Instructions

This assignment is due Sunday, May 06, 2018 at 11:59PM (Central Time). Solutions must be submitted on Gradescope. Your solutions must be typeset. Handwritten solutions will not be graded and will receive a 0.

Late submissions will be accepted within 24 hours after the deadline with a penalty of 25% of the assignment grade. No late submissions will be accepted more than 24 hours after the deadline.

Problem 1 [10 points] Prove that $A$ is decidable if and only if $A \leq_m \varepsilon$. [Hint: Show that if $A$ is decidable, then $A \leq_m \varepsilon$ and if $A \leq_m \varepsilon$, then $A$ is decidable.]

Problem 2 [15 points] Let $\varepsilon_{TM} = \{ \langle M \rangle \mid M$ is a TM and $M$ accepts $\varepsilon \}$. Prove that $A_{TM} \leq_m \varepsilon_{TM}$ by constructing a TM that takes $\langle M, w \rangle$ as input and outputs $\langle M' \rangle$ such that $M$ accepts $w$ if and only if $M'$ accepts $\varepsilon$.

Problem 3 Prove that class $P$ is closed under:

a. [5 points] Union. [Hint: Let $M_1$ and $M_2$ be deterministic TMs that decide languages $A$ and $B$ in polynomial time. Construct a new deterministic TM $M$ that decides $A \cup B$ in polynomial time.]

b. [5 points] Complement. [Hint: Let $M$ be a deterministic TM that decides language $A$ in polynomial time. Construct a new deterministic TM $M'$ that decides $\overline{A}$ in polynomial time.]

Problem 4 [10 points] Prove that class $NP$ is closed under union. [Hint: Let $M_1$ and $M_2$ be nondeterministic TMs that decide languages $A$ and $B$ in polynomial time. Construct a new nondeterministic TM $M$ that decides $A \cup B$ in polynomial time. Alternatively, let $M_1$ and $M_2$ be deterministic TMs that verify languages $A$ and $B$ in polynomial time. Construct a new deterministic TM $M$ that verifies $A \cup B$ in polynomial time.]

Problem 5 [10 points] A cycle in a directed graph is a path that starts and ends at the same vertex.

Let $CYCLE = \{ \langle G, v \rangle \mid G$ is a directed graph that has a cycle starting at vertex $v \}$. 

1
Prove that CYCLE ∈ P. [Hint: Construct a deterministic TM that decides CYCLE in polynomial time and show that your construction is correct.]

**Problem 6 [15 points]** A triangle in an undirected graph is a 3-clique.

Let TRIANGLE = \{\langle G \rangle \mid G \text{ is an undirected graph that has a triangle} \}.

Prove that TRIANGLE ∈ P. [Hint: Construct a deterministic TM that decides TRIANGLE in polynomial time and show that your construction is correct.]

**Problem 7 [15 points]** A dominating set S is a subset of vertices of a graph G such that every vertex not in S is adjacent to at least one vertex in S.

Let DOMINATING-SET = \{\langle G, k \rangle \mid G \text{ is a graph that has a dominating set with } k \text{ vertices} \}.

Prove that DOMINATING-SET ∈ NP. [Hint: Construct a nondeterministic TM that decides DOMINATING-SET in polynomial time and show that your construction is correct. Alternatively, construct a deterministic TM that verifies DOMINATING-SET in polynomial time.]

**Problem 8 [15 points]** Two graphs G and H are said to be isomorphic if the vertices of G may be reordered so that G is identical to H.

Let ISO = \{\langle G, H \rangle \mid G \text{ and } H \text{ are isomorphic graphs} \}.

Prove that ISO ∈ NP. [Hint: Construct a nondeterministic TM that decides ISO in polynomial time and show that your construction is correct. Alternatively, construct a deterministic TM that verifies ISO in polynomial time.]