt{2+3} : \mathrm{int} \quad \Gamma \vdash \texttt{5} : \mathrm{int}}{\Gamma \vdash \texttt{2+3=5} \;:\, ?} \qquad \Gamma \vdash \texttt{1} : \mathrm{int} \quad \Gamma \vdash \texttt{2} : \mathrm{int}}{\Gamma \vdash \texttt{if 2+3=5 then 1 else 2} : ?}$$

Direct Type Inference

$$\dfrac{\dfrac{\Gamma \vdash \texttt{2+3} : \mathrm{int} \quad \Gamma \vdash \texttt{5} : \mathrm{int}}{\Gamma \vdash \texttt{2+3=5} : \mathrm{bool}} \quad \Gamma \vdash \texttt{1} : \mathrm{int} \quad \Gamma \vdash \texttt{2} : \mathrm{int}}{\Gamma \vdash \texttt{if 2+3=5 then 1 else 2} : ?}$$

Direct Type Inference

$$\dfrac{\dfrac{\Gamma \vdash \texttt{2+3} : \text{int} \quad \Gamma \vdash \texttt{5} : \text{int}}{\Gamma \vdash \texttt{2+3=5} : \text{bool}} \quad \Gamma \vdash \texttt{1} : \text{int} \quad \Gamma \vdash \texttt{2} : \text{int}}{\Gamma \vdash \texttt{if 2+3=5 then 1 else 2} : \text{int}}$$

Direct Type Inference

$$\dfrac{\dfrac{\Gamma \vdash \texttt{2+3} : \text{int} \quad \Gamma \vdash \texttt{5} : \text{int}}{\Gamma \vdash \texttt{2+3=5} : \text{bool}} \quad \Gamma \vdash \texttt{1} : \text{int} \quad \Gamma \vdash \texttt{2} : \text{int}}{\Gamma \vdash \texttt{if 2+3=5 then 1 else 2} : \text{int}}$$

```
let type_of (e : exp) : typ option =
  match e with
  | Num i -> Some IntTy
  | Add (e1, e2) -> (match type_of e1, type_of e2 with
                     | Some IntTy, Some IntTy -> Some IntTy
                     | _ -> None)
```

Questions

Nobody has responded yet.

Hang tight! Responses are coming in.

Direct Type Inference

```
let type_of (gamma : context) (e : exp) : typ option =
  match e with
  | Num i -> Some IntTy
  | Add (e1, e2) -> (match type_of e1, type_of e2 with
                     | Some IntTy, Some IntTy -> Some IntTy
                     | _ -> None)
  | Var x -> lookup gamma x
```

Type Inference

$\Gamma \vdash$ `fun f -> fun x -> f x + f 3 : ?`

- Exercise: What type does this term have? How would you figure it out?

Type Inference

$$\frac{\dfrac{\Gamma[\texttt{f} \mapsto ?, \texttt{x} \mapsto ?] \vdash \texttt{f x + f 3} : ?}{\cdots}}{\Gamma \vdash \texttt{fun f -> fun x -> f x + f 3} : ?}$$

Type Inference

$$\dfrac{\dfrac{\dfrac{\dfrac{\texttt{f} : ? \quad \texttt{x} : ?}{\cdots} \qquad \dfrac{\texttt{f} : ? \quad \texttt{3} : ?}{\cdots}}{\Gamma[\texttt{f} \mapsto ?, \texttt{x} \mapsto ?] \vdash \texttt{f x + f 3} : ?}}{\cdots}}{\Gamma \vdash \texttt{fun f -> fun x -> f x + f 3} : ?}$$

Type Inference

$$\dfrac{\dfrac{\dfrac{\mathtt{f} : ? \quad \mathtt{x} : ?}{\cdots} \quad \dfrac{\mathtt{f} : \text{int} \to ? \quad \mathtt{3} : \text{int}}{\cdots}}{\Gamma[\mathtt{f} \mapsto ?, \mathtt{x} \mapsto ?] \vdash \mathtt{f\ x\ +\ f\ 3} : ?}}{\dfrac{\cdots}{\Gamma \vdash \mathtt{fun\ f\ \text{->}\ fun\ x\ \text{->}\ f\ x\ +\ f\ 3} : ?}}$$

