(b). By definition, we should have: $X\cdot\dot\alpha=0$, $X\cdot N\circ\alpha=0$ and $X^*=0$

$X^*=0 \Rightarrow \dot X-(\dot X\cdot N\circ\alpha)N\circ\alpha-(\dot X\cdot\dot\alpha)\dot\alpha+(\dot X\cdot N\circ\alpha)(N\circ\alpha\cdot\dot\alpha)\dot\alpha=0$ (*) Note: $\dot\alpha\perp N\circ\alpha$

$X\cdot\dot\alpha=0 \Rightarrow \dot X\dot\alpha+X\ddot\alpha=0$, $X\cdot N\circ\alpha=0\Rightarrow \dot X\cdot N\circ\alpha+X\cdot N\dot{\circ}\alpha=0$

Plugging into (*): $\dot X+(X\cdot N\dot\circ\alpha)N\circ\alpha+(X\cdot\ddot\alpha)\dot\alpha=0$. 1st order differential equation together with initial condition $X(t_0)=V$. So there exists a unique solution $X(t)$.

Now check $X\cdot\dot\alpha=0$ and $X\cdot N\circ\alpha=0$

$\overset{\circ}{(X\cdot\dot\alpha)}=\dot X\dot\alpha+X\ddot\alpha=X\ddot\alpha-(X\cdot N\dot\circ\alpha)(N\circ\alpha\cdot\dot\alpha)-(X\cdot\ddot\alpha)(\dot\alpha\cdot\dot\alpha)=0$

$\overset{\circ}{(X\cdot N\circ\alpha)}=\dot X\cdot N\circ\alpha+X\cdot(N\dot\circ\alpha)=X\cdot(N\dot\circ\alpha)-(X\cdot N\dot\circ\alpha)(N\circ\alpha\cdot N\circ\alpha)-(X\cdot\ddot\alpha)(\dot\alpha\cdot N\circ\alpha)=0$

domain check same as Thm 1 in chapter as $\|X\|$ is constant

(c) (i) F_α is linear map, If V and W are Fermi parallel along α, then so are V+W and cV ($c\in\mathbb{R}$)

(ii) F_α is one to one and onto: the kernel of F_α is zero because $\|F_\alpha(V)\|=0 \Leftrightarrow V=0$ by (iii).
So F_α is one-to-one from one n-dim vector space to another. But such maps are onto

(iii) $F_\alpha(V)\cdot F_\alpha(W)=V\cdot W$, $\forall V,W\in\dot\alpha(a)^\perp$ because $(X\cdot Y)^* = X^*Y+XY^*=0$, i.e. $X\cdot Y$ is constant

9.1 (a) $\nabla f=(4X_1,6X_2)$ $\quad\nabla_v f(p)=(4,0)\cdot(2,1)=8$

(b) $\nabla f=(2X_1,-2X_2)$ $\quad\nabla_v f(p)=(2,-2)\cdot(\cos\theta,\sin\theta)=2(\cos\theta-\sin\theta)$

(c) $\nabla f=(X_2X_3^2,\ X_1X_3^2,\ 2X_1X_2X_3)$, $\nabla_v f(p)=(1,1,2)\cdot(a,b,c)=a+b+2c$

(d) $\nabla f=(q,2q)$ $\quad\nabla_v f(p)=$ ~~2q·v~~ $2p\cdot v$

9.2 $\nabla_{e_i}f=\left(\frac{\partial f(p)}{\partial X_1},\cdots\frac{\partial f(p)}{\partial X_{n+1}}\right)\cdot(0,\cdots1,\cdots0)=\frac{\partial f(p)}{\partial X_i}$

9.3 (a) $\nabla X_1=(X_2,X_1)$ $\ \nabla X_2=(0,2X_2)$ $\quad\nabla_v X=((0,1)\cdot(0,1),(0,0)\cdot(0,1))=(1,0)$

(b) $\nabla X_1=(0,-1)$ $\ \nabla X_2=(1,0)$. $\nabla_v X=((0,-1)(-\sin\theta,\cos\theta),(1,0)(-\sin\theta,\cos\theta))=(-\cos\theta,-\sin\theta)$

(c) $\nabla X_i=(q,2e_i)$ $\quad\nabla_v X=(2,2,\cdots2)$.

