derive the equation (G) in terms of β_1. (G) itself guarantees β_1 is on S as shown by the proof in Thm of chapter 7, given that $\beta_1(t_0)=p\in S$, $\dot\beta_1(t_0)=\dot\beta_2(t_0)=v\in S_p$.

10.1 $\alpha=(x,y)$ $\dot\alpha=(x',y')$, $\ddot\alpha=(x'',y'')$ $N=(-y',x')$ (due to consistency).

So $k\circ\alpha=\ddot\alpha\cdot N\circ\alpha/\|\dot\alpha\|^2=(-x''y'+y''x')/(x'^2+y'^2)^{3/2}$

10.2 $f=x_2-g(x_1)$, $\cancel{f'=(-g'(x_1),1)}$ $f^{-1}(0)$ can be viewed as $\alpha(t)=\begin{cases}t\\ g(t)\end{cases}$ $t\in I$

By Ex 10.1. curvature of C at point $(t,g(t))=k\circ\alpha=g''(t)/[1+(g'(t))^2]^{3/2}$

10.3 (a) $\nabla=(a,b)$ $X=(+b,-a)$ $\dot\alpha(t)=X(\alpha(t))\Rightarrow\alpha(t)=\begin{pmatrix}+bt+c_1\\-at+c_2\end{pmatrix}$, $\cancel{\text{let } \alpha(0)=\frac{c}{2}\begin{pmatrix}1/a\\1/b\end{pmatrix}\Rightarrow\alpha(t)=\begin{pmatrix}-bt+\frac{c}{2a}\\at+\frac{c}{2b}\end{pmatrix}}$

Since $(a,b)\neq(0,0)$ let $a\neq0$. let $\alpha(0)=\begin{pmatrix}c/a\\0\end{pmatrix}\Rightarrow\begin{pmatrix}c_1=c/a\\c_2=0\end{pmatrix}\Rightarrow\alpha(t)=\begin{pmatrix}+bt+c/a\\-at\end{pmatrix}$ $t\in R$

(b) $\nabla=\left(\frac{2x_1}{a^2},\frac{2x_2}{b^2}\right)$ $X=\left(\frac{2x_2}{b^2},\frac{-2x_1}{a^2}\right)$ $\left.\begin{array}{l}\dot\alpha(t)=X(\alpha(t))\\ \frac{1}{a^2}\alpha_1^2(t)+\frac{1}{b^2}\alpha_2^2(t)=1\end{array}\right\}\Rightarrow\begin{cases}\alpha_1(t)=a\sin\frac{2}{ab}t\\ \alpha_2(t)=b\cos\frac{2}{ab}t\end{cases}$ $t\in R$

(c) $\nabla=(-2ax_1,1)$, $X=(1,2ax_1)$, $\dot\alpha(t)=X(\alpha(t))\Rightarrow\begin{cases}\alpha_1(t)=t+c_1\\ \alpha_2(t)=at^2+2ac_1t+c_2\end{cases}$

$\alpha_2(t)-a(\alpha_1(t))^2=c\Rightarrow c_2=c+ac_1^2$. let $c_1=0$, $c_2=c$. So $\begin{cases}\alpha_1(t)=t\\ \alpha_2(t)=at^2+c\end{cases}$ $t\in R$

(d) $\nabla=(+2x_1,-2x_2)$ $X=(-2x_2,-2x_1)$ $\dot\alpha(t)=X(\alpha(t))$ $\Rightarrow\begin{cases}\alpha_1(t)=\cancel{\cos t}\\ \alpha_2(t)=\cancel{\sin t}\end{cases}$ $\cancel{t\in[0,2\pi)\setminus\left\{\frac{\pi}{2},\frac{3\pi}{2}\right\}}$

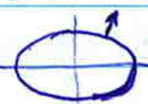
$\alpha_1(t)=\frac{c_1e^{2t}+c_2e^{-2t}}{2}$ $\cancel{\alpha_1(t)=\frac{1}{2}(e^{2t}+e^{-2t}),\ \alpha_2(t)=\frac{c_2}{2}(e^{-2t}-e^{2t})}$ $\alpha_1^2-\alpha_2^2=1$

$\alpha_2(t)=\frac{c_2e^{-2t}-c_1e^{2t}}{2}$ $\forall$ $\cancel{c_1,c_2\in\ldots}$ if ignore $x_1>0$ $c_1c_2=1$