Type Inference

$$\dfrac{\dfrac{\dfrac{\texttt{f} : \text{int} \to ? \quad \texttt{x} : ?}{\cdots} \quad \dfrac{\texttt{f} : \text{int} \to ? \quad \texttt{3} : \text{int}}{\cdots}}{\Gamma[\texttt{f} \mapsto \text{int} \to ?, \texttt{x} \mapsto ?] \vdash \texttt{f x + f 3} : ?}}{\dfrac{\cdots}{\Gamma \vdash \texttt{fun f -> fun x -> f x + f 3} : ?}}$$

Type Inference

$$\dfrac{\dfrac{\dfrac{\dfrac{\texttt{f} : \text{int} \to ? \quad \texttt{x} : \text{int}}{\cdots} \qquad \dfrac{\texttt{f} : \text{int} \to ? \quad \texttt{3} : \text{int}}{\cdots}}{\Gamma[\texttt{f} \mapsto \text{int} \to ?, \texttt{x} \mapsto \text{int}] \vdash \texttt{f x + f 3} : ?}}{\cdots}}{\Gamma \vdash \texttt{fun f -> fun x -> f x + f 3} : ?}$$

Type Inference

$$\dfrac{\dfrac{\dfrac{\mathtt{f} : \mathrm{int} \to ? \quad \mathtt{x} : \mathrm{int}}{\cdots} \quad \dfrac{\mathtt{f} : \mathrm{int} \to ? \quad \mathtt{3} : \mathrm{int}}{\cdots}}{\Gamma[\mathtt{f} \mapsto \mathrm{int} \to \mathrm{int}, \mathtt{x} \mapsto \mathrm{int}] \vdash \mathtt{f\ x\ +\ f\ 3} : \mathrm{int}}}{\dfrac{\cdots}{\Gamma \vdash \mathtt{fun\ f\ \text{->}\ fun\ x\ \text{->}\ f\ x\ +\ f\ 3} : ?}}$$

Type Inference

$$\dfrac{\dfrac{\dfrac{\mathtt{f} : \text{int} \to ? \quad \mathtt{x} : \text{int}}{\cdots} \quad \dfrac{\mathtt{f} : \text{int} \to ? \quad \mathtt{3} : \text{int}}{\cdots}}{\Gamma[\mathtt{f} \mapsto \text{int} \to \text{int}, \mathtt{x} \mapsto \text{int}] \vdash \mathtt{f\ x\ +\ f\ 3} : \text{int}}}{\dfrac{\cdots}{\Gamma \vdash \mathtt{fun\ f\ \text{->}\ fun\ x\ \text{->}\ f\ x\ +\ f\ 3} : (\text{int} \to \text{int}) \to \text{int} \to \text{int}}}$$

Type Inference

- With bound variables, type inference goes both ways:
  - From top to bottom, as we learn the types of literals
  - From bottom to top, as we see how variables are used

- More powerful algorithm: start with some unknown variable τ for the type of the expression, and gather *constraints* on what τ must be

Questions

Nobody has responded yet.

Hang tight! Responses are coming in.

Type Inference

- With bound variables, type inference goes both ways:
  - From top to bottom, as we learn the types of literals
  - From bottom to top, as we see how variables are used

```
fun f -> fun x -> f x + f 3
```

- More powerful algorithm: start with some unknown variable τ for the type of the expression, and gather *constraints* on what τ must be

Constraint-Based Type Inference

- We can do this in two steps:
  - First, introduce *type variables* and gather constraints on them
  - Second, find a solution to the constraints and fill in the variables
- For step 1, we need constraints for each typing rule:

$$\frac{\Gamma \vdash l_1 : \tau_1 \to \tau_2 \quad \Gamma \vdash l_2 : \tau_1}{\Gamma \vdash l_1\ l_2 : \tau_2} \quad \Rightarrow \quad \frac{\Gamma \vdash l_1 : \tau_1 \quad \Gamma \vdash l_2 : \tau_2}{\Gamma \vdash l_1\ l_2 : \tau \mid \{\tau_1 = \tau_2 \to \tau\}}$$