$F(t)=f(\alpha(t))$ $\qquad D_v f=F'(t_0)$

9.5 Let $Y(t)=X(\alpha(t))$ be the vector field tangent to S along α. As $D_vX=(X\circ\alpha)'(t_0)$ where $\alpha: I\to S$ is any parametrized curve in S with $\dot\alpha(t_0)=v$. Then quote the properties i-iii in chapter 8 on page 46. Note in (iii) $\nabla_v(X\cdot Y)$ rather than $D_v(X\cdot Y)$ ($\nabla_v X\cdot Y=(X\cdot Y)'$)

9.4 Same as 9.5. $\nabla_vX=(\dot X\circ\alpha)(t_0)$ $\quad\nabla_v f=\dot F$. Then quote the properties i-iii in chapter 7 on Pg 39

9.6. $X(q)\cdot X(q)=1$ By property iii of ch 9 on Pg 54. $\nabla_vX(q)\cdot X(q)=\nabla_v1=0$ i.e. $\nabla_vX\perp X(p)$

If X is tangent to S, then $\nabla_v X \cdot N(p) = (\dot{X\circ\alpha})(t_0) \cdot N(p) = 0$. So $\nabla_v X = D_v X$. so $D_v X \perp X(p)$

by proof in Thm 1 of chapter 5, then α is ~~unique~~ on S

9.7 (a) "if part": $\forall$ ~~parametric~~ integral curve $\alpha: I \to S$, s.t. $\alpha(t_0) = p$. $\dot\alpha(t) = X(\alpha(t))$ ~~then the geodesic is~~

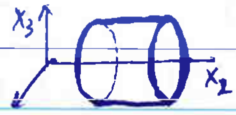
$D_{X(p)} X = 0 \Leftrightarrow \nabla_{X(p)} X \parallel N(p) \Leftrightarrow (\dot{X\circ\alpha}) \parallel N\circ\alpha$ } $\Rightarrow \ddot\alpha \parallel N\circ\alpha \Rightarrow \alpha$ is geodesic.

$\dot\alpha(t) = X\circ\alpha \Rightarrow \ddot\alpha = (\dot{X\circ\alpha})$

"only if" part: $\forall p \in S$ construct integral curve $\alpha: I \to S$, s.t. $\alpha(t_0) = p$, $\dot\alpha(t) = X(\alpha(t))$

as X is tangent to S, α must be on S by proof in Thm 1 in chapter 5. By assumption $\ddot\alpha \in S_\alpha^\perp$ (geodesic)

As $\dot\alpha(t) = X(\alpha(t))$ we have $\ddot\alpha = (\dot{X\circ\alpha}) \in S_\alpha^\perp$ i.e. $(\dot{X\circ\alpha}) \parallel N\circ\alpha \Leftrightarrow D_{X(p)} X = 0$

(b) 

cylinder $X(p) = (p, (0,1,0))$

9.8 (a) $N = (a_1, \dots, a_{n+1})$ $\nabla N_i = 0$ $L_p(v) = 0$

(b) $N = (0, \frac{X_2}{a}, \frac{X_3}{a})$. $\nabla N_1 = (0,0,0)$, $\nabla N_2 = (0, \frac{1}{a}, 0)$, $\nabla N_3 = (0, 0, \frac{1}{a})$, $L_p(v) = -(0, \frac{v_2}{a}, \frac{v_3}{a})$ (let $a > 0$)

9.9 By property (ii) on page 54. $\nabla_v(-N) = \nabla_v(-1)\cdot(N) + (-1)\nabla_v(N) = -\nabla_v N$

9.10 (a) suppose $L^*(e_i) = \sum_{i=1}^n \lambda_i e_i$, then by $L^*(e_i) e_j = e_j L(e_i)$ we have $\lambda_j = e_j \cdot L(e_i)$

So $L^*(e_i) = \sum_{j=1}^n (e_j \cdot L(e_j)) e_j$, $\forall v = \sum_{i=1}^n \alpha_i e_i \in V$, $L^*(v) = \sum_{i=1}^n \alpha_i \sum_{j=1}^n (e_j \cdot L(e_j)) \cdot e_j$.

