8.1 (b) $(fX)' = (\dot{f}X) - [(\dot{fX})\cdot N(\alpha(t))]N(\alpha(t))$

$= \dot{f}X + f\dot{X} - [(\dot{f}X + f\dot{X})\cdot N(\alpha(t))]N(\alpha(t))$ (as $X\cdot N(\alpha(t)) = 0$ by X being tangent to S)

$= \dot{f}X + f\dot{X} - f[\dot{X}\cdot N(\alpha(t))]\cdot N(\alpha(t)) = \dot{f}X + fX'$

8.2 ~~$\alpha(t)$ is always in S, i.e. $\vec{a}\cdot\alpha(t) = b$, i.e. $\dot{\alpha}(t) = 0$. $\dot{\alpha}(t) = 0 \in S^{\perp}_{\alpha(t)}$ So~~

Define Vector field (along α) $\vec{V}(t) = \vec{v}$ on S, As $v \in S_p$.

so V is tangent to S, $\dot{V} = 0$ (and of course along α), So V is parallel along α. So $P_\alpha(v) = (q, v)$

That means parallel transport in an n-plane is path independent

8.3 When $v_1 = (p, 1, 0, 0)$ By example on page 49, $V_1(t) = \dot{\alpha}(t) = (\cos t, 0, -\sin t)$, $V_1(\pi) = (-1, 0, 0)$

when $v_2 = (p, 0, 1, 0)$ Then the vector field $V_2(t) = (0,1,0)$, is parallel to S along α

So $V_2(\pi) = (0,1,0)$. As P_α is linear transform, $P_\alpha(v) = (q, -v_1, v_2, 0)$

8.4 Define geodesic $\alpha(t) = p\cos t + \hat{v}\sin t$, $\dot{\alpha}(t) = -p\sin t + \hat{v}\cos t$. $\alpha(0) = p$, $\alpha(\frac{\pi}{2}) = \hat{v}$

$\dot{\alpha}(0)\cdot v = \|v\|$, As $\dot{\alpha}(\frac{\pi}{2})\cdot P_\alpha(\hat{v}) = -p$ $P_\alpha(\hat{v}) = \|v\|$, and $\|P_\alpha(v)\| = \|v\|$ So $P_\alpha(v) = -p\|v\|$ (By corollary on Pg 48)

Likewise define geodesic $\beta(t) = p\sin t + \hat{w}\cos t$ $P_\beta(-p\|v\|) = w$ $\beta(\frac{\pi}{2}) = p$, $\beta(0) = \hat{w}$

So both $\alpha(\frac{\pi}{2})$ and $\beta(\frac{\pi}{2})$ are on $\{x \in S^2_p \mid p\cdot x = 0\}$. we define geodesic

(by example 3 in ch7 Pg 40) $\nu(t) = \hat{v}\cos t + \sin t\cdot(p\times\hat{v})$, $\nu(0) = \hat{v}$, We now find t_0 s.t. $\nu(t_0) = \hat{w}$

$\hat{v}\cos t_0 + (p\times\hat{v})\sin t_0 = \hat{w} \Rightarrow \hat{v}\cdot\hat{w}\cos t_0 + p\cdot(\hat{v}\times\hat{w})\sin t_0 = 1$. Let the angle

between $\hat{v}$ and $\hat{w}$ be θ, since $p\perp\hat{v}$, $p\perp\hat{w}$, we have either

$\cos\theta\cos t_0 + \sin\theta\sin t_0 = 1$ or $\cos\theta\cos t_0 - \sin\theta\sin t_0 = 1$. But in whichever

case, there must be a solution t_0 ($t_0 = \theta$ or $-\theta$). Check $V(t) = -p\|v\|$ is parallel along $\nu(t)$:

$\dot{\nu}(t)\cdot p\|v\| = 0$ $\forall t$, $\|V(t)\| =$ constant.

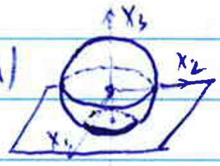
$V(t)\cdot N_{\nu(t)} = \pm p\|v\|\cdot(\hat{v}\cos t + (p\times\hat{v})\sin t) = 0$ So $V(t) \in S_{\nu(t)}$

Therefore $V(t) = -p\|v\|$ is parallel on S^2_p along $\nu(t)$ as $\nu(t)$ is geodesic. (by Corollary Pg 48)

So we finally find a piecewise smooth parametrized curve

$v \to P_\alpha(v) = -p\|v\| \to P_\nu(-p\|v\|) = -p\|v\| \to P_\beta(-p\|v\|) = w$.

8.5 (a)



$S_1 = \{(x - (0,0,\frac{1}{2}))\cdot(0,0,1) = 0\}$.

$S_2 = \{x \mid \|x\|^2 = 1\}$. $\alpha(t) = (\frac{\sqrt{3}}{2}\cos t, \frac{\sqrt{3}}{2}\sin t, \frac{1}{2})$

$X(t) = (\frac{\sqrt{3}}{2}\cos t, \frac{\sqrt{3}}{2}\sin t, 0)$

$X(t)$ is parallel along α as viewed in S_1. But $X(t)$ is not parallel as viewed in S_2 Because $\dot{X}(t) = (-\frac{\sqrt{3}}{2}\sin t, \frac{\sqrt{3}}{2}\cos t, 0) \notin \{\lambda(0,0,1) \mid \lambda \in \mathbb{R}\} = S^{\perp}_{\alpha(t)}$