- $\Gamma \vdash l : \tau \mid S$ means "l has type τ in context Γ, as long as constraints S are satisfied"

Constraint-Based Type Inference

- For step 1, we need constraints for each typing rule:

$$\frac{\Gamma \vdash l_1 : \tau_1 \to \tau_2 \quad \Gamma \vdash l_2 : \tau_1}{\Gamma \vdash l_1\, l_2 : \tau_2} \Rightarrow \frac{\Gamma \vdash l_1 : \tau_1 \quad \Gamma \vdash l_2 : \tau_2}{\Gamma \vdash l_1\, l_2 : \tau \mid \{\tau_1 = \tau_2 \to \tau\}}$$

- $\Gamma \vdash l : \tau \mid S$ means "l has type τ in context Γ, as long as constraints S are satisfied"

```
let get_constraints (gamma : context) (e : exp) : (typ * constraints) =
…
```

Constraint-Based Type Inference: Rules

$$\frac{(n \text{ is a number literal})}{\Gamma \vdash n : \text{int} \mid \{\}} \qquad \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau \mid \{\}}$$

$$\frac{\Gamma \vdash l_1 : \tau_1 \mid S_1 \quad \Gamma \vdash l_2 : \tau_2 \mid S_2}{\Gamma \vdash l_1 + l_2 : \text{int} \mid \{\tau_1 = \text{int}, \tau_2 = \text{int}\} \cup S_1 \cup S_2}$$

$$\frac{\Gamma[x \mapsto \tau_1] \vdash l : \tau_2 \mid S}{\Gamma \vdash (\texttt{fun}\ x : \tau_1\ \texttt{->}\ l) : \tau_1 \to \tau_2 \mid S} \qquad \frac{\Gamma[x \mapsto \tau_1] \vdash l : \tau_2 \mid S \quad \tau_1 \text{ fresh}}{\Gamma \vdash (\texttt{fun}\ x\ \texttt{->}\ l) : \tau_1 \to \tau_2 \mid S}$$

Constraint-Based Type Inference: Rules

$$\frac{\Gamma \vdash l_1 : \tau_1 \mid S_1 \quad \Gamma \vdash l_2 : \tau_2 \mid S_2}{\Gamma \vdash l_1 \mathbin{\texttt{+}} l_2 : \mathrm{int} \mid \{\tau_1 = \mathrm{int}, \tau_2 = \mathrm{int}\} \cup S_1 \cup S_2}$$

$$\frac{\Gamma[x \mapsto \tau_1] \vdash l : \tau_2 \mid S}{\Gamma \vdash (\texttt{fun}\ x\texttt{:}\tau_1\ \texttt{->}\ l) : \tau_1 \to \tau_2 \mid S} \qquad \frac{\Gamma[x \mapsto \tau_1] \vdash l : \tau_2 \mid S \quad \tau_1\ \mathrm{fresh}}{\Gamma \vdash (\texttt{fun}\ x\ \texttt{->}\ l) : \tau_1 \to \tau_2 \mid S}$$

$$\frac{\Gamma \vdash l_1 : \tau_1 \mid S_1 \quad \Gamma \vdash l_2 : \tau_2 \mid S_2 \quad \tau\ \mathrm{fresh}}{\Gamma \vdash l_1\ l_2 : \tau \mid \{\tau_1 = \tau_2 \to \tau\} \cup S_1 \cup S_2}$$

Questions

Nobody has responded yet.

Hang tight! Responses are coming in.