$L^*(v)\cdot w = \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \alpha_i \beta_k (e_i \cdot L(e_j))\cdot(e_j \cdot e_k) = \sum_{i=1}^n \sum_{j=1}^n \alpha_i \beta_j (e_i \cdot L(e_j))$ Let $w = \sum_{i=1}^n \beta_i e_i$

$v \cdot L(w) = \sum_{i,j=1}^n \beta_i \alpha_j L(e_i)\cdot e_j = \sum \alpha_i \beta_j (e_i \cdot L(e_j)) = L^*(v)\cdot w$

So the only possible choice of $L^*(e_i) = \sum_{j=1}^n (e_i \cdot L(e_j)) e_j$ satisfies $v \cdot L(w) = L^*(v)\cdot w$ $\forall v, w \in V$.

(b) if ~~$L^* = L$~~ $L(v) = Av$ $\forall v \in V$. For $\forall w \in V$ $w \cdot L(v) = wAv$,

If we ~~set~~ choose $L^*(v) = A^T v$. then $v \cdot L^*(w) = v \cdot A^T w = wAv = wL(v)$.

As (a) proves L^* is unique and each linear transform corresponds to a unique matrix we know L^* correspond to A'. So $L^* = L \Leftrightarrow A$ is symmetric. So L_p is symmetric by Thm 2 (Pg 58)

9.11 $\forall i \in \{1, \dots, n\}$. $L_p(e_i) = -\nabla_{e_i} N(p) = -((\nabla N_1(p)\cdot e_i), \dots, (\nabla N_{n+1}(p)\cdot e_i))$

$\forall j \in \{1, \dots, n\}$. $\nabla N_j(p) e_i = \frac{\partial N_j}{\partial x_i}\Big|_p = \frac{\partial}{\partial x_i}\left(\frac{1}{\|\nabla f\|}\frac{\partial f}{\partial x_j}\right)\Big|_p = \frac{\partial}{\partial x_i}\left(\frac{1}{\|\nabla f\|}\right)\Big|_p \cdot \frac{\partial f}{\partial x_j}\Big|_p + \frac{1}{\|\nabla f\|}\frac{\partial^2 f}{\partial x_i \partial x_j}$

Since S is n-surface $\left|\frac{\partial}{\partial x_i}\left(\frac{1}{\|\nabla f\|}\right)\Big|_p\right| < \infty$. But $\nabla f(p)/\|\nabla f(p)\| = e_{n+1} \Rightarrow \frac{\partial f}{\partial x_j}\Big|_p = 0$ $\forall j \in \{1 \dots n\}$

So $\nabla N_j(p) e_i = \frac{1}{\|\nabla f\|}\frac{\partial^2 f}{\partial x_i \partial x_j}$

Since $N(p) = e_{n+1}$, and L_p is map $S_p \mapsto S_p$. So $\nabla N_{n+1}(p)\cdot e_i = 0$, thus

$$L_p(e_i) = -\sum_{j=1}^n \frac{1}{\|\nabla f\|}\frac{\partial^2 f}{\partial x_i \partial x_j} e_j$$

By the way, we can prove that $\nabla N_{n+1}(p)\cdot e_i = 0$. First $\frac{\partial f}{\partial x_{n+1}}\Big|_p = \|\nabla f(p)\|$

