3.1 $n=1$ $f=x_1^2-x_2^2$ $f^{-1}(-1)$ is $x_1^2=x_2^2+1$ $\nabla f=(2x_1,-2x_2)$ so $\nabla f\neq 0$, no such p

$f^{-1}(1)$ also doesn't have such p. $f^{-1}(0)$, $\nabla f(0,0)=(0,0)$, $f^{-1}(0)$ is $x_1=\pm x_2$

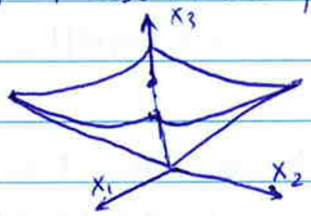
Its tangent space is $\{\lambda(1,1),\lambda(1,-1)|\lambda\in R\}\neq[\nabla f(0,0)]^\perp=R^2$

$n=2$ $f=x_1^2+x_2^2-x_3^2$ $f^{-1}(-1)$ is $x_1^2+x_2^2=x_3^2+1$. $\nabla f\neq 0$ no such p. $f^{-1}(1)$ also no such p

$f^{-1}(0)$: $x_3^2=x_1^2+x_2^2$ at $p=(0,0)$, the tangent space

at $(0,0,0)$ is all vectors $V^{(x_1,x_2,x_3)}$, where V is 45° to x_3 axis

$|V\cdot(0,0,1)/\|V\||=\frac{\sqrt{2}}{2}$ i.e. $x_3^2=x_1^2+x_2^2\neq[\nabla f(0)]^\perp$



3.2 (a) the example in 3.1 with $n=1$, $c=0$. $f^{-1}(0)$: $x_1=\pm x_2$ $(1,1),(1,-1)\in S$, $(1,0)\notin S$

(b) $f(x_1\cdots x_{n+1})=c$. $S=f^{-1}(c)$, tangent space $=R_p^{n+1}$

3.4 $f\circ\alpha=c \Leftrightarrow \frac{d(f(\alpha(t)))}{dt}=0 \Leftrightarrow \nabla f(\alpha(t))\cdot\dot\alpha(t)=0 \Leftrightarrow \dot\alpha\perp\nabla f(\alpha)$ $\forall t$.

3.5 α is integral curve of ∇f $\Rightarrow$ $\dot\alpha=\nabla f(\alpha)$

(a) $\frac{d}{dt}f(\alpha(t))=\nabla f(\alpha(t))\cdot\dot\alpha(t)=\|\nabla f(\alpha(t))\|^2$

(b) $\frac{d}{dt}f(\beta(s_0))=\nabla f(\beta(s_0))\cdot\dot\beta(s_0)=\nabla f(\alpha(t_0))\cdot\dot\beta(s_0)$. As $\|\dot\beta(s_0)\|=\|\dot\alpha(t_0)\|$

it is maximized when $\dot\beta(s_0)=\dot\alpha(t_0)=\nabla f(\alpha(t_0))$, then

$\frac{d}{dt}f(\beta(s_0))=\|\nabla f(\alpha(t_0))\|^2=\frac{d}{dt}f(\alpha(t_0))$ by (a)

4.3 Consider $S=f^{-1}(c)$. $\forall p\in S$. p is an extreme point of g on S.

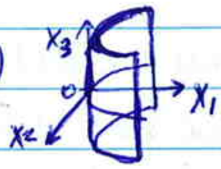
By Lagrange Theorem, $\nabla g(p)=\lambda\cdot\nabla f(p)$ $\forall p\in S$. $\lambda\neq 0$ because $\nabla g(p)\neq 0$ for all $p\in S$

4.4 See http://users.rsise.anu.edu.au/~xzhang/dg_thorpe/monkey.jpg

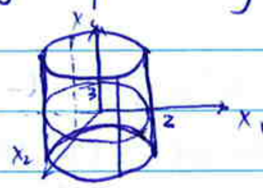
4.5 (a)

the x_2-axis

(b)

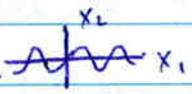


(c)



ellipse on $x_3=0$

4.6



4.7 $g(x_1,x_2,x_3)=f(x_1,(x_2^2+x_3^2)^{1/2})$, then $\frac{\partial g}{\partial x_1}=\frac{\partial f}{\partial x_1}$. Denote $u=(x_2^2+x_3^2)^{1/2}$

then $\frac{\partial g}{\partial x_2}=\frac{\partial f}{\partial u}x_2(x_2^2+x_3^2)^{-1/2}$, $\frac{\partial g}{\partial x_3}=\frac{\partial f}{\partial u}x_3(x_2^2+x_3^2)^{-1/2}$. If $\nabla g(p)=0$, then

$\frac{\partial g}{\partial x_2}=\frac{\partial g}{\partial x_3}=0$, i.e. $0=(\frac{\partial g}{\partial x_2})^2+(\frac{\partial g}{\partial x_3})^2=[\frac{\partial f}{\partial u}]^2=0$. So $\frac{\partial f}{\partial u}=0$. Besides $\frac{\partial f}{\partial x_1}=\frac{\partial g}{\partial x_1}=0$

So $\nabla f=0$ at P, which contradicts with the fact that ∇f ~~is a surface~~ $\neq 0$