Problem 1 Show that any TM can be converted to one in which the head never attempts to move left on the left-most cell of the tape. [Hint: Use a new tape symbol.]

Problem 2 Prove that \( L = \{ \langle M \rangle \mid M \text{ is a DFA that accepts } w^k \text{ whenever it accepts } w \} \) is decidable. [Hint: Your decider should takes \( \langle M \rangle \) as input and construct a new DFA \( M' \). Then, it should use a decider for EQ\_DFA.]

Problem 3 Prove that a language \( L \) is decidable if and only if \( L^c \) is decidable.

Problem 4 Consider the problem of determining whether a computer program written in Python ever prints out “Hello world!” when run on some input \( w \). Prove that this problem is undecidable. Formally, consider the language

\[
HW = \{ \langle P, w \rangle \mid P \text{ is a Python program that, on input } w, \text{ prints Hello world!} \}
\]

and show that it is undecidable. [Hint: Prove this by contradiction. Assume that \( R \) is a decider for \( HW \). Build a new TM \( D \) that decides \( A_{TM} \) using \( R \) as a subroutine. Conclude that since \( A_{TM} \) is undecidable, this is a contradiction so \( HW \) must be undecidable.]

Problem 5 Consider the problem of determining whether a TM \( M \) on input \( w \) ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and prove that it is undecidable. [Hint: Use the result in problem 1 to build a new TM whose head only attempts to move left on the left-most cell of the tape when you want it to. Proceed similarly to problem 4.]

Problem 6 Show that the class of Turing-recognizable languages is not closed under complement.

Problem 7 Consider the language

\[
L = \{ \langle M, w, q \rangle \mid M \text{ is a TM that when run on input } w \text{ never enters state } q \}.
\]

If \( L \) is decidable, describe a TM that decides it. If \( L \) is not decidable, prove it.