ClassCoords

An Interactive Large Scale Graphing Environment

UIC CS523 Term Project

Author: Tom Peterka
Project Contents

1. Final report

2. Teacher materials supplement

3. CD

4. Web site

Final Report

0. Introduction
  0.1 Abstract
  0.2 Background
  0.3 Goals
  0.4 Learning standards
  0.5 Student prerequisites

1. Brief system overview
  1.1 Introduction
  1.2 Usage
  1.3 Hardware

2. Instructional unit
  2.1 Teacher's and students' roles
  2.2 Classroom arrangement
  2.3 Brief description
  2.4 Opening Activity: Find the mall
  2.5 Timeline
  2.6 Presentation of introductory concepts
  2.7 Worksheet
  2.8 Maze game
  2.9 Quiz #1
  2.10 Data collection
  2.11 Graphing data points
  2.12 Graph analysis
  2.13 Reflection
  2.14 Quiz #2
  2.15 Where do we go from here?
  2.16 General assessment strategy

3. Detailed system description
  3.1 System description
  3.2 Block diagram
  3.3 Simulation
  3.4 Other suggested uses and instructional topics
  3.5 Estimated feasibility, cost and market demand

4. Conclusion
  4.1 Summary
  4.2 Suggestions and future work

5. Acknowledgements and references
  5.1 Acknowledgements
  5.2 References
0. Introduction

0.1 Abstract

ClassCoords, an interactive multimedia learning environment to help middle school mathematics students visualize and immerse themselves in a two-dimensional Cartesian coordinate system is proposed. Many math students learn the algorithms for graphing without a sound understanding and visualization of coordinate systems; this often becomes evident only later when they move to higher dimensional spaces. We propose a system to display a coordinate plane onto a fairly large area of the classroom floor, and allow students to walk in it and plot points via a hand-held tracked device. To demonstrate the effectiveness of the system, a sample instructional unit is included with the design, which focuses on introductory graphing concepts and graphing of measured experimental data. The learning activity is supported both by Illinois learning standards as well as the standards of the National Council of Teachers of Mathematics (NCTM), and comes complete with teacher materials, including assessments. The system will be simulated through an application which will be projected via a common PC projector, but without real-time tracking. Finally, feasibility, cost and market analysis is briefly discussed, as well as ideas for other instructional units that could be taught using the system.

0.2 Background

0.2.1 Cognitive theory

The following is a brief listing of some of the cognitive theory guiding principles that form a foundation for the design of the system and the accompanying instructional unit.

Learner centered design [Soloway, 1994]

What interactive learning environment would be complete without citing Elliot Soloway's landmark paper on learner centered design (LCD)? ClassCoords applies Soloway's four principles of LCD as follows:

- understanding
  ClassCoords is an engaging large-scale system that promotes understanding by immersing students in the problem space. The sample instructional unit has activities that specifically were designed to promote conceptual understanding of important concepts through tangible problems, rather than "burying" students with abstractions, formulas, or "number-crunching".
- motivation
  The instructional unit's opening activities are designed with motivation in mind; the primary goals are to capture students' interest and demonstrate the need for coordinate systems.
- diversity
  Diversity is incorporated in the instructional unit rather than in the system itself. A variety of activities are designed: written, oral discussion, active participation,
group and individual. The accompanying grading rubric also distributes the grading among the various activities, accommodating a variety of learning styles.

- **growth**
The simulated application provides only minimal non-fading scaffolds so far, and much more needs to be done in this area. However, in general the system affords room for growth by its applicability to a variety of lessons and units, both in mathematics and science. Some of these are listed in the detailed system description.

Inquiry based learning [Edelson, 1998]

The instructional unit was designed to approximately match Edelson's 3-step model of inquiry based learning. However, like Edelson's "Create-A-World" activity, the correlation between steps and activities can sometimes be a bit blurred.

- **motivate**
The opening two activities are designed to match the first step of Edelson's inquiry based learning model. The goal is to capture students' attention, demonstrate relevance, spark interest or curiosity, and create a place, or "handle" in their knowledge structure for the new unit.

- **acquire**
The acquisition of knowledge roughly occurs during the middle portion of the unit, as students gather experimental data and plot it. Also, a good deal of background information is taught conventionally by the teacher, which students acquire by listening and taking notes.