Constraint-Based Type Inference: Example

```
fun f -> fun x -> f x + f 3
```

Constraint-Based Type Inference: Example

$$\frac{\Gamma[x \mapsto \tau_1] \vdash l : \tau_2 \mid S \qquad \tau_1 \text{ fresh}}{\Gamma \vdash \texttt{fun}\ x\ \texttt{->}\ l : \tau_1 \to \tau_2 \mid S}$$

$\{\} \vdash$ `fun f -> fun x -> f x + f 3` $: ? \mid ?$

Constraint-Based Type Inference: Example

$$\frac{\Gamma[x \mapsto \tau_1] \vdash l : \tau_2 \mid S \quad \tau_1 \text{ fresh}}{\Gamma \vdash \texttt{fun}\ x\ \texttt{->}\ l : \tau_1 \rightarrow \tau_2 \mid S}$$

$$\frac{\{\texttt{f} \mapsto \tau_1\} \vdash \texttt{fun x -> f x + f 3} : ? \mid ?}{\{\} \vdash \texttt{fun f -> fun x -> f x + f 3} : \tau_1 \rightarrow ? \mid ?}$$

Constraint-Based Type Inference: Example

$$\frac{\Gamma[x \mapsto \tau_1] \vdash l : \tau_2 \mid S \quad \tau_1 \text{ fresh}}{\Gamma \vdash \texttt{fun}\ x\ \texttt{->}\ l : \tau_1 \to \tau_2 \mid S}$$

$$\cfrac{\cfrac{\{\texttt{f} \mapsto \tau_1, \texttt{x} \mapsto \tau_2\} \vdash \texttt{f x + f 3} : ? \mid ?}{\{\texttt{f} \mapsto \tau_1\} \vdash \texttt{fun x -> f x + f 3} : \tau_2 \to ? \mid ?}}{\{\} \vdash \texttt{fun f -> fun x -> f x + f 3} : \tau_1 \to \tau_2 \to ? \mid ?}$$

Constraint-Based Type Inference: Example

$$\frac{\Gamma[x \mapsto \tau_1] \vdash l : \tau_2 \mid S \quad \tau_1 \text{ fresh}}{\Gamma \vdash \texttt{fun}\ x\ \texttt{->}\ l : \tau_1 \to \tau_2 \mid S}$$

$$\cfrac{\cfrac{\Gamma \vdash \texttt{f x + f 3} : ? \mid ?}{\{\texttt{f} \mapsto \tau_1\} \vdash \texttt{fun x -> f x + f 3} : \tau_2 \to ? \mid ?}}{\{\} \vdash \texttt{fun f -> fun x -> f x + f 3} : \tau_1 \to \tau_2 \to ? \mid ?}$$

$$\Gamma = \{\texttt{f} \mapsto \tau_1, \texttt{x} \mapsto \tau_2\}$$

Constraint-Based Type Inference: Example

Exercise: Which rule would we apply next?

$$\dfrac{\dfrac{\Gamma \vdash \texttt{f x + f 3} : ? \mid ?}{\{\texttt{f} \mapsto \tau_1\} \vdash \texttt{fun x -> f x + f 3} : \tau_2 \to ? \mid ?}}{\{\} \vdash \texttt{fun f -> fun x -> f x + f 3} : \tau_1 \to \tau_2 \to ? \mid ?}$$

$$\Gamma = \{\texttt{f} \mapsto \tau_1, \texttt{x} \mapsto \tau_2\}$$

Constraint-Based Type Inference: Example

$$\frac{\Gamma \vdash l_1 : \tau_1 \mid S_1 \quad \Gamma \vdash l_2 : \tau_2 \mid S_2}{\Gamma \vdash l_1 + l_2 : \text{int} \mid \{\tau_1 = \text{int}, \tau_2 = \text{int}\} \cup S_1 \cup S_2}$$

$$\frac{\dfrac{\Gamma \vdash \texttt{f x + f 3} : ? \mid ?}{\{\texttt{f} \mapsto \tau_1\} \vdash \texttt{fun x -> f x + f 3} : \tau_2 \to ? \mid ?}}{\{\} \vdash \texttt{fun f -> fun x -> f x + f 3} : \tau_1 \to \tau_2 \to ? \mid ?}$$

