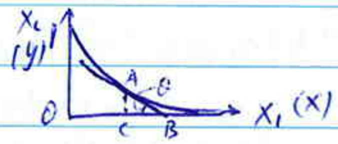
14.21 (a) $x'^2+y'^2 = 1-e^{-2t}+e^{-2t} = 1 = \|\alpha'(t)\|^2 = \|\alpha'(t)\|$



(b) $-tg\theta = y'/x' = -e^{-t}/\sqrt{1-e^{-2t}}$. So $\sin\theta = e^{-t}$

$|AB| = y/\sin\theta = e^{-t}/e^{-t} = 1$.

(c) $k = -y''/y = -e^{-t}/e^{-t} = -1$ by Ex 14.20(b) and α being unit speed.

15.1 For $(x_1, x_2, \cdots x_n, 0) \in R^n$. (Equatorial hyperplane). Solve

$\|t(x_1,\cdots,x_n,0)+(1-t)(0,\cdots 0,-1)\|=1$, i.e. $\|(tx_1,\cdots,tx_n,t-1)\|=1$

So $t^2(x_1^2+\cdots+x_n^2)+(t-1)^2=1$. If $t\neq 0$, then $t=2(\sum_{i=1}^n x_i^2+1)^{-1}$

So $\varphi(x_1,\cdots x_n,0) = (2x_1,\cdots,2x_n,1-\sum_{i=1}^n x_i^2)/(\sum_{i=1}^n x_i^2+1)$

15.2 For $(x_1\cdots x_n,-1)$. Solve $\|(1-t)(0,0,\ldots 0,1)+t(x_1\cdots x_n,-1)\|=1$, so $t=4(4+\sum_{i=1}^n x_i^2)^{-1}$

So $\varphi(x_1\cdots x_n,-1) = (4x_1,\cdots,4x_n,\sum_{i=1}^n x_i^2-4)/(\sum_{i=1}^n x_i^2+4)$

15.3 (a) If $v(t)\in f^{-1}(c)$. let $(\alpha(t),s(t)) = \psi|_V^{-1}(v(t))$. so

$f(v(t)) = f(\psi(\alpha(t),s(t))) = s(t) = c$. So $v(t) = \psi(\alpha(t),c) = \varphi\circ\alpha + c\cdot N\circ\alpha$.

If $\beta_q(s)=v(t)$, i.e. $\varphi(q)+sN(q) = \varphi(\alpha(t))+cN(\alpha(t))$. then since there is a smooth inverse of $\psi|_V$, so $q=\alpha(t)$. $s=c$. then

$v'(t)\cdot\beta_q'(s) = N(q)\cdot((\dot\varphi\circ\alpha)(t)+c\cdot(\dot N\circ\alpha)(t)) = N(\alpha(t))((\dot\varphi\circ\alpha)(t)+c\cdot(\dot N\circ\alpha)(t))$

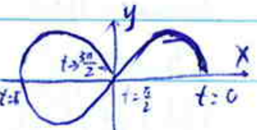
As $\|N(\alpha(t))\|\equiv 1$ so $N(\alpha(t))\cdot\dot N\circ\alpha(t)=0$. By definition, $(N\circ\alpha)(t)\cdot(\dot\varphi\circ\alpha)(t)=0$

So $v'(t)\cdot\beta_q'(s)=0$. i.e. $f^{-1}(c)$ are everywhere orthogonal to the lines $\beta_q(s)$.

(b) By (a) the vector part of $\nabla f(\psi(q,s)) = \lambda\cdot\beta_q'(s) = \lambda N(q)$.

But $\frac{\partial f}{\partial s}=1$, i.e. $\nabla f\cdot\frac{\partial\psi}{\partial s} = \nabla f\cdot N(q)=1$ So $\lambda=1$ $\nabla f(g)=(g,N(q))$, $g=\psi(q,s)$

15.4 $(x(t),y(t)) = (2\cos t, \sin 2t)$ $t\in(0,\frac{3\pi}{2})$



$(x'(t),y'(t)) = (-2\sin t, 2\cos 2t)\neq(0,0)$ obviously one to one

But when $t\to\frac{3\pi}{2}$, the curve approaches its own point $(0,0)$ crossed at $t=\frac{\pi}{2}$, so not n-surface.

15.5 $\forall(\overset{R^{n+1}}{p},\overset{R^{n+1}}{v})\in T(S)$, $f(p)=c$, $v\cdot N(p)=0$. $J=\begin{pmatrix}\nabla f^T & 0\\ sth. & N(p)\end{pmatrix} = \nabla f\cdot N(p)\neq 0$.

So $T(S)$ is $2n$-surface in R^{2n+2}

15.6 $\forall(p,v)\in T(S)$. $f(p)=c$, $v\cdot N(p)=0$ $v\cdot v=0$ $J=\begin{pmatrix}\nabla f^T & 0\\ \beta & N(p)\\ 0 & 2v\end{pmatrix}$. If $\alpha_1\begin{pmatrix}\nabla f\\ 0\end{pmatrix}+\alpha_2\begin{pmatrix}\beta\\ N(p)\end{pmatrix}+\alpha_3\begin{pmatrix}0\\ 2v\end{pmatrix}=0$

then $\left.\begin{aligned}\alpha_1\nabla f+\alpha_2\beta=0\\ \alpha_2 N(p)+\alpha_3\cdot 2v=0\end{aligned}\right\} \Rightarrow \alpha_2=-2v\cdot N(p)=0 \Rightarrow \alpha_3=0 \Rightarrow \alpha_1=0$ So independent, thus $T_1(S)$ is $(2n-1)$-surface in R^{2n+2}