- **refine**
Students refine their knowledge by exploring, applying, and reflecting according to Edelson. This roughly occurs in the middle and end of the unit as data is graphed and linear relationships are explored. Opportunities for reflection exist throughout the unit.

How Students Learn [Pellegrino, 2002]

Several of Pellegrino's ideas are incorporated into the design of ClassCoords.

- **facts within structured framework**
The instructional unit is structured so that activities are performed within a context. For example, background and relevancy are stressed before any graphing is actually done. Graphing of linear data is done in the context of experimentation. At the end of the unit, other types of non-linear graphs are briefly explored. In this way, "placeholders" are created within a framework, and details are stored in their "place".

- **teaching depth over breadth**
The instructional unit originally included linear equations, their properties, and the correlation between geometric and algebraic representation. However, the study of linear equations is quite involved, and would have required a more superficial coverage of all the sub-topics. This was not desired, so linear equations were
deferred and introductory concepts are covered in more depth instead. In this way the focus remains on conceptual understanding and visualization.

- variety of learning strategies
A variety of activities are included with the unit, utilizing a variety of learning strategies. In this way we hope to reach the maximum number of learners and learning styles.

- assessment centered
Assessments are built into the sample unit throughout, rather than one large unit test at the end.

- community centered
ClassCoords affords collaboration because of its large scale. The entire classroom becomes a community of learners, and can also be divided into smaller groups.

- reflection
The system and accompanying instructional unit afford many opportunities for reflection through discussion, in-class demonstration of problem solutions, and worksheets.

0.2.2 Current tools for teaching graphing

- paper and pencil, chalkboard, overhead transparencies, etc.

These are the traditional lecture / presentation materials for the teacher as transmitter model. While appropriate for many situations, the problem is when traditional technology is used exclusively and all other methods are ignored.

- graphing calculator

This is a handheld calculator with graphic ability and graphing software applications. ClassCoords is essentially a large-scale introductory graphing calculator, large enough for people to walk around in, with a direct tracked
interface. Like ClassCoords, the graphing calculator is only a tool, and is only as useful as the learning activity with which it is used. The graphing calculator is a non-collaborative tool, used from roughly 7th grade through high school, and is most effective in high school levels where advanced features are maximized. Ideally, students should progress in graphing technology from ClassCoords to graphing calculators.

- **geoboard**
  This is a manipulative for graphing polygons, consisting of pegs for vertices and rubber bands for edges. It is useful for hands-on activities, not restricted to graphing, but has limited resolution. Its use is predominantly in earlier elementary grades.

- **masking tape or erasable marker on the floor, or seats used as coordinate locations**
  These are also examples of tools which are similar to ClassCoords, where the goal is large-scale immersion into the coordinate system. The coordinate system covers a sizeable portion of the classroom floor, and the differences lie in the method of marking the grid. (In the case of students' seats as point locations, students would typically stand in order to represent a point which is to be marked.)

### 0.3 Goals

**Project assignment**

To design a multimedia interactive learning environment for some problem domain with K-12 education using non-traditional technology (ie, interface should not be the typical desktop computer). The product should include both a description of the proposed system along with a description of an instructional unit using the system.

**Problem statement**

To design an interactive multimedia learning environment to help 6th and 7th grade math students visualize a two-dimensional coordinate system and use the system to graph experimental data points.

### 0.4 Learning standards
The following are standards that apply only to the instructional unit described below. The system can be used with other instructional units to satisfy other standards.

0.4.1 Illinois Learning Standards

Principles

The Illinois Learning Standards list several principles that are common to all curricula. ClassCoords along with the proposed learning activity satisfy the following:

- Application of learning
- Solving problems
- Communicating
- Using Technology
- Working on Teams
- Making connections

Standards

The following are the applicable individual Illinois math standards.

State goal 8: Use algebraic and analytical methods to identify and describe patterns and relationships in data, solve problems, and predict results.
B. Interpret and describe numerical relationships using tables, graphs and symbols
8.B.3 Use graphing technology and algebraic methods to analyze and predict linear relationships and make generalizations from linear patterns

State goal 10: Collect, organize, and analyze data using statistical methods; predict results; and interpret uncertainty using concepts of probability.
A. Organize, describe and make predictions from existing data.
10.A.3a Construct, read and interpret tables, graphs (including circle graphs) and charts to organize and represent data
B. Formulate questions, design data collection methods, gather and analyze data and communicate findings
10.B.3 Formulate questions (e.g., relationships between car age and mileage, average incomes and years of schooling), devise and conduct experiments or simulations, gather data, draw conclusions and communicate results to an audience using traditional methods and contemporary technologies.