$$\Gamma = \{\texttt{f} \mapsto \tau_1, \texttt{x} \mapsto \tau_2\}$$

Constraint-Based Type Inference: Example

$$\frac{\Gamma \vdash l_1 : \tau_1 \mid S_1 \quad \Gamma \vdash l_2 : \tau_2 \mid S_2}{\Gamma \vdash l_1 + l_2 : \text{int} \mid \{\tau_1 = \text{int}, \tau_2 = \text{int}\} \cup S_1 \cup S_2}$$

$$\frac{\dfrac{\dfrac{\Gamma \vdash \texttt{f x} : ? \mid ? \quad \Gamma \vdash \texttt{f 3} : ? \mid ?}{\Gamma \vdash \texttt{f x + f 3} : \text{int} \mid \{? = \text{int}, ? = \text{int}, ?\}}}{\{\texttt{f} \mapsto \tau_1\} \vdash \texttt{fun x -> f x + f 3} : \tau_2 \to ? \mid ?}}{\{\} \vdash \texttt{fun f -> fun x -> f x + f 3} : \tau_1 \to \tau_2 \to ? \mid ?}$$

$$\Gamma = \{\texttt{f} \mapsto \tau_1, \texttt{x} \mapsto \tau_2\}$$

Constraint-Based Type Inference: Example

$$\frac{\Gamma \vdash l_1 : \tau_1 \mid S_1 \quad \Gamma \vdash l_2 : \tau_2 \mid S_2 \quad \tau \text{ fresh}}{\Gamma \vdash l_1\ l_2 : \tau \mid \{\tau_1 = \tau_2 \rightarrow \tau\} \cup S_1 \cup S_2}$$

$$\cfrac{\cfrac{\cfrac{\Gamma \vdash \texttt{f x} : ? \mid ? \quad \Gamma \vdash \texttt{f 3} : ? \mid ?}{\Gamma \vdash \texttt{f x + f 3} : \text{int} \mid \{? = \text{int}, ? = \text{int}, ?\}}}{\{\texttt{f} \mapsto \tau_1\} \vdash \texttt{fun x -> f x + f 3} : \tau_2 \rightarrow ? \mid ?}}{\{\} \vdash \texttt{fun f -> fun x -> f x + f 3} : \tau_1 \rightarrow \tau_2 \rightarrow ? \mid ?}$$

$$\Gamma = \{\texttt{f} \mapsto \tau_1, \texttt{x} \mapsto \tau_2\}$$

Constraint-Based Type Inference: Example

$$\frac{\Gamma \vdash l_1 : \tau_1 \mid S_1 \quad \Gamma \vdash l_2 : \tau_2 \mid S_2 \quad \tau \text{ fresh}}{\Gamma \vdash l_1\ l_2 : \tau \mid \{\tau_1 = \tau_2 \to \tau\} \cup S_1 \cup S_2}$$

$$\cfrac{\cfrac{\cfrac{\Gamma \vdash \texttt{f} : ? \mid ? \quad \Gamma \vdash \texttt{x} : ? \mid ?}{\Gamma \vdash \texttt{f x} : \tau_3 \mid \{? = ? \to \tau_3, ?\}} \quad \Gamma \vdash \texttt{f 3} : ? \mid ?}{\Gamma \vdash \texttt{f x + f 3} : \text{int} \mid \{? = \text{int}, ? = \text{int}, ?\}}}{\cfrac{\{\texttt{f} \mapsto \tau_1\} \vdash \texttt{fun x -> f x + f 3} : \tau_2 \to ? \mid ?}{\{\} \vdash \texttt{fun f -> fun x -> f x + f 3} : \tau_1 \to \tau_2 \to ? \mid ?}}$$

