11.6 $\alpha(t)=(2\sin t, 1+2\cos t)$ $\underbrace{l(C)=l(\alpha)}=\int_0^{2\pi}\|\dot\alpha(t)\|dt=\int_0^{2\pi}\|(2\cos t,-2\sin t)\|dt=4\pi$

11.7 $\alpha(t)=(\sqrt{1+t^2},t)$, $t\in(-\sqrt3,\sqrt3)$, $l(C)=l(\alpha)=\int_{-\sqrt3}^{\sqrt3}\|(t(1+t^2)^{-1/2},1)\|dt$
$=2\int_0^{\sqrt3}\sqrt{1+t^2/(1+t^2)}\,dt$

11.8 $\alpha(t)=(\frac{2}{3}t^{\frac{3}{2}},t)$ $t\in(0,3)$ $l(C)=l(\alpha)=\int_0^3\|(t^{\frac12},1)\|dt=\int_0^3\sqrt{t+1}\,d(t+1)=14/3$

11.9 If $\alpha(t)$ $t\in(a,b)$ is consistent with N, then $\beta(t)\triangleq\alpha(-t)$ $t\in(-b,-a)$ is consistent with $-N$
$(\dot\alpha_1(t),\dot\alpha_2(t))^T=R_{-\pi/2}(N_1(\alpha(t)),N_2(\alpha(t)))^T$. for $\forall t\in(a,b)$
$(\dot\beta_1(t),\dot\beta_2(t))^T=(-\dot\alpha_1(-t),-\dot\alpha_2(-t))^T=R_{-\pi/2}(-N_1(\alpha(-t)),-N_2(\alpha(-t)))^T$
$\int_{-b}^{-a}\alpha(-t)dt=\int_a^b\alpha(t)dt$ So $l(C)=l(\tilde C)$

11.10 (a) $\int_a^b|k\circ\alpha(t)|dt=\int_a^b|\ddot\alpha\cdot N(\alpha(t))|dt=\int_a^b\|\ddot\alpha(t)\|dt$. If β is reparametrization of α. $\beta:(c,d)\to C$
$\beta=\alpha\circ h$. (Since α,β are both one-to-one, such h must exist, $h(t)\triangleq\alpha^{-1}(\beta(t))$) since both α and β are ~~smooth~~ regular. ($\dot\alpha\neq0$, $\dot\beta\neq0$) h must be differentiable
$\dot\beta(t)=\dot\alpha(h(t))\cdot h'(t)$. But $\|\dot\alpha\|=\|\dot\beta\|=1$, so $\|h'(t)\|\equiv1$. But h' is continuous.
So $h'\equiv1$ or $h'\equiv-1$. In whichever case $\ddot\beta(t)=\ddot\alpha(h(t))\cdot(h'(t))^2=\ddot\alpha(h(t))$
so $\int_c^d|k\circ\beta(t)|dt=\int_c^d\|\ddot\beta(t)\|dt=\begin{cases}\int_c^d\|\ddot\alpha(h(t))\|\,h'(t)\,dt & \text{if } h'\equiv1\\ \int_d^c\|\ddot\alpha(h(t))\|\cdot h'(t)\,dt & \text{if } h'\equiv-1\end{cases}$
$=\int_a^b\ddot\alpha(u)\,du$, $u\triangleq h(t)$.

(b) By Ex 10.6. $l(N\circ\alpha)=\int_a^b\|\dot N\circ\alpha\|dt=\int_a^b\|-k\circ\alpha\cdot\dot\alpha\|dt=\int_a^b|k\circ\alpha|\,dt$.

11.11 (a) $d(f+g)(v)=\nabla(f+g)\cdot v=\nabla f\cdot v+\nabla g\cdot v=df(v)+dg(v)$ $\forall v\triangleq(p,v)\in R_p^{n+1}$, $p\in U$
(b) $d(fg)(v)=\nabla(fg)\cdot v=$ ~~$(\nabla f\,g)$~~ $\nabla f(p)\cdot g(p)\cdot v+\nabla g(p)\cdot f(p)\cdot v$
So $dfg=g\,df+f\,dg$
(c) $d(h\circ f)(v)=\nabla(h\circ f)\cdot v=h'(f(p))\nabla f(p)\cdot v$, So $d(h\circ f)=(h'\circ f)\,df$

11.12 (a) $\int_\alpha(x_2dx_1-x_1dx_2)=\int_0^{2\pi}[2\sin t(-2\sin t)-2\cos t(2\cos t)]dt=-8\pi$
(b) $\int_C(-x_2dx_1+x_1dx_2)=\int_0^{2\pi}[(-b\sin t)(-a\sin t)+(a\cos t)(b\cos t)]dt=2\pi ab$
(c) $\int_\alpha\sum_{i=1}^{n+1}x_idx_i=f(\alpha(\pi))-f(\alpha(0))=\frac12(n+1)$, where $f(x)=\frac12\sum_{i=1}^{n+1}x_i^2$: $df(v)=\nabla f(p)\cdot v=\sum_{i=1}^{n+1}p_iv_i$
$df(X_j)=p_j$, So $df=\sum_{i=1}^{n+1}x_idx_i$

11.13 $w(\dot\alpha(t))=\sum_{i=1}^{n+1}f_i(\alpha(t))\,\dot\alpha_i(t)=\sum_{i=1}^{n+1}f_i(\alpha(t))\cdot\frac{dx_i}{dt}$ So $\int_\alpha w=\int_a^b\sum_{i=1}^{n+1}(f_i\circ\alpha)\frac{dx_i}{dt}dt$