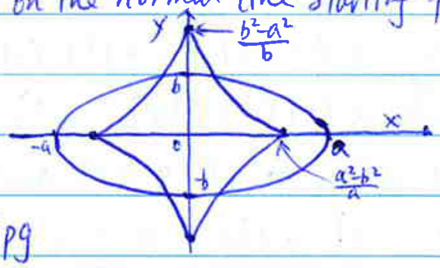
that first the restriction of X to C is a tangent vector field on C, because $X\cdot\nabla f_i = (\nabla f_1\times\nabla f_2)\cdot\nabla f_i = 0$ for $i=1,2$.
So $(f_i\circ\alpha)'(t) = \nabla f_i(\alpha(t))\cdot\dot\alpha(t) = \nabla f_i(\alpha(t))\cdot X(\alpha(t)) = 0$. So $f_i\circ\alpha$ is constant, thus $\alpha(t)\in C$.

~~To make~~ α map I onto C, ~~first of all~~ iff C ~~must be~~ is connected. The proof is similar to the Thm 1 in Chapter 11, except the construction of rectangle B, we now construct $\Delta = \{p_0 + s_1v_1+s_2v_2+ru \mid |s_i|<\varepsilon_i, |r|<\varepsilon_3\}$ where $v_i=\nabla f_i(p_0)$, $u = X(p_0)$. The $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are chosen ~~as follows. First~~, so that $J\vec f(p)$ is fully ranked for all $p\in A$. This is possible because $J\vec f(p_0)$ is fully ranked. Denote $g_r(s_1,s_2) = \vec f(p_0+s_1v_1+s_2v_2+ru)$, then $Jg_r = \begin{pmatrix}\nabla f_1\\ \nabla f_2\end{pmatrix}(v_1,v_2) = \begin{pmatrix}\nabla f_1\\ \nabla f_2\end{pmatrix}(\nabla f_1,\nabla f_2)$. As $\mathrm{rank}(P'P) = \mathrm{rank}(P) = \mathrm{rank}(P')$, $\mathrm{rank}\begin{pmatrix}\nabla f_1\\ \nabla f_2\end{pmatrix}_p = 2$ for $\forall p\in\Delta$. So Jg_r is fully ranked for all $p\in\Delta$. Applying ~~Inverse function Thm~~ Lagrange Mean Value Thm, if r is chosen such that $\exists s_1,s_2$ s.t. $\vec f(p_0+s_1v_1+s_2v_2+ru) = \binom{c_1}{c_2}$, then such s_1,s_2 are unique in $(-\varepsilon_1,\varepsilon_1)\times(-\varepsilon_2,\varepsilon_2)$.
Now let $\gamma(t) = p_0 + h_0(t)u + h_1(t)v_1 + h_2(t)v_2$, where $h_0(t) = (\gamma(t)-p_0)\cdot u/\|u\|^2$. $h_i(t) = (\gamma(t)-p_0)\cdot v_i/\|v_i\|^2$ $(i=1,2)$. h_1, h_2, h_0 are all smooth and $h_1(0)=h_2(0)=h_0(0)=0$. $h_0'(0) = \dot\gamma(0)\cdot u/\|u\|^2 = 1$ by definition that $\dot\gamma(0) = X(p_0)=u$. So we can choose $t_1<0<t_2$ (small enough) s.t. $h_0'(t)>0$, set $r_1=h_0(t_1)$, $r_2=h_0(t_2)$, then for $\forall r\in(r_1,r_2)$, $\exists!\, t\in(t_1,t_2)$, s.t. $h_0(t)=r$. Now construct $B = \{p_0+ru+s_1v_1+s_2v_2 \mid r_1<r<r_2, |s_i|<\varepsilon_i\ (i=1,2)\}$.
$\forall p_0+ru+s_1v_1+s_2v_2 \in B\cap C$, then it's belonging to $B \Rightarrow \exists!\, t\in(t_1,t_2)$, $h_0(t)=r$.
Its belonging to $C \Rightarrow \exists!\, s_1,s_2$ s.t. $p_0+ru+s_1v_1+s_2v_2\in C$. Now that we know $p_0+h_0(t)u+h_1(t)v_1+h_2(t)v_2 \in \gamma\subseteq C$, so $s_1=h_1(t)$, $s_2=h_2(t)$. So $p_0+ru+s_1v_1+s_2v_2\in\gamma$.
i.e. $C\cap B\subseteq\gamma$

16.1 (a) By using the rest of Ex 10.4 (b) at $p=(a\cos t, b\sin t)$, the curvature for outward orientation is $k(p) = -ab\left(\frac{a^2}{b^2}x_2^2+\frac{b^2}{a^2}x_1^2\right)^{-\frac{3}{2}}$, $N = (a^2\sin^2 t+b^2\cos^2 t)^{-\frac12}(b\cos t, a\sin t)$.
By applying Thm 1, the focal point on the normal line starting from p is $p+\frac{1}{k(p)}N(p)$
$= \left(\frac{a^2-b^2}{a}\cos^3 t, \frac{b^2-a^2}{b}\sin^3 t\right)$

(b) For example of $a=2$, $b=1$, see http://rsise.anu.edu.au/~xzhang/reading/ex1601.jpg



16.2 (a) Only need to prove that for q sufficiently close to p, $N(p)$ and $N(q)$ are not parallel in R^2. Otherwise, for $\forall k\in Z^+$, $\exists q_k\in C\cap\varepsilon(p,\varepsilon)$ (the ε-ball about p), such that $N(p)$ and $N(q_k)$ are parallel. But as N is smooth, $\boxed{\text{So } \|N(p)-N(q_k)\| = 0 \text{ or } 2}$, in the neighbourhood close enough to p, $\|N(p)-N(q_k)\|$ must be less than an arbitrary small positive number. So $N(q_k)=N(p)$, so $\cancel{(N(q_k)-N(p))}\; JN(p+\lambda_k(q_k-p))\cdot\frac{(q_k-p)}{\|q_k-p\|} = 0$ $(\lambda_k\in(0,1))$. As $(q_k-p)/\|q_k-p\|\in S^1$ which is compact,