10.5 $h(t_0) = (\alpha(t_0) - p) \cdot N(p) = (p-p) \cdot N(p) = 0$ $\quad h'(t_0) = (\dot\alpha(t_0)) \cdot N_p = 0$.

$h''(t_0) = \ddot\alpha(t_0) \cdot N(p) = k(p)$ because $\|\dot\alpha(t_0)\| = 1$

10.6 (a) As $\|\dot\alpha\| = \text{const}$ $\dot\alpha \cdot \ddot\alpha = 0$ But $\dot\alpha \cdot N \circ \alpha = 0$ and $\{v \mid v \cdot \dot\alpha = 0\}$ is one dimensional (as C is in 2D plane) So $\ddot\alpha = \lambda N \circ \alpha$, $\lambda = \ddot\alpha \cdot N \circ \alpha = k \circ \alpha$, So $\dot T = \ddot\alpha = (k \circ \alpha) \cdot (N \circ \alpha)$

(b) $\|N\| = 1$. So $(N \circ \alpha) \cdot (\dot N \circ \alpha) = 0$. But $(N \circ \alpha) \cdot \dot\alpha = 0$. and we are in 2-D plane

so $\dot N \circ \alpha = \lambda \dot\alpha$ $\quad \lambda = \dot N \circ \alpha \cdot \dot\alpha$ Besides, as $\dot\alpha \cdot N \circ \alpha = 0$ we have

$\ddot\alpha \cdot N \circ \alpha + \dot\alpha \cdot \dot N \circ \alpha = 0$ So $\lambda = -\ddot\alpha \cdot N \circ \alpha = -k \circ \alpha$.

Thus. $\dot N \circ \alpha = -(k \circ \alpha) \cdot \dot\alpha = -(k \circ \alpha) \cdot T$.

10.7 (a) $\|\dot\alpha\| = 1 \Rightarrow \dot\alpha \ddot\alpha = 0 \Rightarrow T \perp N$. $B \perp N$ and $B \perp T$ are by definition of B (cross product)

(b) $\dot T = \ddot\alpha \,/\!/\, N(t) = \ddot\alpha / \|\ddot\alpha\|$ So $\dot T = \|\ddot\alpha\| \cdot N$ so $k \triangleq \|\ddot\alpha\|$

$\dot B = \dot T \times N + T \times \dot N = T \times \dot N$ So $\dot B \perp T$. $\dot B \perp \dot N$ But we know $N \perp T$

and $\|N\| = 1 \Rightarrow \dot N \perp N$. As we are in 3-D space $\dot B = -\tau \cdot N$ wher $\tau: I \to R$

$\tau(t) = -\dot B(t) \cdot N(t)$ so $\tau(t)$ is smooth.

$\dot N \perp N$. We know $B \perp N$, $T \perp N$ and $B \perp T$. So there exist $\lambda_1, \lambda_2 : I \to R$

$\dot N = \lambda_1 B + \lambda_2 T$ $\qquad \lambda_1 = \dot N \cdot B = -N \cdot \dot B = \tau$ (Since $N \cdot B = 0 \Rightarrow \dot N B + N \dot B = 0$)

$\lambda_2 = \dot N \cdot T = -N \cdot \dot T = -k$ (since $N \cdot T = 0 \Rightarrow \dot N \cdot T + N \dot T = 0$)

So $\dot N = \tau B - k T$.

10.8 By definition of circle of curvature. $C_p = O_p$. $\dot\alpha(0) \in C_p$. $\dot\beta(0) \in O_p$. C_p and O_p are one dimensional. $\|\dot\alpha(0)\| = \|\dot\beta(0)\| = 1$. So $\dot\alpha(0) = \pm\dot\beta(0)$. Furthermore $\dot\alpha(0), \dot\beta(0)$ are both consistent with $N(p)$ and $N_1(p)$ resp. ($N(p)$ and $N_1(p)$ are orientation norms of C and O). But $N_1(p) = N(p)$

Thus $\dot\alpha(0) = \dot\beta(0)$

As ~~$\ddot\alpha \perp \dot\alpha \Rightarrow \ddot\alpha /\!/ N(p)$~~ $\quad \ddot\alpha \cdot N(p) = -\nabla_{\dot\alpha(0)} N \cdot \dot\alpha(0)$ by Thm 1 of chapter 9

~~$\ddot\beta \perp \dot\beta \Rightarrow \ddot\beta /\!/ N(p)$~~ $\quad \ddot\beta \cdot N_1(p) = -\nabla_{\dot\beta(0)} N_1 \cdot \dot\beta(0)$

But $\dot\alpha(0) = \dot\beta(0)$ and by definition of circle of curvature, $\nabla_{\dot\alpha(0)} N = \nabla_{\dot\beta(0)} N_1$

So $\ddot\alpha(0) N(p) = \ddot\beta(0) \cdot N_1(p)$ (*) ~~But $N = N_1(p)$~~ As $\ddot\alpha /\!/ N(p)$, suppose

$\ddot\alpha(0) = \lambda_1 N(p)$, suppose $\ddot\beta(0) = \lambda_2 N_1(p)$ similarly as $\ddot\beta /\!/ N_1(p)$

So $\quad \lambda_1 = \ddot\alpha(0) \cdot N(p) \overset{by (*)}{=} \ddot\beta(0) \cdot N_1(p) = \lambda_2$, $\ddot\alpha(0) = \lambda_1 N(p) = \lambda_2 N_1(p) = \ddot\beta(0)$

as $N_1(p) = N(p)$

10.9 "only if": $O: \|x - q\|^2 = r^2$. $p \in O \Rightarrow \|p - q\|^2 = r^2 \Rightarrow f(0) = \|p-q\|^2 - r^2 = 0$

$C_p = O_p \Rightarrow$ the normal vector of O at $p = 2(p-q) \perp O_p = C_p = \{\lambda \dot\alpha(0) \mid \lambda \in R\}$