Problem 1. Prove that $L = \{\langle M \rangle \mid M$ is a DFA that accepts $w^*$ whenever it accepts $w\}$ is decidable.

Problem 2. Prove that a language $L$ is decidable if and only if $L^c$ is decidable.

Problem 3. Consider the problem of determining whether a computer program written in Python ever prints out “Hello world!” when run on some input $w$. Prove that this problem is undecidable. Formally, consider the language

$\{\langle P, w \rangle \mid P$ is a Python program that, on input $w$, prints Hello world! $\}$

and show that it is undecidable.

Problem 4. Consider the problem of determining whether a TM $M$ on input $w$ ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and prove that it is undecidable.

Problem 5. Show that the class of Turing-recognizable languages is not closed under complement.

Problem 6. Consider the language

$L = \{\langle M, w, q \rangle \mid M$ is a TM that when run on input $w$ never enters state $q\}$.

If $L$ is decidable, describe a TM that decides it. If $L$ is not decidable, prove it by giving a reduction from an undecidable language $L'$. That is, show $L' \leq L$.

Problem 7. In class, we proved that $A_{\text{TM}} \leq \text{HALT}_{\text{TM}}$ (although we didn’t use the terminology of reductions). Prove that $\text{HALT}_{\text{TM}} \leq A_{\text{TM}}$.

Problem 8. Prove that $EQ_{\text{CFG}}$ is co-Turing-recognizable by describing a TM that recognizes the complement.

Problem 9. Prove that $EQ_{\text{CFG}}$ is undecidable.

Problem 10. Use the results of Problems 8 and 9 to show that $EQ_{\text{CFG}}$ is not Turing-recognizable.