Second. $\frac{\partial}{\partial x_i} \|\nabla f\|\Big|_p = \frac{\partial}{\partial x_i}\left[\sum_{k=1}^{n+1}\left(\frac{\partial f}{\partial x_k}\right)^2\right]^{-\frac{1}{2}}\Big|_p = \left[\sum_{k=1}^{n+1}\left(\frac{\partial f}{\partial x_k}\right)^2\right]^{-\frac{3}{2}} \cdot \frac{-1}{2} \cdot \sum_{k=1}^{n+1} 2 \frac{\partial f}{\partial x_k}\frac{\partial^2 f}{\partial x_k \partial x_i}\Big|_p$. But $\frac{\partial f}{\partial x_k}\Big|_p = \begin{cases} 0 & k=1,\cdots,n \\ \|\nabla f\| & k=n+1 \end{cases}$

$= -\|\nabla f\|^{-3} \cdot \|\nabla f\| \frac{\partial^2 f}{\partial x_{n+1}\partial x_i} = -\|\nabla f\|^{-2} \frac{\partial^2 f}{\partial x_{n+1}\partial x_i}$ So L_p is Symmetric

So $\nabla N_{n+1}(p) \cdot e_i = -\|\nabla f\|^{-2} \frac{\partial^2 f}{\partial x_{n+1}\partial x_i} \|\nabla f\| + \|\nabla f\|^{-1} \frac{\partial^2 f}{\partial x_{n+1}\partial x_i} = 0$.

9.12 (a) Suppose a parametrized curve $\alpha: I \to S$. $\alpha(t_0) = p$. $\dot\alpha(t_0) = X(\alpha(t_0))$

$\nabla_{X(p)} Y = \dot{Y\circ\alpha}$, $\nabla_{X(p)} Y \cdot N(p) = \dot{Y\circ\alpha} \cdot N\circ\alpha$.

But as Y is tangent to S. $(Y\circ\alpha)\cdot(N\circ\alpha) = 0 \Rightarrow (\dot{Y\circ\alpha})(N\circ\alpha) + (Y\circ\alpha)(\dot{N\circ\alpha}) = 0$

So $\nabla_{X(p)} Y \cdot N(p) = -(Y\circ\alpha)\cdot(\dot{N\circ\alpha})\Big|_{t_0} = -Y(p)\cdot L_p(X(p))$

Similarly, one can prove $\nabla_{Y(p)} X \cdot N(p) = -X(p) \cdot L_p(Y(p))$

By Thm 2 (Pg 55) $L_p(X(p)) \cdot Y(p) = X(p) \cdot L_p(Y(p))$ Thus $\nabla_{X(p)} Y \cdot N(p) = \nabla_{Y(p)} X \cdot N(p)$

(b) by (a) obvious

9.13. For $\forall v$, define a parametrized curve $\alpha: I \to U$, $\alpha(t_0) = p$. $\dot\alpha(t_0) = v$. For $\forall \varepsilon$ there exists a δ s.t. $\|X(p+v) - X(p) - X'(p)(v)\| / \|v\| < \varepsilon$, $\forall \|v\| < \delta$. As α is continuous, there exists $\delta_1 > 0$ s.t. $\forall t \in (t_0 - \delta_1, t_0 + \delta_1)$: $\|\alpha(t) - \alpha(t_0)\| < \delta$. thus

$$\|X(p + \alpha(t) - \alpha(t_0)) - X(p) - X'(p)(\alpha(t) - \alpha(t_0))\| / \|\alpha(t) - \alpha(t_0)\| < \varepsilon$$

i.e. $\|X(\alpha(t)) - X(\alpha(t_0)) - X'(p)(\alpha(t) - \alpha(t_0))\| / \|\alpha(t) - \alpha(t_0)\| < \varepsilon$.

i.e. $\lim_{t\to t_0} \|X(\alpha(t)) - X(\alpha(t_0)) - X'(p)(\alpha(t) - \alpha(t_0))\| / \|\alpha(t) - \alpha(t_0)\| = 0$ $(*)$

Notice $\lim_{t\to t_0} \frac{\alpha(t) - \alpha(t_0)}{t - t_0} = \dot\alpha(t) = v$ So $\lim_{t\to t_0} \frac{\|\alpha(t) - \alpha(t_0)\|}{|t - t_0|} = \|v\|$ (1)