0.4.2 NCTM Standards

The National Council of Teachers of Mathematics (NCTM) also have a set of principles which are common to all their standards. The following technology principle is satisfied:

Technology Principle
"Students can learn more mathematics more deeply with the appropriate and responsible use of technology. They can make and test conjectures. They can work at higher levels of generalization or abstraction... Technology cannot replace the mathematics teacher, nor can it be used as a replacement for basic understandings and intuitions. The teacher must make prudent decisions about when and how to use technology and should ensure that the technology is enhancing students’ mathematical thinking”

The following are the applicable NCTM math standards:

Standards for grades 6-8
"...students will learn significant amounts of algebra and geometry throughout grades 6, 7, and 8. Moreover, they will see algebra and geometry as interconnected with each other... They will have experience with both the geometric representation of algebraic ideas, such as visual models of algebraic identities, and the algebraic representation of geometric ideas, such as equations for lines represented on coordinate grids. ... They also will relate algebraic and geometric ideas to other topics—for example, ...when they represent an approximate line of fit for a scatterplot both geometrically and algebraically" 

Algebra
• identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.
• use graphs to analyze the nature of changes in quantities in linear relationships

Geometry
• precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties;
• use coordinate geometry to represent and examine the properties of geometric shapes;
• use visual tools such as networks to represent and solve problems;
• use geometric models to represent and explain numerical and algebraic relationships;

Representation
• create and use representations to organize, record, and communicate mathematical ideas;
• select, apply, and translate among mathematical representations to solve problems;
• use representations to model and interpret physical, social, and mathematical phenomena.

0.5 Student prerequisites

Integer and fractional arithmetic, ie, familiarity with positive and negative whole numbers and fractions and arithmetic operations on them is assumed. No prior experience with graphs or graphing is assumed.
1. Brief System Overview

1.1 Introduction

Students are immersed in a two-dimensional coordinate system that is displayed on a large portion of the classroom floor. Initial approximations indicate a 9' x 12' display. Practically any lesson in number sense, algebra, geometry, or data analysis that can profit from a visual representation would benefit this tool. For example, points can be located, distances measured, experimental data plotted, simple relations and functions investigated, geometric definitions can be studied, and linear transformations can be performed. Although the system is ideal for a large number of math and science topics, one particular math instructional activity is included with the system to demonstrate its viability.

1.2 Usage

A bright neon-colored sphere ("the ball") resembling a billiard ball with control buttons, is the primary interface. The ball would be carried to the desired location and an "enter" button would be pressed to enter the current point onto the grid. The ball is tracked via an optical position tracking system, so that its 2d position within the coordinate system is always known. The current position is displayed in real-time on the floor as the ball is being moved, so the user always knows where the current position is and can make adjustments accordingly. The system and ball are linked via a wireless connection. Certain control functions such as reading/writing graph files, system setup and initialization, etc. are performed from a control "console", which is simply a typical desktop PC interface (display, keyboard, mouse) running an application program with the usual GUI interface elements. Typically only the teacher would interact with the "console".

1.3 Hardware

The system is controlled by a Windows Pentium-class PC. A single projector is the display medium, and the projector will be biased toward the front of the display area, so that shadows are thrown behind a user, rather than directly under him. An overhead camera provides optical tracking of the ball, and a wireless mouse inside the ball notifies the PC application that the user has "entered" a coordinate point. See section 3 for a complete description of the system.
2. Instructional Unit

The following is a description of a sample instructional unit using ClassCoords. Complete materials for the unit can be found in the teacher materials supplement.

2.1 Teacher's and students' roles

The ClassCoords system is a tool for teachers and students to visualize coordinate systems, similar to the blackboard or overhead projector. The main difference between the system and other more traditional tools is the ability for teacher and students to immerse themselves in the coordinate plane, due to the size and location of the display and the real time tracking mechanism. The teacher's role as instructor and facilitator has not changed with this system, however. Hopefully the teacher will be comfortable with the direct interface to the system, and will welcome the system as another tool to help make mathematics less abstract and more tangible.

