For Ex 14.15 - 14.18, there's no need to check N or $-N$.

14.15 $J\varphi = \begin{pmatrix} -a\sin\theta\sin\phi & a\cos\theta\cos\phi \\ a\cos\theta\sin\phi & a\sin\theta\cos\phi \\ 0 & -a\sin\phi \end{pmatrix} = (E_1 . E_2)$, $N = (\cos\theta\sin\phi, \sin\theta\sin\phi, \cos\phi)$

$L_p(E_1(p)) = \nabla_{(p,1,0)} N = -\frac{\partial N}{\partial \theta} = (\sin\theta\sin\phi, -\cos\theta\sin\phi, 0) = \frac{-1}{a} E_1(p)$

$L_p(E_2(p)) = -\nabla_{(p,0,1)} N = -\frac{\partial N}{\partial \phi} = (\cos\theta\cos\phi, \sin\theta\cos\phi, -\sin\phi) = \frac{1}{a} E_2$. So $k(p) = \frac{-1}{a^2}$

14.16 $J\varphi = \begin{pmatrix} 0 & -\sin\theta \\ 0 & \cos\theta \\ 1 & 0 \end{pmatrix}$ $N = (\cos\theta, \sin\theta, 0)$ $L_p(E_1(p)) = (0,0,0) = 0 \cdot E_1(p)$ So $k(p) = 0$

14.17 ~~$L_p(E_1$~~ $J\varphi = \begin{array}{c} \begin{array}{cc} E_1 & E_2 \end{array} \\ \begin{pmatrix} \cos\theta & -t\sin\theta \\ \sin\theta & t\cos\theta \\ 0 & 1 \end{pmatrix} \end{array}$ $N = (\sin\theta, -\cos\theta, t)/\sqrt{t^2+1}$

$L_p(E_1(p)) = (-\sin\theta\, t\,(t^2+1)^{\frac{-3}{2}}, \cos\theta\, t\,(t^2+1)^{\frac{-3}{2}}, (t^2+1)^{\frac{-3}{2}})$

$L_p(E_2(p)) = (\cos\theta\,(t^2+1)^{\frac{-1}{2}}, \sin\theta\,(t^2+1)^{\frac{-1}{2}}, 0)$ $\det[E_i(p) . E_j(p)] = t^2+1$.

$\det[L_p(E_i(p)) \cdot E_j(p)] = \begin{vmatrix} 0 & (t^2+1)^{-1/2} \\ (t^2+1)^{-1/2} & 0 \end{vmatrix} = -(t^2+1)^{-1}$ So $k(p) = -(t^2+1)^{-2}$

14.18 $J\varphi = \begin{pmatrix} \cosh t & 0 \\ \sinh t\cos\theta & -\cosh t\sin\theta \\ \sinh t\sin\theta & \cosh t\cos\theta \end{pmatrix}$ $N = (\sinh t, -\cosh t\cos\theta, -\cosh t\sin\theta)/\sqrt{\cosh 2t}$

Using the fact that $\nabla_{e_i} E_j = (\frac{\partial^2 \varphi_1}{\partial x_i \partial x_j}, \cdots, \frac{\partial^2 \varphi_{n+1}}{\partial x_i \partial x_j})$,

we have $\nabla_{e_1} E_1 = (\sinh t, \cosh t\cos\theta, \cosh t\sin\theta)$ $\nabla_{e_1} E_2 = \nabla_{e_2} E_1 = (0, -\sinh t\sin\theta, \sinh t\cos\theta)$

$\nabla_{e_2} E_2 = (0, -\cosh t\cos\theta, -\cosh t\sin\theta)$

$\det[E_i(p) \cdot E_j(p)] = \cosh 2t \cdot \cosh^2 t$. $\det[\nabla_{e_i} E_j \cdot N(p)] = -\cosh^2 t/\cosh 2t$

So $k(p) = -(\cosh 2t)^{-2}$

14.19 $J\varphi = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2x & 2y & 2z \end{pmatrix}$ $N = (2x, 2y, 2z, -1)/\sqrt{1+4(x^2+y^2+z^2)}$. ~~$L_p(E_i(p)) = \frac{\partial N}{\partial x}$~~ $H\varphi_1 = H\varphi_2 = H\varphi_3 = 0$

$H\varphi_4 = \operatorname{diag}(2,2,2)$ So $\nabla_{e_i} E_i = (0,0,0,2)$ for $i = 1,2,3$. $\nabla_{e_i} E_j = (0,0,0,0)$ for $i \neq j$.

$\det[E_i(p) \cdot E_j(p)] = \begin{vmatrix} 1+4x^2 & 4xy & 4xz \\ 4xy & 1+4y^2 & 4yz \\ 4xz & 4yz & 1+4z^2 \end{vmatrix} = 1+4(x^2+y^2+z^2)$.

$\det[\nabla_{e_i} E_j \cdot N(p)] = -8(1+4(x^2+y^2+z^2))^{-3/2}$, So $k(p) = -8(1+4(x^2+y^2+z^2))^{-5/2}$

14.20 (a) $J\varphi = \begin{pmatrix} x' & 0 \\ y'\cos\theta & -y\sin\theta \\ y'\sin\theta & y\cos\theta \end{pmatrix}$ $N = (y', -x'\cos\theta, -x'\sin\theta)/(y'^2+x'^2)^{1/2}$

$H\varphi_1 = \begin{pmatrix} x'' & 0 \\ 0 & 0 \end{pmatrix}$, $H\varphi_2 = \begin{pmatrix} y''\cos\theta & -y'\sin\theta \\ -y'\sin\theta & -y\cos\theta \end{pmatrix}$, $H\varphi_3 = \begin{pmatrix} y''\sin\theta & y'\cos\theta \\ y'\cos\theta & -y\sin\theta \end{pmatrix}$

$\nabla_{e_1} E_1 = (x'', y''\cos\theta, y''\sin\theta)$ $\nabla_{e_1} E_2 = \nabla_{e_2} E_1 = (0, -y'\sin\theta, y'\cos\theta)$. $\nabla_{e_2} E_2 = (0, -y\cos\theta, -y\sin\theta)$

So $\det[E_i(p) \cdot E_j(p)] = \begin{vmatrix} x'^2+y'^2 & 0 \\ 0 & y^2 \end{vmatrix} = y^2(x'^2+y'^2)$

$\det[\nabla_{e_i} E_j \cdot N(p)] = \begin{vmatrix} x''y' - x'y'' & 0 \\ 0 & x'y \end{vmatrix} / (x'^2+y'^2) = (x''y' - x'y'')x'y/(x'^2+y'^2)$

So $k(p) = x'(x''y' - x'y'')/y(x'^2+y'^2)^2$

(b) If $\|\alpha(t)\| = 1$, then $x'^2+y'^2 = 1$ $\ddot{\alpha} \cdot \dot{\alpha} = 0$, i.e. $x''x' + y''y' = 0$

So $x'x''y' = -y''y'^2 = -y''(1-x'^2)$. So $k(p) = \frac{1}{y}(-y'' + y''x'^2 - x'^2y'') = \frac{-1}{y} y''$