11.14 If C is connected, then there is a one-to-one, global parametrization $\alpha(t): [a,b] \to C$.
$\int_C w_X = \int_a^b X(\alpha(t))\cdot\dot{\alpha}(t)\,dt = \int_a^b \|\dot{\alpha}(t)\|\,dt = \ell(C)$ (as X is rotating $\nabla f/\|\nabla f\|$ by $-\pi/2$)
If C is not connected, then the above is true for each segment, so globally holds too

11.15 Treat α as $\dot{\alpha}$, then $\dot{\alpha}(t) = (\cos\theta(t), \sin\theta(t))$ by proof in Thm 3 ($\theta(t) \triangleq \theta_0 + \int_{t_0}^t \eta(\beta(t))dt$
As for uniqueness: If $\theta_1(t)$ and $\theta_2(t)$ satisfy: $\cos\theta_1(t) \equiv \cos\theta_2(t)$, $\sin\theta_1(t) \equiv \sin\theta_2(t)$
$\theta_1(t_0) = \theta_2(t_0) = \theta_0$, then $\theta_1(t) = \theta_2(t)$ for all $t \in I$. Proof: By first two equations
$\sin(\theta_1(t) - \theta_2(t)) = 0$ so $\cos(\theta_1(t) - \theta_2(t))\cdot(\dot{\theta}_1(t) - \dot{\theta}_2(t)) = 0$.
But $\sin(\theta_1 - \theta_2) = 0 \Rightarrow \cos(\theta_1 - \theta_2) \neq 0$, so $\dot{\theta}_1(t) - \dot{\theta}_2(t) = 0$ So $\theta_1(t) \equiv \theta_2(t)$ as it holds for $t=0$.

11.16 Let $\beta(t) = f(t)\cdot\alpha(t)$. Define $\varphi_1(t) = \varphi_1(a) + \int_{\alpha_t}\eta$, $\varphi_2(t) = \varphi_2(a) + \int_{\beta_t}\eta$
$\varphi_1(a)$ is chosen so that $\alpha(a)/\|\alpha(a)\| = (\cos\varphi_1(a), \sin\varphi_1(a))$ and $\varphi_1(a) \in [0, 2\pi)$
$\varphi_2(a)$ - - - - - - $\beta(a)/\|\beta(a)\| = (\cos\varphi_2(a), \sin\varphi_2(a))$ and $\varphi_2(a) \in [0, 2\pi)$
As $\beta(a)/\|\beta(a)\| = \alpha(a)/\|\alpha(a)\|$ since $f>0$ and such choice of $\varphi_1(a), \varphi_2(a)$ is unique, we have
$\varphi_1(a) = \varphi_2(a)$. Furthermore, by proof in Thm 3,
$\alpha(t)/\|\alpha(t)\| = (\cos\varphi_1(t), \sin\varphi_1(t))$. $\beta(t)/\|\beta(t)\| = (\cos\varphi_2(t), \sin\varphi_2(t))$
As $\alpha(t)/\|\alpha(t)\| \equiv \beta(t)/\|\beta(t)\|$, $\cos\varphi_1(t) \equiv \cos\varphi_2(t)$, $\sin\varphi_1(t) \equiv \sin\varphi_2(t)$
and $\varphi_1(a) = \varphi_2(a)$. Same as the proof of uniqueness in Ex 11.15 we have
$\varphi_1(t) \equiv \varphi_2(t)$, $k(\alpha) = \frac{1}{2\pi}(\varphi_1(b) - \varphi_1(a)) = \frac{1}{2\pi}(\varphi_2(b) - \varphi_2(a)) = k(\beta)$. Now may need piece-
Let $f = \|\alpha\|^{-1}$ (As $\|\alpha\| \neq 0$), then $k(\alpha) = k(\alpha/\|\alpha\|)$. wise, but still true
Actually no need of α being closed and $f(a) = f(b)$. $\int_\alpha \eta \equiv \int_\beta \eta$.

11.17 Since by Ex 11.16, α and $\alpha/\|\alpha\|$ have the same winding number, it is now equivalent to proving that with $\varphi(t,u)$ redefined as $\hat{\varphi}(t,u) = \varphi(t,u)/\|\varphi(t,u)\|$, the result holds. Now $\|\hat{\varphi}_u(t)\| = 1$ for all u, and t, and $\varphi(t,u)/\|\varphi(t,u)\|$ is ~~continuous~~ continuous as $\|\varphi(t,u)\|$ is continuous
and $\hat{\varphi}_u(t)$ is smooth on each $[t_i, t_{i+1}]$, $\hat{\varphi}_u(a) = \hat{\varphi}_u(b)$.
As $[a,b]\times[0,1]$ is compact, and $\hat{\varphi}$ is continuous, $\hat{\varphi}$ must be uniform continuous, i.e.
$\forall \varepsilon_1 > 0\, \exists \delta_1 > 0\, \forall \|(t_1,u_1) - (t_2,u_2)\| < \delta_1$, $\|\hat{\varphi}(t_1,u_1) - \hat{\varphi}(t_2,u_2)\| < \varepsilon_1$. Specifically, let $t_1 = t_2$
$\|\hat{\varphi}(t,u_1) - \hat{\varphi}(t,u_2)\| < \varepsilon_1$ i.e. $\hat{\varphi}(t,u_1)\cdot\hat{\varphi}(t,u_2) \geq 1 - \frac{\varepsilon_1^2}{2} = 1 - \varepsilon_2$ ($\varepsilon_2 \triangleq \frac{\varepsilon_1^2}{2}$) $\forall |u_1 - u_2| < \varepsilon_1$ (*)
Define $\theta_u(t) = \theta_u(a) + \int_{\hat{\varphi}_u}\eta$, $\theta_x(t) = \theta_x(a) + \int_{\hat{\varphi}_x}\eta$. $\forall u \in [0,1]$, $x \in (u-\varepsilon_1, u+\varepsilon_1)\cap[0,1]$
$\theta_u(a)$ is chosen so that $\hat{\varphi}_u(a) = (\cos\theta_u(a), \sin\theta_u(a))$. Likewise, $\hat{\varphi}_x(a) = (\cos\theta_x(a), \sin\theta_x(a))$
and $\theta_u(a), \theta_x(a) \in [0, 2\pi)$ unique. For $\forall t$, by proof in Thm 3,
$\hat{\varphi}_u(t) = (\cos\theta_u(t), \sin\theta_u(t))$, $\hat{\varphi}_x(t) = (\cos\theta_x(t), \sin\theta_x(t))$