$\lim_{t\to t_0} (X(\alpha(t)) - X(\alpha(t_0)))/(t - t_0) = \nabla_v X$ (by definition of $\nabla_v X$) (2)

As $X'(p)$ is a linear map, ~~suppose its corresponding matrix is A~~, thus

If $\lim_{i\to\infty} v_i = v$ then $\lim_{i\to\infty} A(v_i) = A(v)$ use basis expression must finite dimensional

$\lim_{t\to t_0} \frac{X'(p)(\alpha(t) - \alpha(t_0))}{t - t_0} = \lim_{t\to t_0} X'(p)\left(\frac{\alpha(t) - \alpha(t_0)}{t - t_0}\right)$ (since $X'(p)$ is linear)

$= X'(p)\left(\lim_{t\to t_0} \frac{\alpha(t) - \alpha(t_0)}{t - t_0}\right) = X'(p)(v)$ (3)

Plugging (1) (2) (3) into $(*)$ $\|\nabla_v X - X'(p)(v)\| / \|v\| = 0$ i.e. $\nabla_v X = X'(p)(v)$

9.14 $L_p(p, v) \overset{\text{def of } L_p}{=} -\nabla_v N(p) \overset{\text{def. of } \tilde N}{=} -\nabla_v \tilde N(p) \overset{\text{result of 9.13}}{=} -\tilde N'(p)(v)$

9.15 (a) $\dot{\tilde\alpha}(t) = X(\tilde\alpha(t)) \Leftrightarrow \begin{cases} \dot x_1 = u_1, \cdots, \dot x_{n+1} = u_{n+1} \\ \dot u_k = -(u_1, \cdots, u_{n+1}) \cdot \left(\sum_{i=1}^{n+1} u_i \frac{\partial N_1}{\partial x_i}, \cdots, \sum_{i=1}^{n+1} u_i \frac{\partial N_{n+1}}{\partial x_i}\right) N_k = -N_k \sum_{i,j=1}^{n+1} u_i u_j \frac{\partial N_j}{\partial u_i} \end{cases}$ (denote $\alpha = (x_1, \cdots, x_{n+1})$, $\tilde\alpha = (\alpha, u_1, \cdots, u_{n+1})$)

So $\ddot x_k + N_k \cdot \sum_{i,j=1}^{n+1} \dot x_i \dot x_j \frac{\partial N_j}{\partial x_i} = 0$ which is the same as (G) +

Then follow the proof in the theorem of chapter 7, α is a geodesic of S. (Note: $\alpha \subset S$ is assumed)

Note the equation $\dot{\tilde\alpha}(t) = X(\tilde\alpha(t))$ is 1st order differential system in u and x so unique solution

(b) $X(\beta(t)) = \dot\beta(t) \Leftrightarrow \{\dot\beta_1 = \beta_2$ and $\dot\beta_2 = -(\beta_2 \cdot \nabla_{\beta_2} N) N(\beta_1)$ As in (a), we can derive the equation (G) in terms of β_1. (G) itself guarantees β_1 is on S as shown by the proof in Thm of chapter 7, given that $\beta_1(t_0) = p \in S$, $\dot\beta_1(t_0) = \beta_2(t_0) = v \in S_p$.

10.1 $\alpha = (x, y)$ $\dot\alpha = (x', y')$, $\ddot\alpha = (x'', y'')$ $N = (-y', x')$ (due to consistency).

So $k \circ \alpha = \ddot\alpha \cdot N \circ \alpha / \|\dot\alpha\|^2 = (-x''y' + y''x') / (x'^2 + y'^2)^{3/2}$

10.2 $f : X_2 - g(X_1)$, ~~$f' = (-g'(X_1), 1)$~~ $f^{-1}(0)$ can be viewed as $\alpha(t) = \begin{cases} t \\ g(t) \end{cases}$ $t \in I$

By Ex 10.1. curvature of C at point $(t, g(t)) = k \circ \alpha = g''(t) / [1 + (g'(t))^2]^{3/2}$

