Revising the Pareto Chart

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The Pareto chart is a bar chart of frequencies sorted by frequency. The most popular incarnation of the chart puts the highest bars on the left and includes a line showing the scores produced by adding the heights in order from left to right. This chart is used widely in quality control settings to identify critical factors leading to failure or defects in a process. This article presents revisions that remedy problems with the chart and improve its usability in diagnostic settings.

KEY WORDS: Acceptance intervals; Bar charts; Multinomial distribution; Quality charts; Rank charts.

1. INTRODUCTION

Figure 1 shows a Pareto chart. The chart in the figure is adapted from an example in Tague (2004), published by the American Society for Quality. The horizontal axis represents six different categories of customer complaints. The left vertical axis represents the counts of complaints in each category. The right vertical axis represents cumulative counts expressed as percents of total count.

The motivation for the Pareto chart can be traced to Juran (1951, p. 39), who observed:

> It is seen from these instances that there is some universal principle which underlies all these cases. The losses are *never uniformly distributed* over the quality characteristics. Rather, the losses are *always maldistributed* in such a way that a small percentage of the quality characteristics always contributes a high percentage of the quality loss.

Juran added in a note,

> The economist Pareto found that wealth was nonuniformly distributed in the same way. Many other instances can be found—the distribution of crime among criminals, the distribution of accidents among hazardous processes, etc.

Although the first edition of Juran's *Quality Control Handbook* did not include the exact form of the chart in Figure 1, later editions did. Juran added the cumulative line at the top of the chart in order to make it easier to judge Pareto's "80/20 rule." Juran (1975) explained that he generalized Vilfredo Pareto's observation that 80% of income in Italy was limited to 20% of the population. Juran characterized the principle as the "vital few and trivial many" among manufacturing defects; 80% of defects in a process seemed to be accounted for by 20% of the causes. The 80/20 rule is a consequence of the scale-invariance of power distributions (Newcomb 1881; Zipf 1935). Pareto's choice of critical fractiles is, of course, arbitrary. Other choices could have fit the distribution as accurately, but might not have been as memorable.

The type of Pareto chart shown in Figure 1 is widely used in Total Quality Management, Six Sigma, ISO9000, and other approaches to quality assurance. If Google search frequencies are an indication of popularity, it is one of the most popular charts in the world. The Pareto chart has several problems that limit its usefulness for quality applications, however.

First, the vertical axes for the plot are ill-defined. It is meaningless to superimpose a density and a distribution. More generally, it is meaningless to plot a derivative and an integral in the same frame. Some applications remedy this problem by making dual vertical scales (count on the left and cumulative count on the right, or percent on the left and cumulative percent on the right). Dual scale plots are confusing, however, and the rationale for aligning the scales in this particular way is arbitrary.

Second, forcing the counts for the bars and cumulative line to align (as in Figure 1) limits the effective range of the graph. When there are many categories, the cumulative line forces the bars toward the bottom of the frame and makes it difficult to distinguish the bars. This is a well-known problem; experts advise users to truncate the plot by making an "Other" category at the right end of the horizontal scale, but this is rather ad hoc.

Third, there is no theoretical justification for representing the

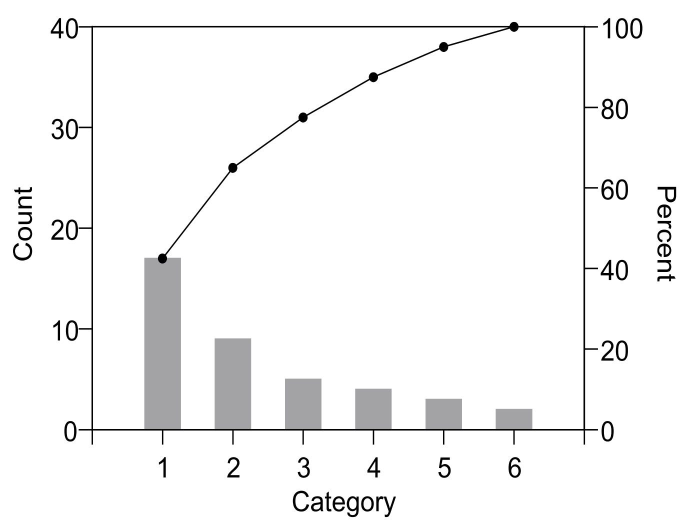

Figure 1. Pareto chart.