$$\Gamma = \{\texttt{f} \mapsto \tau_1, \texttt{x} \mapsto \tau_2\}$$

Constraint-Based Type Inference: Example

$$\dfrac{\dfrac{\dfrac{\overline{\Gamma \vdash \texttt{f} : \tau_1 \mid \{\}} \quad \overline{\Gamma \vdash \texttt{x} : \tau_2 \mid \{\}}}{\Gamma \vdash \texttt{f x} : \tau_3 \mid \{? = ? \to \tau_3, ?\}} \quad \Gamma \vdash \texttt{f 3} : ? \mid ?}{\Gamma \vdash \texttt{f x + f 3} : \text{int} \mid \{\tau_3 = \text{int}, ? = \text{int}, ?\}}}{\dfrac{\{\texttt{f} \mapsto \tau_1\} \vdash \texttt{fun x -> f x + f 3} : \tau_2 \to ? \mid ?}{\{\} \vdash \texttt{fun f -> fun x -> f x + f 3} : \tau_1 \to \tau_2 \to ? \mid ?}}$$

$$\Gamma = \{\texttt{f} \mapsto \tau_1, \texttt{x} \mapsto \tau_2\}$$

Constraint-Based Type Inference: Example

$$\dfrac{\dfrac{\dfrac{\overline{\Gamma \vdash \texttt{f} : \tau_1 \mid \{\}} \quad \overline{\Gamma \vdash \texttt{x} : \tau_2 \mid \{\}}}{\Gamma \vdash \texttt{f x} : \tau_3 \mid \{\tau_1 = \tau_2 \to \tau_3\}} \quad \Gamma \vdash \texttt{f 3} : ? \mid ?}{\Gamma \vdash \texttt{f x + f 3} : \text{int} \mid \{\tau_3 = \text{int}, ? = \text{int}, ?\}}}{\dfrac{\{\texttt{f} \mapsto \tau_1\} \vdash \texttt{fun x -> f x + f 3} : \tau_2 \to ? \mid ?}{\{\} \vdash \texttt{fun f -> fun x -> f x + f 3} : \tau_1 \to \tau_2 \to ? \mid ?}}$$

$$\Gamma = \{\texttt{f} \mapsto \tau_1, \texttt{x} \mapsto \tau_2\}$$

Constraint-Based Type Inference: Example

$$
\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \mathtt{f} : \tau_1 \mid \{\}} \quad \dfrac{}{\Gamma \vdash \mathtt{x} : \tau_2 \mid \{\}}}{\Gamma \vdash \mathtt{f\ x} : \tau_3 \mid \{\tau_1 = \tau_2 \to \tau_3\}} \quad \dfrac{\dfrac{}{\Gamma \vdash \mathtt{f} : \tau_1 \mid \{\}} \quad \dfrac{}{\Gamma \vdash \mathtt{3} : \mathrm{int} \mid \{\}}}{\Gamma \vdash \mathtt{f\ 3} : \tau_4 \mid \{\tau_1 = \mathrm{int} \to \tau_4\}}}{\dfrac{\dfrac{\Gamma \vdash \mathtt{f\ x\ +\ f\ 3} : \mathrm{int} \mid \{? = \mathrm{int}, ? = \mathrm{int}, ?\}}{\{\mathtt{f} \mapsto \tau_1\} \vdash \mathtt{fun\ x\ \text{->}\ f\ x\ +\ f\ 3} : \tau_2 \to ? \mid ?}}{\{\} \vdash \mathtt{fun\ f\ \text{->}\ fun\ x\ \text{->}\ f\ x\ +\ f\ 3} : \tau_1 \to \tau_2 \to ? \mid ?}}
$$

$$\Gamma = \{\mathtt{f} \mapsto \tau_1, \mathtt{x} \mapsto \tau_2\}$$

Constraint-Based Type Inference: Example

$$\dfrac{\dfrac{\dfrac{\overline{\Gamma \vdash \texttt{f} : \tau_1 \mid \{\}} \quad \overline{\Gamma \vdash \texttt{x} : \tau_2 \mid \{\}}}{\Gamma \vdash \texttt{f x} : \tau_3 \mid \{\tau_1 = \tau_2 \rightarrow \tau_3\}} \quad \dfrac{\overline{\Gamma \vdash \texttt{f} : \tau_1 \mid \{\}} \quad \overline{\Gamma \vdash \texttt{3} : \text{int} \mid \{\}}}{\Gamma \vdash \texttt{f 3} : \tau_4 \mid \{\tau_1 = \text{int} \rightarrow \tau_4\}}}{\Gamma \vdash \texttt{f x + f 3} : \text{int} \mid \{\tau_3 = \text{int}, \tau_4 = \text{int}, \tau_1 = \tau_2 \rightarrow \tau_3, \tau_1 = \text{int} \rightarrow \tau_4\}}}{\dfrac{\{\texttt{f} \mapsto \tau_1\} \vdash \texttt{fun x -> f x + f 3} : \tau_2 \rightarrow ? \mid ?}{\{\} \vdash \texttt{fun f -> fun x -> f x + f 3} : \tau_1 \rightarrow \tau_2 \rightarrow ? \mid ?}}$$

