there must be a subsequence of $(q_k - p)/\|q_k - p\|$ which converges to v ($\|v\|=1$). Without loss of generality, we assume that subsequence is $\{q_k\}$ itself. Let $k \to \infty$, We have $\nabla_v N = 0$, because $\lim_{k\to\infty} \frac{q_k - p}{\|q_k - p\|} = v$. $\lim_{k\to\infty} p + \lambda_k (q_k - p) = p$ as $q_k \to p$. $\nabla_v N = 0$ contradicts with the assumption that the curvature $k(p) \neq 0$. So for $q \in C$ sufficiently close to p, the normal lines to C at p and q intersects at some point $h(q) \in \mathbb{R}^2$

(b) First derive $h(q)$. $p + S_1 \cdot N(p) = q + S_2 \cdot N(q)$. Suppose there is a local parametrization of C about p: $\alpha(t): I \to C$, $\alpha(t_0) = p$, and suppose I is small enough s.t. $\forall t \in I$, $\alpha(t)$ satisfies (a). So to derive $h(\alpha(t))$, suppose $\alpha(t) + S_2 N(\alpha(t)) = \alpha(t_0) + S_1 N(\alpha(t_0))$ ($S_1, S_2 \in \mathbb{R}$). Multiply both sides by $\dot\alpha(t_0)$ and Notice $N(\alpha(t_0)) \cdot \dot\alpha(t_0) = 0$, so

$\alpha(t) \cdot \dot\alpha(t_0) = \cancel{\alpha(t_0)\dot\alpha(t)} + S_2 N(\alpha(t)) \cdot \dot\alpha(t_0) = \alpha(t_0)\dot\alpha(t_0)$. By assumption $N(\alpha(t))$ is not parallel with $\dot\alpha(t_0)$, so $N(\alpha(t)) \cdot \dot\alpha(t_0) \neq 0$ So $S_2 = \frac{\alpha(t_0)\cdot\dot\alpha(t_0) - \alpha(t)\dot\alpha(t_0)}{N(\alpha(t)) \cdot \dot\alpha(t_0)}$

So $h(\alpha(t)) = \frac{1}{N(\alpha(t)) \cdot \dot\alpha(t_0)} \left[\alpha(t) \cdot \left(N(\alpha(t)) \cdot \dot\alpha(t_0)\right) + N(\alpha(t)) \cdot \left(\alpha(t_0)\dot\alpha(t_0) - \dot\alpha(t_0)\alpha(t)\right) \right]$

Both numerator and denominator $\to 0$ as $t \to t_0$. So using L'Hospital's rule, the derivative of denominator is $\dot{N\circ\alpha}(t) \cdot \dot\alpha(t_0)$ which equals $-k(t_0)\|\dot\alpha(t_0)\|^2$

the derivative of numerator is $\dot\alpha(t) \cdot \left(N(\alpha(t)) \cdot \dot\alpha(t_0)\right) + \alpha(t)\left[\dot{N\circ\alpha}(t) \cdot \dot\alpha(t_0)\right]$

$+ \dot{N\circ\alpha}(t) \cdot \left(\alpha(t_0) \cdot \dot\alpha(t_0)\right) - \dot{N\circ\alpha}(t_0) \cdot \left[\dot\alpha(t_0) \cdot \alpha(t)\right] - N(\alpha(t)) \cdot \left[\dot\alpha(t_0) \cdot \dot\alpha(t)\right]$

when $t = t_0$, it is equal to $\alpha(t_0) \cdot \left(-k(t_0)\|\dot\alpha(t_0)\|^2\right) - k(t_0)\dot\alpha(t_0)\left(\alpha(t_0) \cdot \dot\alpha(t_0)\right)$

$+ k(t_0)\dot\alpha(t_0)\left[\dot\alpha(t_0) \cdot \alpha(t_0)\right] - N(\alpha(t_0)) \cdot \|\dot\alpha(t_0)\|^2$

So $\lim_{t\to t_0} h(\alpha(t)) = -\|\dot\alpha(t_0)\|^2 \left(N\circ\alpha(t_0) + k(t_0) \cdot \alpha(t_0)\right) / \left[-k(t_0)\|\dot\alpha(t_0)\|^2\right]$

$= \alpha(t_0) + \frac{1}{k(t_0)}(N\circ\alpha)(t_0)$ $\quad (\alpha(t_0) = p)$

By Thm 1, this is the focal point of C along the normal line through p.

16.3 (a) $\dot\alpha(t) = \varphi'(t) + \frac{1}{k^2(t)}\left[(\dot{N\circ\varphi})(t)\,k(t) - k'(t)\,(N\circ\varphi)(t)\right]$

As $(\dot{N\circ\varphi})(t) = -L_p(\dot\varphi(t)) = -k(t)\dot\varphi(t)$.

So $\dot\alpha(t) = \frac{-1}{k^2(t)}k'(t)(N\circ\varphi)(t)$, So $\dot\alpha(t) = 0$ iff $k'(t) = 0$

(b) As $\dot\alpha(t)$ is parallel to $(N\circ\varphi)(t)$ and by definition $\alpha(t)$ is on the normal line to Image φ at $\varphi(t)$ and by Thm 1, $\alpha(t)$ is the focal locus of φ. So the latter is tangent ~~at~~ $\alpha(t)$ to the focal locus of φ

(c) The sum is $Q = \int_{t}^{t_2} \|\dot\alpha(t_1)\|\,dt_1 + \|\alpha(t) - \varphi(t)\|$. Suppose $k'(t) = b\|k'(t)\|$ and $k(t) = a\|k(t)\|$ where $a, b \in \{\pm1\}$ as both $k(t)$ and $k'(t)$ keep their sign for $t \in (t_1, t_2)$. So $\frac{d}{dt}Q = -\|\dot\alpha(t)\| + \frac{d}{dt}\left|\frac{1}{k(t)}\right| = -\|\dot\alpha(t)\| + a\frac{d}{dt}\frac{1}{k(t)}$

$= \frac{-1}{k^2(t)}ab\,k'(t) + a\frac{-1}{k^2(t)}k'(t)$. Notice that if $a \cdot b = 1$ then the conclusion in this exercise doesn't hold. Otherwise if $k'(t) \cdot k(t) < 0$. $\frac{d}{dt}Q = 0$ so $Q =$ constant