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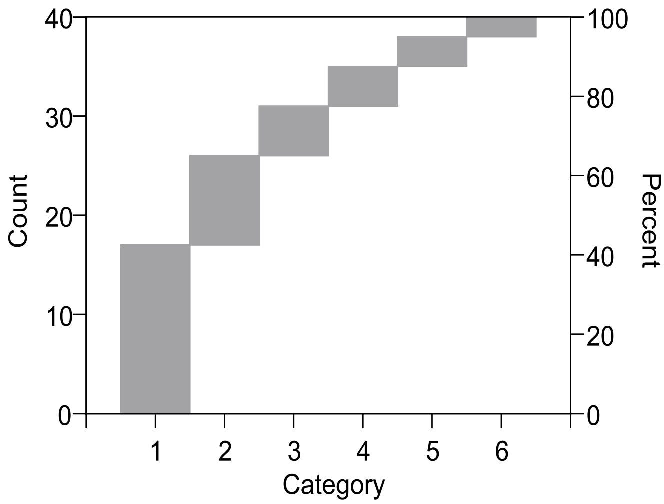

Figure 2. Juran's revised pareto chart.

cumulative frequencies with an interpolated line element. Since the categories cannot be assumed to be equally spaced on a scale, we are not justified in interpreting the overall slope or segment slopes in this line. For similar reasons, we are not justified in looking for "kinks" in this line to detect breakpoints or subgroups of problem categories.

Finally, the chart makes it difficult to evaluate the sample distribution of frequencies because there is no reference distribution in the plot. To apply Juran's logic of attacking the "vital few," we need to know if our data plausibly fit a Pareto or similar power distribution. If the frequencies are essentially random, then there is little programmatic justification for attacking the problems with the largest frequency first, especially when costs of remedies are not uniform.

2. REVISION

Juran solved one of these problems in a simple modification of his chart. As Figure 2 shows, Juran reshaped the Pareto diagram to be a cumulative bar chart. This appealing simplification was not widely adopted by the quality community, even though it more closely achieves Juran's goals than the version in Figure 1. Juran's revision eliminates the dual-scale problem, but it does not take care of the other two problems mentioned above.

2.1 Acceptance Intervals

Recall that the motivation for the Pareto chart was Juran's interpretation of Pareto's rule, that a small percent of defect types causes most defects. To identify this situation, Juran sorted the Pareto chart by frequency. Post-hoc sorting of unordered categories by frequency induces a downward trend on random data, however, so care is needed in interpreting heights of bars. Why not provide acceptance limits derived from assuming that all categories are equally probable before sorting occurs?

There are asymptotic results for the order statistics of a multinomial distribution (Ivchenko 1971), but it is simple to compute exact integer values on small samples with Monte Carlo. For each sample, we assign 40 defects randomly to the six categories in

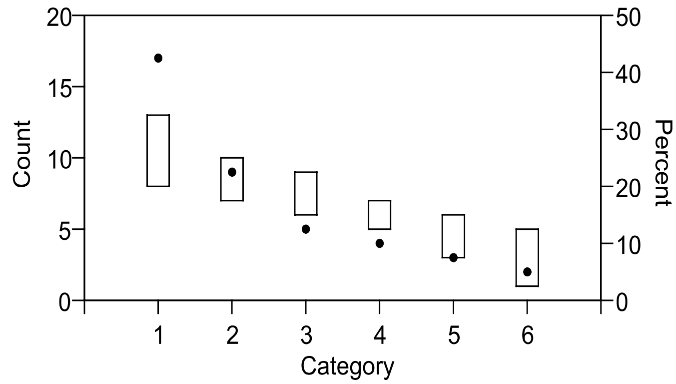

Figure 3. Pareto dot plot with acceptance intervals.

Figure 1. Then we sort the six categories by frequency. We do this a few thousand times and compute the lower $\alpha/2$ and upper $(1-\alpha/2)$ frequency values for each of the six categories. We continue sampling until none of the order statistics change. The process converges quickly for small samples because it is measured on the integers. Figure 3 shows the result (for $\alpha = 0.05$). We see that the first category falls outside the acceptance interval. We conclude that our efforts should be devoted to reducing Category 1 defects.

We use dots and intervals instead of bars because the acceptance intervals are closed on the positive integers. Consider the first category. It is impossible for the lower bound of this interval to be less than n/k, where n is the total count and k is the number of categories. And it is impossible for its upper bound to be greater than the total count. This bounding is a consequence of the post-hoc sorting that is the basis for the Pareto plot. Thus, it is useful to see both lower and upper bounds represented in the graph in order to assess the overall multinomial distribution.