10.4 (a) $k=0$ as $\ddot\alpha=0$. (b) $\alpha=\begin{pmatrix}a\sin 2t/ab\\ b\cos 2t/ab\end{pmatrix}$, $\dot\alpha=\begin{pmatrix}2/b\cos(2t/ab)\\-2/a\sin(2t/ab)\end{pmatrix}$, $\ddot\alpha=\begin{pmatrix}-4/ab^2\sin(2t/ab)\\-4/a^2b\cos(2t/ab)\end{pmatrix}$

$N=\lambda\begin{pmatrix}2/a\sin(2t/ab)\\2/b\cos(2t/ab)\end{pmatrix}$ $\cancel{=\frac{1}{\sqrt{a^2+b^2}}\begin{pmatrix}b\sin(2t/ab)\\a\cos(2t/ab)\end{pmatrix}}$, $k(p)=\frac{\ddot\alpha\cdot N\circ\alpha}{\|\dot\alpha\|^2}$ $\cancel{=\frac{-4/ab}{4(a^2+b^2)/a^2b^2}=\frac{-ab}{a^2+b^2}}$ $\cancel{\ddot\alpha\cdot N\circ\alpha=\frac{-4}{ab\sqrt{a^2+b^2}}}$

$\cancel{\text{So } k(p)=}$ $\|\dot\alpha\|^2=\frac{4}{a^2b^2}\left(a^2\cos^2\frac{2t}{ab}+b^2\sin^2\frac{2t}{ab}\right)$ $\cancel{\text{So } k(p)=}$



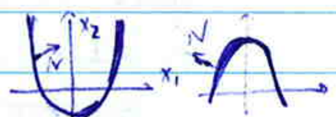
So $k<0$

$\ddot\alpha\cdot N\circ\alpha=\frac{-4}{a^2b^2}\begin{pmatrix}a\sin\frac{2t}{ab}\\b\cos\frac{2t}{ab}\end{pmatrix}\cdot\frac{2}{ab}\begin{pmatrix}b\sin\frac{2t}{ab}\\a\cos\frac{2t}{ab}\end{pmatrix}\Big/\frac{2}{ab}\sqrt{a^2\cos^2\frac{2t}{ab}+b^2\sin^2\frac{2t}{ab}}$

So $k(p)=\frac{\ddot\alpha\cdot N\circ\alpha}{\|\dot\alpha\|^2}=-ab\left(a^2\cos^2\frac{2t}{ab}+b^2\sin^2\frac{2t}{ab}\right)^{-3/2}$ If $a=b=r$. then $k(p)=-\frac{1}{r}$.

$=-ab\left(\frac{a^2}{b^2}x_2^2+\frac{b^2}{a^2}x_1^2\right)^{-3/2}$

(c) Use Ex 10.2. $\cancel{k\circ\alpha=}$ $g(t)=at^2+c$, $g'(t)=2at$. $g''(t)=2a$

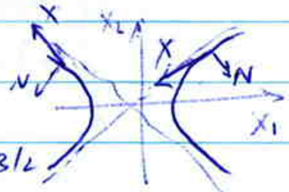


$k\circ\alpha=2a/(1+4a^2t^2)^{3/2}=2a/(1+4a^2x_1^2)^{3/2}$

(d) Use Ex 10.1 $\cancel{\alpha(t)=\begin{pmatrix}\cos t\\ \operatorname{tg} t\end{pmatrix}\ \dot\alpha(t)=\begin{pmatrix}\sin t/\cos^2 t\\1/\cos^2 t\end{pmatrix}\ \ddot\alpha(t)=\frac{1}{\cos^3 t}\begin{pmatrix}1+\sin^2 t\\2\sin t\end{pmatrix}}$

$\cancel{k\circ\alpha=-\cos^3 t/(1+\sin^2 t)^{3/2}=-(x_1^2+x_2^2)^{3/2}\cdot\operatorname{sgn}(x_1)}$

In general for $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $k=-ab/(a^2\operatorname{tg}^2 t+b^2\sec^2 t)^{3/2}$



$\alpha(t)=\frac{1}{2}(c_1e^{2t}+c_2e^{-2t},c_2e^{-2t}-c_1e^{2t})^T$, $\dot\alpha(t)=(c_1e^{2t}-c_2e^{-2t},-c_2e^{-2t}-c_1e^{2t})^T$

$\ddot\alpha(t)=2(c_1e^{2t}+c_2e^{-2t},c_2e^{-2t}-c_1e^{2t})$ So $k\circ\alpha=8/[2(e^{4t}+e^{-4t})]^{3/2}$

$k=1/(x_1^2+x_2^2)^{3/2}$, So curve is always turning (according to X) towards N