$= \frac{1}{4}\left(\|d\varphi(v+w)\|^2 - \|d\varphi(v-w)\|^2\right) = \frac{1}{4}\lambda^2(p)\left(\|v+w\|^2 - \|v-w\|^2\right) = \lambda^2(p)\cdot v\cdot w.$

15.13 Let $\tilde{S} = \{q \in S \mid q$ can be joined to p by a continuous curve in $S\}$. Let $S = f^{-1}(c)$.
First $\tilde{S}$ is obviously connected. $\forall q_1, q_2 \in \tilde{S}$, just concatenate their curve joining p will yield a continuous curve between q_1 and q_2. Since $\tilde{S} \subseteq S$, so $\forall q \in \tilde{S}$, $\nabla f(q) \neq 0$. Now we only need to prove that ~~there~~ there is an open set ~~s.t.~~ U, s.t. ~~$\tilde{S} = \{x \in U \mid f(x) = c\}$~~. $\tilde{S} = \{x \in U \mid f(x) = c\}$. We mimic the proof of Thm 3.
For each $q \in \tilde{S} \subseteq S$, let $\varphi_q : U_q \to S$ be a local parametrization of S whose image contains q and let $\psi_q : U_q \times R \to R^{n+1}$ be defined by $\psi_q(r,s) = \varphi_q(r) + s N(\varphi_q(r))$, where N is the orientation of S. Then as in the proof of Thm2, we can find an open set V_q about $(\varphi_q^{-1}(q), 0)$ in $U_q \times R$ s.t. $\psi_q|_{V_q}$ maps V_q one to one onto and open set W_q in R^{n+1}, and $(\psi_q|_{V_q})^{-1} : W_q \to V_q$ is smooth. Furthermore by shrinking V_q if necessary, we may assume that $\psi_q(r,s) \in S$ for $(r,s) \in V_q$ iff $s = 0$. $\boxed{\text{By shrinking } V_q \text{ further, we may assume that } V_q = U_q' \times I \text{ where } U_q' \subseteq U_q,\ I \subseteq R,\ 0 \in I,\ \varphi_q^{-1}(q) \in U_q',\ U_q' \text{ open},\ I \text{ open},\ U_q' \text{ connected}}$ ~~Since V_q is an open set,~~ then for any $u \in W_q \cap S$, let $(\psi_q|_{V_q})^{-1}(u) = (r,0)$ must be ~~there must be a unique~~ [illegible] $r \in U_q'$ ~~s.t. $\psi_q(r,0) = u$~~. Since U_q' is open and connected ~~connected~~, there is a smooth curve $\alpha(t) : [a,b] \to U_q'$ s.t. ~~$\alpha(t) \in V_q$~~. $\alpha(a) = \varphi_q^{-1}(q)$, $\alpha(b) =$ ~~$(r,0)$~~ r, (actually we should define $\alpha(t)$ in an open set containing $[a,b]$). Since ~~$\alpha(t) \in V_q$~~ As $\alpha(t) \in U_q'$ and $0 \in I$, we have $\beta(t) \triangleq \psi_q(\alpha(t); 0) \in W_q \cap S$. So $\beta(b) = u$ is connected to $\beta(a) = q$ through a continuous curve on S, so $u \in \tilde{S}$. In other words, for $\forall q \in \tilde{S}$, there is an open set W_q about q, s.t. $W_q \cap S \subseteq \tilde{S}$.
Now we define $U = \bigcup_{q \in \tilde{S}} W_q$ which is open, ~~then $\tilde{S} \subseteq U$ by definition~~.
① $\forall x \in \tilde{S}$, we have $x \in U$, $f(x) = c$. So $x \in \{x \in U \mid f(x) = c\}$, so $\tilde{S} \subseteq \{x \in U \mid f(x) = c\}$
② $\forall x \in \{x \in U \mid f(x) = c\}$, there must be a $q \in \tilde{S}$, s.t. $x \in W_q$. As $x \in S$, so $x \in W_q \cap S \subseteq \tilde{S}$. Thus, $\{x \in U \mid f(x) = c\} \subseteq \tilde{S}$.
Hence $\tilde{S} = \{x \in U \mid f(x) = c\}$, i.e. $\tilde{S}$ is n-surface.

15.14 Suppose $\alpha(t_1) = \alpha(t_2)$ for some $t_1 \neq t_2 \in I$. As the maximal integral curve of X through $\alpha(t_1)$ is unique, ~~then~~ denoted as $\beta(t)$ and $\beta(0) = \alpha(t_1)$, then $\alpha(t) = \beta(t - t_1)$ and $\alpha(t) = \beta(t - t_2)$ for all $t \in I$. Setting $\tau = t_2 - t_1$, we have $\alpha(t) = \beta(t - t_1) = \beta(t + \tau - t_2) = \alpha(t + \tau)$ for all t such that both t and $t + \tau \in I$.
Thus if α is not one to one then it is periodic.
To prove that the maximal integral curve X through $\alpha(t_1)$ is ~~unique~~ constrained in C, we notice