so $(p-q)\cdot\dot\alpha(0)=0$ so $f'(0)=2(\alpha(0)-q)\dot\alpha(0)=2(p-q)\cdot\dot\alpha(0)=0$.

By Thm 1 of chapter 9. $\ddot\alpha(t_0)\cdot N(p)=-\nabla_{\dot\alpha(0)}N(p)\cdot\dot\alpha(0)$ (N, N_1 are orientation of C and O resp.)

$N(p)=N_1(p)=\lambda(p-q)/r$ ($\lambda=\pm1$ which determines orientation) $\lambda=1$ outwards $\lambda=-1$ inwards

$\nabla_V N(p)=\nabla_V N_1(p)=\lambda\frac{1}{r}V$.

So $\ddot\alpha(t_0)\lambda(p-q)/r=\ddot\alpha(t_0)\cdot N(p)=-\nabla_{\dot\alpha(0)}N(p)\cdot\dot\alpha(0)=-\nabla_{\dot\alpha(0)}N_1(p)\cdot\dot\alpha(0)=-\lambda\frac{1}{r}\dot\alpha(0)\cdot\dot\alpha(0)=\frac{-\lambda}{r}$

So $\ddot\alpha(p-q)=-1$. So $f''(0)=2+2(p-q)\ddot\alpha(0)=0$

"If part" $f(0)\overset{=0}{\Rightarrow}\|p-q\|=r^2$ So $p\in O$. ①

$f'(0)=0\Rightarrow(p-q)\cdot\dot\alpha(0)=0$ As we are in 2D, and $p-q\in O_p^\perp$. So $\dot\alpha(0)\in O_p$.

But $\dot\alpha(0)\in C_p$ as well and O_p and O_p are both one dimensional, So $O_p=C_p$ ②; then we can easily choose an orientation of O such that its orientation at p is the same as C's ③

$f''(0)\Rightarrow(p-q)\cdot\ddot\alpha(0)=-1$ ~~$\forall v\in C_p$, i.e. $V=\mu\dot\alpha(0)$, N~~

Since $\nabla_V N\cdot N=0$ and $N\perp\dot\alpha(0)$ So $\nabla_{\dot\alpha(0)}N=a\cdot\dot\alpha(0)$ $a\in R$ as we are in 2D

$a=\nabla_{\dot\alpha(0)}N\cdot\dot\alpha(0)=-\ddot\alpha(0)N(p)=-\ddot\alpha(0)N_1(p)=-\lambda(p-q)/r\cdot\ddot\alpha(0)=\frac{\lambda}{r}$,

So $\nabla_{\dot\alpha(0)}N=\frac{\lambda}{r}\dot\alpha(0)$. But $\nabla_{\dot\alpha(0)}N_1=\frac{\lambda}{r}\dot\alpha(0)$ By Example in chapter 9 on page 56

So $\nabla_{\dot\alpha(0)}N=\nabla_{\dot\alpha(0)}N_1$. Furthermore, $\forall v\in C_p$, v must be $v=\mu\dot\alpha(0)$ $\mu\in R$.

But $\nabla_V N=\nabla_{\mu\dot\alpha(0)}N=\mu\cdot\nabla_{\dot\alpha(0)}N=\mu\nabla_{\dot\alpha(0)}N_1=\nabla_{\mu\dot\alpha(0)}N_1=\nabla_V N_1$ ④

Combining ①–④. O is circle of curvature of C at p.

10.10 $\dot\alpha(t)=(\cos\theta(t),\sin\theta(t))$ As $\alpha(t)$ is local parametrization of C

$N(\alpha(t))=(-\sin\theta(t),\cos\theta(t))$. $\ddot\alpha(t)=(-\sin\theta(t)\cdot\dot\theta(t),\cos\theta(t)\dot\theta(t))$. As α is unit speed, $k\circ\alpha=\ddot\alpha(t)\cdot N(\alpha(t))=\dot\theta(t)\overset{\triangle}{=}d\theta/dt$.

11.1 $l(\alpha)=\int_0^2\|(2t,3t^2)\|dt=\int_0^2\sqrt{4+9t^2}\,t\,dt\overset{u=t^2}{=}\frac{1}{2}\int_0^4\sqrt{4+9u}\,du$

$=\frac{1}{18}\int_0^4\sqrt{4+9u}\,d(4+9u)=\frac{1}{18}\frac{2}{3}(4+9u)^{\frac{3}{2}}\Big|_0^4=\frac{10}{27}(10\sqrt{10}-1)$

11.2 $l(\alpha)=\int_{-1}^{1}\|(-3\sin3t,3\cos3t,4)\|dt=10$

11.3 $l(\alpha)=\int_0^{2\pi}\|-2\sqrt2\sin2t,2\cos2t,2\cos2t\|dt=\int_0^{2\pi}2\sqrt2\,dt=4\pi\sqrt2$

11.4 $l(\alpha)=\int_0^{2\pi}\|(-\sin t,\cos t,-\sin t,\cos t)\|dt=2\sqrt2\pi$

11.5. $\alpha(t)=(12t,-5t)$ $t\in(-1,1)$ $l(C)=l(\alpha)=\int_{-1}^{1}13\,dt=26$ Ref Ex 11.9 changes sign

Actually, don't bother with orientation and $\dot\alpha$ compliance, because $l(C)\geq0$. and ~~orientation only~~