Instructions

This assignment is due Sunday, April 22, 2018 at 11:59PM (Central Time). Solutions must be submitted on Gradescope. Your solutions must be typeset. Handwritten solutions will not be graded and will receive a 0.

Late submissions will be accepted within 24 hours after the deadline with a penalty of 25% of the assignment grade. No late submissions will be accepted more than 24 hours after the deadline.

Problem 1 Answer the following questions about the following DFA $M$ and give reasons for your answers.

![DFA Diagram]

a. [2 points] Is $\langle M, \text{abaab} \rangle \in A_{\text{DFA}}$?
b. [2 points] Is $\langle M, \text{bab} \rangle \in A_{\text{DFA}}$?
c. [2 points] Is $\langle M \rangle \in A_{\text{DFA}}$?
d. [2 points] Is $\langle M \rangle \in E_{\text{DFA}}$?
e. [2 points] Is $\langle M, M \rangle \in E_{\text{DFA}}$?

Problem 2 Closure properties of decidable languages.

a. [5 points] Prove that the class of decidable languages is closed under concatenation. [Hint: Let $M_1$ and $M_2$ be TMs that decide languages $A$ and $B$. Construct a new TM $M$ to decide $A \circ B$. TM $M$ will take as input some string $w = w_1w_2 \cdots w_n$ where each $w_i \in \Sigma$ and will have to divide $w$ into two pieces $x = w_1w_2 \cdots w_k$ and $y = w_{k+1}w_{k+2} \cdots w_n$ for some $0 \leq k \leq n$ and check that $x \in A$ and $y \in B$. Make sure that $M$ tries all $n+1$ possible $x$ and $y$.]
b. [10 points] Prove that the class of decidable languages is closed under Kleene star. [Hint: Let $M_1$ decide language $A$. Construct a new TM $M$ to decide $A^*$. Recall that for string $w$ to be in $A^*$, there must be a division of $w$ into $k$ pieces $w = w_1w_2 \cdots w_k$ for some $k \geq 0$ such that each $w_i \in A$. TM $M$ will have to try all possible divisions for all values of $k$ up to some number. If $|w| = n$, think about which values of $k$ the TM $M$ needs to consider. This problem is tricky!]

Problem 3 Closure properties of Turing-recognizable languages.

a. [10 points] Prove that the class of Turing-recognizable languages is closed under concatenation. [Hint: This is similar to the previous problem but now you have the issue that the TMs $M_1$ or $M_2$ may not halt on some division of $w$ into $x$ and $y$, but will halt and accept on some other division. Have $M$ first write down all of the possible splits and then simulate $M_1$ and $M_2$ on each of the possible $x$ and $y$ in “parallel” by performing one step of the simulation of each TM at a time.]

b. [15 points] Prove that the class of Turing-recognizable languages is closed under Kleene star. [Hint: Use the hints for Problems 2b and 3a.]

Problem 4 [10 points] Let $A = \{\langle R, S \rangle \mid R$ and $S$ are regular expressions such that $L(R) \subseteq L(S)\}$. Prove that $A$ is decidable by giving a TM that decides it. [Hint: Your TM should construct one or more DFAs and use a decider for a language we’ve already shown to be decidable as a subroutine.]

Problem 5 [10 points] Let $B = \{\langle R \rangle \mid R$ is a regular expression describing a language containing at least one string $w$ that has 111 as a substring (i.e., $w = x111y$ for some $x$ and $y$)$\}$. Prove that $B$ is decidable by giving a decider for it. [Hint: Your TM should construct one or more DFAs and use a decider for a language we’ve already shown to be decidable as a subroutine.]

Problem 6 [10 points] Prove that $EQ_{CFG}$ is undecidable. [Hint: Give a reduction from $ALL_{CFG}$.]

Problem 7 [10 points] Let $C = \{\langle M \rangle \mid M$ is a TM that accepts $w^e$ iff it accepts $w\}$. Prove that $C$ is undecidable. [Hint: Give a reduction from $A_{TM}$.]

Problem 8 [10 points] Consider the problem of determining whether a Turing machine $M$ on an input $w$ ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.