3. APPLICATIONS

Acceptance limits can save us remedial effort and/or force us to collect more data before we attack problems. Figure 4 shows a Pareto dot plot of the frequences of attributed causes of patient falls in a hospital. The source of these data is a Pareto chart on the Web site of the University of Texas Health Sciences Center at Houston. As the dot plot reveals, there's little reason to apply

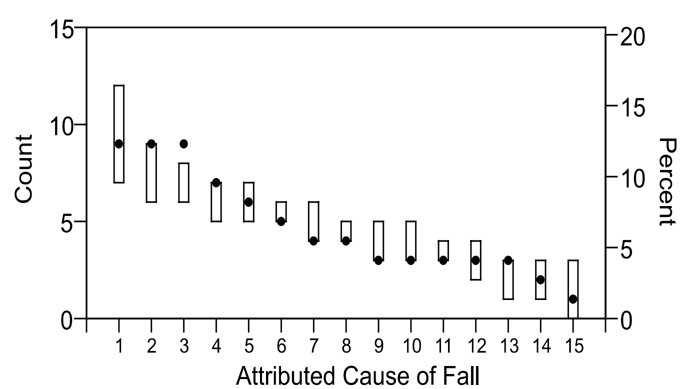

Figure 4. Pareto dot plot of hospital falls.

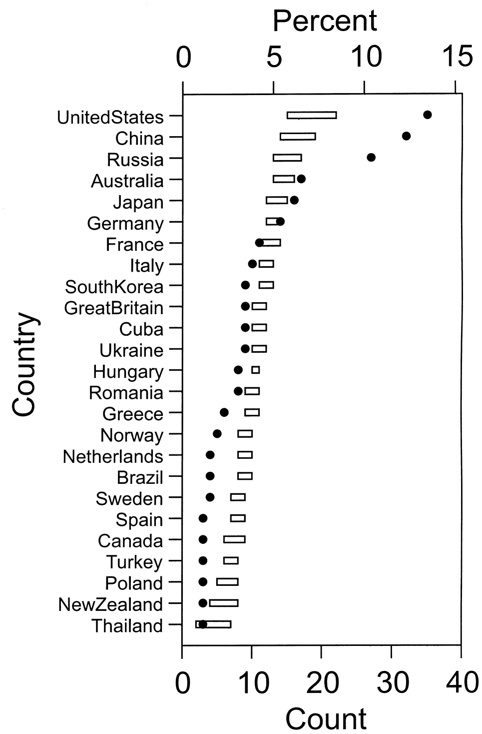

Figure 5. Pareto dot plot of 2004 Summer Olympics gold medals.

the Pareto principle here. And it is the third category, not the first, that falls outside of the acceptance limits.

What does a Pareto dot plot look like for data that *do* fit a (discretized) Pareto distribution? Figure 5 shows a transposed Pareto dot plot of the number of gold medals awarded to each of the top 25 countries participating in the 2004 Summer Olympics. The first five countries are outside the acceptance intervals.

4. CONCLUSION

The Pareto dot plot has been designed to address a particular applied problem in quality assurance. In practice, data on frequencies of failure are collected for a set of discrete factors. These frequencies are sorted and problems associated with the largest frequencies are attacked first. The justification for this procedure is Juran's observation that most of the failures in manufacturing processes are caused by a few factors. Implicit additional assumptions are that the cost of remedies may be ignored (as being uniform across alternatives) and that side effects of remedies (on other failures, on efficiencies, etc.) may be ignored. In such a context, it is important to assess whether the trend in failures observed is an artifact of the post-hoc sorting used in making a Pareto chart. If it appears to be an artifact, then other considerations should be made before attacking the problems. The Pareto dot plot makes this strategy possible.

The procedure developed here is easy to implement in computer software and may be readily added to the list of charts available in statistics packages used in the quality field. If widely used, the Pareto dot plot could help prevent wasted effort and could encourage practitioners to collect enough data to devise an effective solution strategy.

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REFERENCES

Ivchenko, G. (1971), "On Limit Distributions for the Order Statistics of the Multinomial Distribution," *SIAM Theory of Probability and its Applications*, 16, 102–115.

Juran, J. M. (1951), "The Economics of Quality," in *Quality Control Handbook*, ed. J. M. Juran, New York: McGraw-Hill, pp. 1–41.

——— (1975), "The Non-Pareto Principle: Mea Culpa," *Quality Progress*, 8, 8–9.

Newcomb, S. (1881), "Note on the Frequency of the Use of Digits in Natural Numbers, *American Journal of Mathematics*, 4, 39–40.

Tague, N. R. (2004), *The Quality Toolbox* (2nd ed.), Milwaukee, WI: ASQ Quality Press.

Zipf, G. K. (1935), *The Psycho-Biology of Language*, Boston: Houghton-Mifflin.