Students hopefully will also have a positive experience with the system. It affords a large-scale graphical aid, as well as the opportunity to get out of their seats and walk around within the display. The direct interface should be easy for students to learn to operate. The instructional unit provides an opportunity for individual, small group, and whole class work. There are also a variety of opportunities for students and groups to demonstrate their solutions in class using the system, as well as more traditional paper and pencil problem solving. Therefore, students will have the opportunity to use a variety of methods in a variety of settings to grasp coordinate system graphing concepts.

2.2 Classroom arrangement

Prior to beginning, the chairs in the classroom should be arranged around 3 sides of the classroom periphery, leaving the center area open for the display of the coordinate system. The instructor should expect and encourage action within the classroom, ie, students getting up and walking around within the display space, discussing their observations, etc (in an orderly fashion of course). The system and instructional unit afford an engaging and active classroom setting. There are also parts of the lesson which need to be taught more traditionally, with the instructor presenting necessary background information. At these times, the teacher will demonstrate concepts using the system and/or other tools in a more common "lecture" mode.

2.3 Brief description

The instructional unit consists of three main phases. First, two opening activities are performed design to "hook" students' curiosity and help them realize the need for coordinate systems. Then a data collection activity follows, which includes graphing of the resulting data points using the system. Finally, a data analysis phase follows, which focuses on high-level conceptual properties of linear
relationships. Best-fit lines are constructed through the students’ data points, and properties such as linearity, increasing and decreasing (positive and negative slope) are discussed at a conceptual graphical level. This conceptual "introduction" to linear relationships will serve as a precursor to the algebraic study of linear equations which students will encounter in the following years. Optionally, students will interpolate previously unmeasured data points graphically and then experimentally verify their accuracy. To conclude, students will be introduced briefly to some non-linear relationships, in order to frame their learning in a larger context. Each of the three phases will take approximately one week, and the entire unit is estimated to take three weeks. It is geared for 5th, 6th, or 7th grades, or roughly middle school (although the term "middle school" is subject to debate), but the exact scheduling is up to local school discretion.

2.4 Opening Activity: Find the mall

The class will be divided into 3 teams, and each team will be further divided into two halves, writers and readers. The readers of each team will be outside in the hall for a few minutes, while the writers remain in the room. For each of the 3 teams, the location of a mall will be displayed on the floor using the class coordinates system. The writers will have a few minutes to write down instructions (form included) to explain to their readers the location of the point. Then the point will disappear, and the readers will rejoin the class, try to follow their group’s instructions, and locate the mystery point. Each team will have the same problem to solve, but under different circumstances: the first will locate the mall on the floor with no grid and no reference point. The second will locate the point with a reference point in the center (the location of the school) and a set of axes. The third will locate the mall with a reference point and a rectangular grid.

The entire class will regroup and discuss their findings, and a reflection sheet will be filled out. The teacher will explain that the activity was not a contest, but was meant to demonstrate that a coordinate system is helpful for describing locations. In fact, this is the entire motivation for using a coordinate system; locations are very difficult to describe without one. The mall metaphor will lead in to a brief discussion of Rene Descartes, who is said to have discovered the Cartesian coordinate system while lying ill in his bed. As he lay, he watched a fly on the ceiling and described its position by counting the lines formed by intersecting rectangular ceiling tiles.

The goals of the opening activity are:
• create need for information

Students should want to learn about graphing in a Cartesian coordinate system because it can be used as a tool to solve problems such as the navigation exercises demonstrated above. However, they should realize that coordinate systems have a structure, language, and rules that need to be mastered in order to be able to use them.

• demonstrate relevance

Being able to give and follow navigational directions is something students can relate to (especially to the mall). They should see that a frame of reference (eg a coordinate system) is useful for finding one's way around.

• generate curiosity

The opening activities should be fun and engaging in order to spark students' curiosity. Motivation is a major factor here, and can affect the outcome of the following weeks of work.

2.5 Timeline

In order for students to use metacognitive thinking, ie, monitor and reflect on their own learning, they need a bird's-eye-view of what they have done, are currently doing, and will be doing. The timeline (included) will be projected periodically showing the entire graphing unit and the current status. The teacher should display this at the start of each class or at the start or end of each activity, and indicate what the class has accomplished, the current status, and what is coming up.

2.6 Presentation of introductory concepts

The teacher will demonstrate the following concepts with the system by displaying a ready-made example coordinate system on the floor (included). The teacher will point out the key elements below while walking around in the coordinate system.