10.3 (a) $\nabla = (a, b)$ $X = (+b, -a)$ $\dot\alpha(t) = X(\alpha(t)) \Rightarrow \left\{ \alpha(t) = \begin{pmatrix} +bt + c_1 \\ -at + c_2 \end{pmatrix} \right.$, ~~$\alpha(0) = \frac{c}{2}\begin{pmatrix} 1/a \\ 1/b \end{pmatrix} \Rightarrow \alpha(t) = \begin{pmatrix} -bt + \frac{c}{2a} \\ at + \frac{c}{2b} \end{pmatrix}$~~

Since $(a, b) \neq (0, 0)$ let $a \neq 0$. let $\alpha(0) = \begin{pmatrix} c/a \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} c_1 = c/a \\ c_2 = 0 \end{pmatrix} \Rightarrow \alpha(t) = \begin{pmatrix} +bt + c/a \\ -at \end{pmatrix}$ $t \in R$

(b) $\nabla = \left(\frac{2X_1}{a^2}, \frac{2X_2}{b^2}\right)$ $X = \left(\frac{2X_2}{b^2}, \frac{-2X_1}{a^2}\right)$ $\left.\begin{matrix} \dot\alpha(t) = X(\alpha(t)) \\ \frac{1}{a^2}\alpha_1^2(t) + \frac{1}{b^2}\alpha_2^2(t) = 1 \end{matrix}\right\} \Rightarrow \begin{cases} \alpha_1(t) = a \sin \frac{2}{ab} t \\ \alpha_2(t) = b \cos \frac{2}{ab} t \end{cases}$ $t \in R$

(c) $\nabla = (-2aX_1, 1)$, $X = (1, 2aX_1)$, $\dot\alpha(t) = X(\alpha(t)) \Rightarrow \begin{cases} \alpha_1(t) = t + c_1 \\ \alpha_2(t) = at^2 + 2ac_1t + c_2 \end{cases}$

$\alpha_2(t) - a(\alpha_1(t))^2 = c \Rightarrow c_2 = c + a c_1^2$. let $c_1 = 0$, $c_2 = c$. So $\begin{cases} \alpha_1(t) = t \\ \alpha_2(t) = at^2 + c \end{cases}$ $t \in R$

(d) $\nabla = (+2X_1, -2X_2)$ $X = (-2X_2, -2X_1)$ $\dot\alpha(t) = X(\alpha(t))$ $\Rightarrow \begin{cases} \alpha_1(t) = \cancel{\cos t} \\ \alpha_2(t) = \cancel{\sin t} \end{cases}$ ~~$t \in [0, 2\pi)$~~ $\setminus \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

$\alpha_1(t) = \frac{c_1 e^{2t} + c_2 e^{-2t}}{2}$ ~~$\alpha_1(t) = \frac{1}{2}(e^{2t} + e^{-2t})$, $\alpha_2(t) = \frac{c_2}{2}(e^{-2t} - e^{2t})$~~ $\alpha_1^2 - \alpha_2^2 = 1$

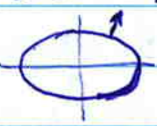
$\alpha_2(t) = \frac{c_2 e^{-2t} - c_1 e^{2t}}{2}$ $\forall$ ~~$c_1, c_2 \in \ldots$~~ if ignore $x_1 > 0$ $c_1c_2 = 1$

10.4 (a) $k = 0$ as $\ddot\alpha = 0$. (b) $\alpha = \begin{pmatrix} a \sin 2t/ab \\ b \cos 2t/ab \end{pmatrix}$, $\dot\alpha = \begin{pmatrix} 2/b \cos(2t/ab) \\ -2/a \sin(2t/ab) \end{pmatrix}$, $\ddot\alpha = \begin{pmatrix} -4/ab^2 \sin(2t/ab) \\ -4/a^2b \cos(2t/ab) \end{pmatrix}$