$$\Gamma = \{\texttt{f} \mapsto \tau_1, \texttt{x} \mapsto \tau_2\}$$

Constraint-Based Type Inference: Example

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \mathtt{f} : \tau_1 \mid \{\}} \quad \dfrac{}{\Gamma \vdash \mathtt{x} : \tau_2 \mid \{\}}}{\Gamma \vdash \mathtt{f\ x} : \tau_3 \mid \{\tau_1 = \tau_2 \to \tau_3\}} \quad \dfrac{\dfrac{}{\Gamma \vdash \mathtt{f} : \tau_1 \mid \{\}} \quad \dfrac{}{\Gamma \vdash \mathtt{3} : \text{int} \mid \{\}}}{\Gamma \vdash \mathtt{f\ 3} : \tau_4 \mid \{\tau_1 = \text{int} \to \tau_4\}}}{\Gamma \vdash \mathtt{f\ x\ +\ f\ 3} : \text{int} \mid S_1}}{\dfrac{\{\mathtt{f} \mapsto \tau_1\} \vdash \mathtt{fun\ x\ ->\ f\ x\ +\ f\ 3} : \tau_2 \to ? \mid ?}{\{\} \vdash \mathtt{fun\ f\ ->\ fun\ x\ ->\ f\ x\ +\ f\ 3} : \tau_1 \to \tau_2 \to ? \mid ?}}$$

$$\Gamma = \{\mathtt{f} \mapsto \tau_1, \mathtt{x} \mapsto \tau_2\}$$

$$S_1 = \{\tau_3 = \text{int}, \tau_4 = \text{int}, \tau_1 = \tau_2 \to \tau_3, \tau_1 = \text{int} \to \tau_4\}$$

Constraint-Based Type Inference: Example

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \texttt{f} : \tau_1 \mid \{\}} \quad \dfrac{}{\Gamma \vdash \texttt{x} : \tau_2 \mid \{\}}}{\Gamma \vdash \texttt{f x} : \tau_3 \mid \{\tau_1 = \tau_2 \to \tau_3\}} \quad \dfrac{\dfrac{}{\Gamma \vdash \texttt{f} : \tau_1 \mid \{\}} \quad \dfrac{}{\Gamma \vdash \texttt{3} : \text{int} \mid \{\}}}{\Gamma \vdash \texttt{f 3} : \tau_4 \mid \{\tau_1 = \text{int} \to \tau_4\}}}{\Gamma \vdash \texttt{f x + f 3} : \text{int} \mid S_1}}{\dfrac{\{\texttt{f} \mapsto \tau_1\} \vdash \texttt{fun x -> f x + f 3} : \tau_2 \to \text{int} \mid S_1}{\{\} \vdash \texttt{fun f -> fun x -> f x + f 3} : \tau_1 \to \tau_2 \to \text{int} \mid S_1}}
$$

$$\Gamma = \{\texttt{f} \mapsto \tau_1, \texttt{x} \mapsto \tau_2\}$$

$$S_1 = \{\tau_3 = \text{int}, \tau_4 = \text{int}, \tau_1 = \tau_2 \to \tau_3, \tau_1 = \text{int} \to \tau_4\}$$

Questions

Nobody has responded yet.

Hang tight! Responses are coming in.

HW7 Overview

- Implement type inference
- get_constraints function implements inference rules
- There's also a "constraint solving" section; for now, it just works