• x-y axes, Cartesian coordinates, rectangular coordinates
• labeling
• origin
• positive and negative x,y directions
• quadrants
• coordinate naming, ordered pairs
• domain and range, independent and dependent variable

2.7 Worksheet
This is a homework assignment to review these key concepts (included); class will be notified to be sure to complete it, not only for a grade but also for the opportunity to use ClassCoords system the next day (motivation). The class will review the completed worksheets the following day; volunteers will be able to demonstrate their answers in ClassCoords. This will also serve as an introduction to students' use of the system. Teacher will demonstrate key usage concepts, and students will have the chance to plot points from their completed worksheets.

2.8 Maze game

For homework, students will be given a worksheet showing a maze on a coordinate system, and will be asked to generate a path of \((x,y)\) points in order to successfully complete the maze. The following day, several students can demonstrate their solutions using the ClassCoords.

2.9 Quiz #1

An in-class quiz will be performed to assess students grasp of concepts so far. Concepts may be reviewed and reinforced at the teacher's discretion based on results.

2.10 Data collection

The second main phase of the graphing unit consists of gathering experimental data to be used in the last phase, data analysis. In order to make the lesson more tangible for students, they will collect real data, as opposed to sample data provided from a textbook or worksheet. They will work collaboratively, and have a choice of simple experiments to perform. They will record their data in forms that are provided in the teacher materials. All experiments are designed to produce linear results, however actual results may vary due to inaccuracies. These will be discussed during the data analysis phase.

Teams of 3 students will be formed, either voluntarily, randomly, or assigned by the teacher (one team may have 2 or 4 students). Each team will perform one experiment from the suggested list below. Experiments are to be done in class, and do not require a dedicated laboratory or any prerequisite science concepts.

Not all need to be performed; teachers may use their discretion based on availability of materials, etc. More than one group can perform the same experiment, or alternatively the entire class can perform the same. Hopefully there are enough ideas here for the teacher to exercise his/her own judgement. See the teacher materials for detailed descriptions of the following:

- #1: volume vs. height for water in a graduated cylinder
- #2: diameter vs. perimeter for round objects
- #3: Fahrenheit vs. Celsius temperature of various objects
- #4: spring balance extension vs. number of marbles
- #5: distance vs. time for battery operated toy
• #6: distance traveled vs. starting height for a toy car released from an incline
• #7: mass vs volume of water
• #8: mass in ounces vs. mass in grams of various objects
• #9: length vs. #pennies for small nails

A data entry form is included, which has room to describe the experiment textually and graphically, and a table to enter data values.

2.11 Graphing data points

Data values will be graphed in ClassCoords by each group in turn, and stored by the teacher as computer files for future reference.

2.12 Graph analysis

The following properties are discussed using the students' graphs as examples:
• increasing / decreasing
• linearity
• steep / gradual slope
• passing through origin (simplification of intercepts)

2.13 Reflection

The steps of the process which were performed in this part of the unit are abstracted as:
• experiment
• collect
• analyze
Analysis of other sample data sets is done with a worksheet.

2.14 Quiz #2

An in-class quiz covering new concepts is included.

2.15 Where do we go from here?

The following topics are optional, but recommended if time permits.

• Experimental validation of interpolation and extrapolation

Time permitting, students will re-group into their experiment groups, and take turns finding interpolated / extrapolated points using ClassCoords. They then will re-construct their experiment and attempt to test the accuracy of these points.

• non-linear relationships
Several examples of non-linear graphs are included to provide some context and help students build a larger conceptual framework for their graphing knowledge. Also, hopefully their interest to learn more will be stimulated.

2.16 General assessment strategy

Assessment was designed into the unit and runs throughout, as is evidenced by the large number of worksheets, quizzes, etc. The assessment strategy is to develop a portfolio for each student for this project, consisting of several small snapshots of their progress as opposed to one all-or-nothing grade. See the grading rubric for an outline of the assessment items.
3. Detailed System Description

3.1 System description

3.1.1 Computer

A Standard PC Pentium-class desktop computer running Windows 98 or later is utilized. Graphics acceleration is not mandatory because the 2d display graphics are not hardware intensive. A small number of objects (less than 100 points, lines, and text labels) are produced per frame. Other more demanding graphics features such as polygons, shading, texturing, etc. are not used for this simple application.