$N = \lambda \begin{pmatrix} 2/a \sin(2t/ab) \\ 2/b \cos(2t/ab) \end{pmatrix}$ ~~$= \frac{1}{\sqrt{a^2+b^2}} \begin{pmatrix} b \sin(2t/ab) \\ a \cos(2t/ab) \end{pmatrix}$~~, $k(p) = \frac{\ddot\alpha \cdot N \circ \alpha}{\|\dot\alpha\|^2}$ ~~$= \frac{-4/ab}{4(a^2+b^2)/a^2b^2} = \frac{-ab}{a^2+b^2}$~~ ~~$\ddot\alpha \cdot N\circ\alpha = \frac{-4}{ab\sqrt{a^2+b^2}}$~~

$\|\dot\alpha\|^2 = \frac{4}{a^2b^2}\left(a^2 \cos^2 \frac{2t}{ab} + b^2 \sin^2 \frac{2t}{ab}\right)$

$\ddot\alpha \cdot N \circ \alpha = \frac{-4}{a^2b^2} \begin{pmatrix} a \sin \frac{2t}{ab} \\ b \cos \frac{2t}{ab} \end{pmatrix} \cdot \frac{2}{ab} \begin{pmatrix} b \sin \frac{2t}{ab} \\ a \cos \frac{2t}{ab} \end{pmatrix} / \frac{2}{ab}\sqrt{a^2 \cos^2 \frac{2t}{ab} + b^2 \sin^2 \frac{2t}{ab}}$

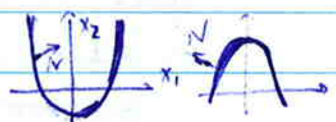


So $k < 0$

So $k(p) = \frac{\ddot\alpha \cdot N \circ \alpha}{\|\dot\alpha\|^2} = -ab\left(a^2 \cos^2 \frac{2t}{ab} + b^2 \sin^2 \frac{2t}{ab}\right)^{-3/2}$ If $a = b = r$, then $k(p) = -\frac{1}{r}$.

$= -ab\left(\frac{a^2}{b^2} X_2^2 + \frac{b^2}{a^2} X_1^2\right)^{-3/2}$

(c) Use Ex 10.2. ~~$k \circ \alpha =$~~ $g(t) = at^2 + c$, $g'(t) = 2at$. $g''(t) = 2a$

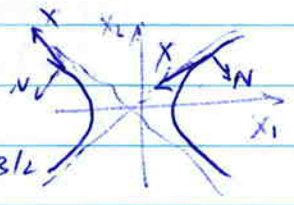


$k \circ \alpha = 2a / (1 + 4a^2t^2)^{3/2} = 2a / (1 + 4a^2X_1^2)^{3/2}$

(d) Use Ex 10.1 ~~$\alpha(t) = \begin{pmatrix} \cos t \\ tg t \end{pmatrix}$ $\dot\alpha(t) = \begin{pmatrix} \sin t / \cos^2 t \\ 1/\cos^2 t \end{pmatrix}$ $\ddot\alpha(t) = \frac{1}{\cos^3 t}\begin{pmatrix} 1 + \sin^2 t \\ 2 \sin t \end{pmatrix}$~~

~~$k \circ \alpha = -\cos^3 t / (1 + \sin^2 t)^{3/2} = -(X_1^2 + X_2^2)^{3/2} \cdot \operatorname{sgn}(X_1)$~~

In general for $\frac{X^2}{a^2} - \frac{y^2}{b^2} = 1$ $k = -ab / (a^2 tg^2 t + b^2 \sec^2 t)^{3/2}$



$\alpha(t) = \frac{1}{2}(c_1e^{2t} + c_2e^{-2t}, c_2e^{-2t} - c_1e^{2t})^T$, $\dot\alpha(t) = (c_1e^{2t} - c_2e^{-2t}, -c_2e^{-2t} - c_1e^{2t})^T$

$\ddot\alpha(t) = 2(c_1e^{2t} + c_2e^{-2t}, c_2e^{-2t} - c_1e^{2t})$ So $k \circ \alpha = 8 / [2(e^{4t} + e^{-4t})]^{3/2}$

$k = 1 / (X_1^2 + X_2^2)^{3/2}$, So curve is always turning (according to X) towards N