as $S_{1\alpha(t)}^{\perp} = S_{2\alpha(t)}^{\perp} \Leftarrow S_{1\alpha(t)} = S_{2\alpha(t)}$

(b) X is parallel along α in $S_1 \Leftrightarrow \dot{X}(t) \in S_{1\alpha(t)}^{\perp} \Leftrightarrow \dot{X}(t) \in S_{2\alpha(t)}^{\perp} \Leftrightarrow X$ is parallel in S_2

(c) α is geodesic in $S_1 \Leftrightarrow \ddot{\alpha}(t) \in S_{1\alpha(t)}^{\perp} \Leftrightarrow \ddot{\alpha}(t) \in S_{2\alpha(t)}^{\perp} \Leftrightarrow \alpha$ is geodesic in S_2

8.6 (a) Let $Y(u) = X(h(u))$

$\dot{X}(t) \in S_{\alpha(t)}^{\perp} \Rightarrow \dot{Y}(u) = \dot{X}(h(u))\, h'(u) \in S_{\alpha(h(u))}^{\perp} = S_{\beta(u)}^{\perp}$.

As $h'(t) \neq 0$. $h(t)$ is monotonic. so there is $\underbrace{h^{-1}}_{(h^{-1})' \neq 0}$ $X = Y \circ h^{-1}$. So the same proof as above goes

therefore it is iff.

(b) ~~X~~ First by (a), $X \circ h$ is parallel along $\alpha \circ h$. If $\alpha(t_1) = p$, $\alpha(t_2) = q$.

Let $h(u_1) = t_1$, $h(u_2) = t_2$. So $\alpha(h(u_1)) = p$, $\alpha(h(u_2)) = q$.

$X(h(u_1)) = X(t_1)$, $X(h(u_2)) = X(t_2)$. Besides h is monotonic so $h: [u_1, u_2] \to [t_1, t_2]$

Thus, $X \cdot h$ transports p at u_1 to q at u_2

(c) $\forall v$. If $\alpha(t_1) = p$, $\alpha(t_2) = q$ X is parallel to S along α. $X(t_1) = v$. $X(t_2) = u$ (i.e. $P_\alpha(v) = u$)

$\beta(t) \triangleq \alpha(-t)$. As $\dot{X}(t) \in S_{\alpha(t)}^{\perp}$ ~~So $\dot{X}(-t) \in S_{\alpha(t)}^{\perp}$~~ Let $Y(t) = X(-t)$, then

$\dot{Y}(t) = -\dot{X}(-t) \in S_{\alpha(-t)}^{\perp} = S_{\beta(t)}^{\perp}$ ~~So $Y(t)$ is~~ Besides as $X(t) \cdot N_{\alpha(t)} = 0$

$Y(t) \cdot N_{\beta(t)} = X(-t) \cdot N_{\alpha(-t)} = 0$ So $Y(t)$ is parallel along $\beta(t)$

$\beta(-t_2) = q$. $\beta(-t_1) = q$. $Y(-t_2) = X(t_2) = u$. $Y(-t_1) = X(t_1) = v$

So u is transported to v along $\beta(t)$ from q at $-t_2$ to p at $-t_1$, i.e. parallel transport from q to p along $\alpha(-t)$ is the inverse of parallel transport from p to q along α

8.7 (i) γ is α concatenates with β. If P_α, P_β correspond to A and B respectively, then P_γ correspond to $A \cdot B$, which is also nonsingular

(ii) By the third question in Ex 8.6, P_α^{-1} is the parallel transport along $\alpha(-t)$, $\beta(t) \triangleq \alpha(-t)$ $t \in [-b, -a]$, P_β corresponds to A^{-1}

8.8 We use X^* to denote $X'(t)$, the Fermi derivative.

(a) i $(X+Y)^* = (X+Y)' - [(X+Y)'(t) \cdot \dot{\alpha}(t)]\dot{\alpha}(t)$ by $(X+Y)' = X' + Y'$

$= X' - [X'(t) \cdot \dot{\alpha}(t)]\dot{\alpha}(t) + Y' - [Y'(t) \cdot \dot{\alpha}(t)]\dot{\alpha}(t) = X^* + Y^*$

ii $(fX)^* = (fX)' - [(fX)' \dot{\alpha}(t)]\dot{\alpha}(t)$ (by $(fX)' = f'X + fX'$)

$= (f'X + fX') - [(f'X + fX') \cdot \dot{\alpha}(t)]\dot{\alpha}(t)$ (by $X(t) \dot{\alpha}(t) = 0$)

$= f'X + f\{X' - [X' \cdot \dot{\alpha}(t)]\dot{\alpha}(t)\} = f'X + fX^*$

iii $(X \cdot Y)' =$ ~~$(X \cdot Y)' - [(XY)' \dot{\alpha}(t)]\dot{\alpha}(t)$ (by $(XY)' = X'Y + XY'$)~~

~~$= X'Y + XY' - [(X'Y + XY') \cdot \alpha(t)]\dot{\alpha}(t)$~~

$= X'Y + Y'X$

by $\dot{\alpha}(t) Y \overset{\text{!!}}{=} \dot{\alpha}(t) X(t) = 0$

$X^* Y + X Y^* = [X' - (X'(t) \cdot \dot{\alpha}(t))\dot{\alpha}(t)] Y + X \cdot [Y' - (Y'(t) \cdot \dot{\alpha}(t))\dot{\alpha}(t)] = X'Y + XY'$