More demands are placed on the system by the tracking unit, which requires CPU computational power and memory to be able to recognize the ball location quickly enough to produce graphics frames at an acceptable rate during the time that the ball is in motion. The goal is to reduce lag time between the user moving the ball and the display being refreshed, and this will primarily depend on the computation of the ball coordinates from camera input. No prototype system exists yet, so exact system requirements are not known, but an initial guess is to use the current state-of-the-art PC configuration, such as a 1.5 GHz Pentium IV with 512 MB RAM.

The camera input for the tracking system will be a "FireWire" bus, so a Firewire adapter card will need to be included.

3.1.2 Tracking system

Optical tracking is used because of its low cost and the low-resolution requirements of the system. A camera is mounted directly over the center of the display area, and is connected to the PC via a IEEE 1394 FireWire connection. The FireWire bandwidth is 400 Mbps, or nearly 20 frames per second at 800x600 resolution. The camera is constantly photographing the display space, and sending .jpg images to the PC. A typical firewire camera with 1280x1024 resolution and firewire PC adapter card costs approximately $1700. [www.edmundoptics.com]
The "ball" is a bright neon color that hopefully does not appear anywhere else in the display space. For example, no student should be wearing that color. Pattern recognition software is used to detect the pixels in the .jpg images corresponding to the ball color and to scale and translate those pixel coordinates into graphing space coordinates. The image analysis is a demanding process, and will likely be the determining factor in the amount of "lag" between the movement of the ball and the updating of the image. According to Greg Dawe of EVL, a group of graduate students are currently working on this application, and have just recently begun to get the efficiency close to a "usable" range. We are assuming that with continued increase in computing power and continued progress in software, this tracking method will work dependably in the near future (1 year). The advantage is the low cost, as opposed to wireless electromagnetic tracking systems that can cost from $50,000 on up. [ascension-tech.com]

3.1.3 Display system

A single projector is used to display the output onto the floor. Other display technologies were considered to attempt to solve the occlusion problems that result from top-projecting an image. These ranged from expensive to impractical to impossible, and finally the decision was made to bias the projector near the front of the display space, so that shadows are thrown behind a user, and to simply allow these shadows. Really, this was the only choice, as all others were impossible, especially when economics were taken into consideration. The display specifications are listed below:

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>projector resolution</td>
<td>800x600</td>
</tr>
<tr>
<td>floor display size</td>
<td>12’x9’ (144” x 108&quot;)</td>
</tr>
<tr>
<td>pixel size on floor</td>
<td>.18&quot;</td>
</tr>
<tr>
<td>projector brightness</td>
<td>2600 lumens</td>
</tr>
<tr>
<td>sample projector model</td>
<td>Mitsubishi S490</td>
</tr>
<tr>
<td>estimated projector cost</td>
<td>$5000</td>
</tr>
<tr>
<td>projector throw distance</td>
<td>approx. 23’</td>
</tr>
<tr>
<td>mirror size</td>
<td>approx. 4.5’ x 6’</td>
</tr>
<tr>
<td>estimated mirror cost</td>
<td>$500</td>
</tr>
</tbody>
</table>

"Throw distance" is the distance from the projector to the image or screen. A mirror will be used to fold the "throw distance" of the projector, making the
required floor to ceiling height approx. 11', which can be accommodated in a classroom with some modifications, which are discussed below. The required size of the mirror is given in the table above.

3.1.4 Input system

The "ball" has a 3d radio wireless mouse inside of it, and the "enter" button on the ball actuates the mouse button. In this way, a common stock component is used, and the ball is just a cover for the mouse. The reason why the ball is necessary at all is to provide a brightly colored target for the tracking camera. Positional input from the mouse is ignored, as it is taken from the tracking system instead, and only the event of the mouse button being pressed is used. The wireless mouse has a base station connected to the PC's mouse port. In a quick review of available products, we have found mice with a range of 5 meters, or about 15 feet, for about $50. This range is almost enough, as the PC is probably located off in the corner of the room, but let us assume other slightly longer range products are available. A range of 25-30 ft. would be ideal. Some modification to the housing of the mouse will be necessary to fit into the ball. The total cost of the ball and mouse inside is estimated to be $200.

3.1.5 Room requirements

- room size and layout

Assuming a typical classroom size of approximately 20' x 30' and a ceiling height of 10', the room could be arranged as follows:
A side view using a typical ceiling height of 10 feet is shown below:

As seen above, modifications need to be made to the floor and ceiling to house the projector and mirror. These are needed to achieve an approximate 11' throw distance from the projector to the center of the mirror. Of course, a taller ceiling height would help, but we used 10' as a typical ceiling. (Classrooms with lower ceilings would require even more modification.) The dimensions given above are very rough, based on projector specifications, but without doing detailed calculations. We are estimating carpentry to cost $10,000, but this may vary depending on the original construction and ease of creating the required recessed areas.

- lighting
Ambient lighting needs to be dim to avoid "washing out" the projected image. Typically classrooms have florescent lighting which cannot be dimmed. The solution is to re-wire the room lights for several zones on several switches, so that only a few lights in the student seating areas are on while ClassCoords is in use. In this way, the projection area is dimly lit, while there is still some ambient light for students to read or take notes.

- floor material

A white, flat (non-glossy) floor surface is ideal, but not usually found in a classroom. Glossy tile or carpet is the norm. We did not spend much effort researching floor finishes, but at least should point out that flooring needs to be considered. An example of a low-cost solution is a portable mat that can be rolled up and stored when not needed, or even large white paper that can be disposed of.

All together, we are budgeting $15,000 for the total cost of room modifications. (carpentry, electrical, floor covering)

3.2 Block diagram

The following is a diagram of the entire system:
3.3 Simulation

The enclosed CD contains an application to simulate ClassCoords. It can run on any PC (Win98 or newer). Tracking is not simulated; mouse input is used instead. Also included is complete source code in C++/OpenGL, a readme file with installation and execution details, and sample graph files for the instructional unit. See the CD for complete details.

3.4 Other suggested uses and instructional topics

The following is a brief list of other mathematics topics that would be enhanced through the use of ClassCoords.

statistics

- scatterplots
- mean, median, mode, range
- quartiles, inter-quartile range

gamey

- parallel, perpendicular, intersecting lines
- angles, triangles, right triangles
- circles and simple polygons
- linear transformations - scaling, rotation, translation
- other coordinate systems - polar coordinates

algebra

- relations and functions
- higher degree functions (eg quadratic, conic sections)
- systems of equations
- inequalities

3.5 Estimated feasibility, cost and market demand

A formal market analysis is beyond the scope of this project, so the following is based on only the feedback we received from the two math teachers and several students we interviewed, and our own expectations. We believe ClassCoords is a viable project that will prove useful. Graphing is an abstract concept, and ClassCoords makes it more tangible by providing a large-scale immersive model. The students and teachers we talked to were excited about the system. The next marketing step would be to secure the interest of school administrators and school boards, ie., the decision-makers responsible for purchasing ClassCoords.

We believe there will be significant demand for the product, and have made design decisions to keep the cost within reach of a school's technology budget. The project is feasible and relies solely on technology that is currently available
or very near-term, primarily using stock components which can be purchased "off-the-shelf" from any number of suppliers. Estimated costs are outlined below:

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost $</th>
</tr>
</thead>
<tbody>
<tr>
<td>projector</td>
<td>5000</td>
</tr>
<tr>
<td>mirror</td>
<td>500</td>
</tr>
<tr>
<td>camera</td>
<td>1700</td>
</tr>
<tr>
<td>ball / 3d mouse</td>
<td>200</td>
</tr>
<tr>
<td>room renovation</td>
<td>15000</td>
</tr>
<tr>
<td>PC</td>
<td>1500</td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>1100</td>
</tr>
<tr>
<td>Grand total</td>
<td>25000</td>
</tr>
</tbody>
</table>

So, in our estimation, the system can be deployed in a typical classroom for approximately $25,000. Of this total, the system cost is $10,000 and room renovations account for the remaining $15,000. These numbers are within the realm of many schools’ annual technology budgets, especially considering that renovation dollars usually are drawn from a separate fund than technology dollars.
4. Conclusion

4.1 Summary

We have presented a detailed design for ClassCoords, a large-scale floor-display immersive coordinate system. What started as a vague idea has evolved into a complete design package including relevant background and learning standards, sample instructional unit, and system specifications. The instructional unit is an introduction to graphing in a Cartesian coordinate system, containing engaging activities based on modern cognitive theories. In fact, the entire premise of the system is to engage learners by immersing them in the coordinate space.

Another feature of the instructional unit is its high-level, introductory, conceptual nature. This is also intentional. Originally, we began designing a unit for 8th graders, complete with algebraic linear equations, etc., but decided to move it ahead one or two grade levels (6th or 7th grade) and make it more introductory in nature. Too often students are overwhelmed by complex graphing tasks when they have not mastered graphing fundamentals. This was another main premise of this project: The first step in learning graphing is to develop a visualization of the coordinate system. At the end we do introduce some more difficult graph analysis concepts such as best-fit line, slope and intercept (at a conceptual level only). Not all students may grasp these immediately, but there will be ample opportunities to study these in later grades. At least students are being exposed to these concepts at this early stage, so they will not be seeing them for the first time in later grade levels when they can be developed more analytically.

Finally, we provided a fairly detailed high-level system description and specifications. This includes cost estimates, which are affordable for two reasons. First, we use stock hardware components that are readily available and affordable. Second, we made explicit decisions along the way to choose affordable solutions to problems, even if we had to trade-off some features. For example, it would have been ideal to have a display completely free from occlusions. However, the costs would have been staggering, so we opted for a single projector and a shadow cast behind the user where we deemed it would not be objectionable. The choice of a low-cost camera-based tracking system vs. a high-cost electromagnetic tracking system was made for similar reasons. Our thinking is that an affordable, if not perfect, system was still better than no system at all, because a high price tag has been the end of many a good idea.

4.2 Suggestions and future work

To date, most of the work on this project has been design, theory, and simulation. The next logical steps are to build and test a prototype system. Hardware components need to be procured, telemetry calculations need to be done more meticulously, and the components need to be installed in a real classroom. The simulation, which will evolve into the actual application, needs improvement in the user interface and in functionality. The tracking system needs to be developed, primarily the 2d image recognition software, and then it
A few practical suggestions have evolved during the course of the project, but have not been implemented due to time constraints. The following is a "do-list" of those items.

- The aspect ratio of the simulation was originally 1:1. However, since most projectors have an aspect ratio of 4:3, the application should be re-sized to these proportions.
- The present color scheme for the simulation is dark colors on a white background. The reason for this is to be able to economically print the final project report and lesson plans on paper. However, for classroom projection, light colors on a black background are better to minimize reflection and prevent "washing out" of the image.
- More scaffolding and examples of a few difficult concepts in the lesson plans, especially domain, range, independent and dependent variables. These suggestions were provided by Daniel Moll, an elementary mathematics teacher who reviewed the lesson plans and provided comments (see acknowledgements). I took some of these math concepts for granted and expected one or two simple exercises to adequately cover them. Mr. Moll's commentary is based on over 25 years of math teaching experience, and he made many constructive suggestions for which I am grateful. Some of the smaller ones were implemented in time, but some of these larger changes will require more re-organization of the lesson plans than time will allow. We leave them as future work.
5. Acknowledgements and References

5.1 Acknowledgements

I would like to thank the following individuals for their help and advice.

Daniel Moll (jr. high mathematics teacher, Hillside School District 93, Hillside IL) for his review of the project report and lesson plans and for his thoughtful comments and suggestions. Melinda Peterka (4th and 5th grade gifted education mathematics and science teacher, Americana School, Queen Bee School District 16, Glendale Heights IL) for her input throughout the course of the project. Prof. Tom Moher (University of Illinois at Chicago UIC) for his guidance to help narrow the project down to a workable scope. Greg Dawe (Electronics Visualization Lab EVL at UIC) for taking time out of his busy schedule to discuss hardware considerations of the project and for suggesting hardware solutions that are practical and affordable.

5.2 References

Learning standards

- Illinois learning standards
  http://www.isbe.state.il.us/ils/Default.htm
- NCTM standards
  http://www.nctm.org/standards

Teaching graphing

- Teaching graphing in middle school math
  http://www.middleweb.com/Graphing.html
- Math Forum - Ask Dr. Math
  http://mathforum.org/dr.math/problems/bridge3.29.96.html
- E-Example 6.2.1 Learning about Rate of Change in Linear Functions
- NCTM Illuminations Cartesian Graphing
  http://illuminations.nctm.org/lessonplans/6-8/cartesian/
- The Math Forum - Math Library - Graphing of Data
  http://mathforum.org/library/topics/graphing_data/

Cognitive theory


Math textbooks


Hardware

• http://www.usnews.com/usnews/nycu/tech/articles/011224/24display.htm
• http://www.ascension-tech.com
• http://www.edmundoptics.com
• http://www.projectorcentral.com
• http://www.gamenationtv.com/hardware/mouse.shtml