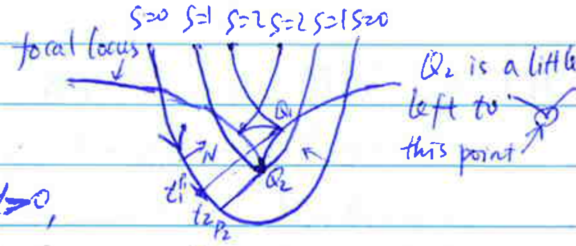

An example of $ab=1$ is the ~~fo~~ parabola. If we paramefrize by $\varphi(t)=(t,\frac{1}{2}t^2)$ $t\in(-\infty, 0)$
then $\dot\varphi(t)=(1,t)$, $N=\frac{1}{\sqrt{t^2+1}}(-t,+1)$, $k(t)=((1+t^2)^{3/2}>0$, $k'>0$,
then $|P_1Q_1|+|\text{length of } \alpha\ \alpha(t_1)\to\alpha(t_2)| > |P_2Q_2|+0$. can't be constant.
So we will need $kk'\leq 0$, like what the Figure 16.6 shows

16.4 (a) Let $\psi(s,t)=\varphi(t)+sN(\varphi(t))$. For each $s<\frac{1}{k(t_0)}$, $\varphi_s(t_0)$ is not a focal point by Thm 1, so $I_s\neq\phi$. If $t_0\in I_s$, then $\varphi_s'(t_0)\neq 0$. As φ_s' is continuous, there must be $\varepsilon>0$ s.t. $\forall t\in(t_0-\varepsilon,t_0+\varepsilon)$, $\varphi_s'(t)\neq 0$, i.e. $t\in I_s$. Thus I_s is open.

(b) Suppose $\varphi(t)$ is unit speed, which doesn't lose generality as the conclusion only takes care of t_0. $\varphi_s(t)=\varphi(t)+sN(\varphi(t))$, $k(t_0)=\ddot\varphi(t_0)\cdot N(\varphi(t_0))$
$\varphi_s'(t)=\varphi'(t)+s(\dot{N}\circ\varphi)(t)$ As $\varphi_s'(t)\cdot(N\circ\varphi)(t)=(\varphi'(t)+s(\dot N\circ\varphi)(t))(N\circ\varphi)(t)$
by definition $\varphi'(t)\cdot(N\circ\varphi)(t)=0$. $\|(N\circ\varphi)(t)\|=\text{const} \Rightarrow (\dot N\circ\varphi)(N\circ\varphi)=0$, so $\varphi_s'(t)\cdot(N\circ\varphi)(t)=0$
~~So $N_s(\varphi_s(t))=N(\varphi(t))$~~. To check the direction, we notice that
$\varphi_s'(t)\cdot\varphi'(t)=(\varphi'(t)+s\cdot(\dot N\circ\varphi)(t))\cdot\varphi'(t)=\|\varphi'(t)\|^2+s(-k\|\varphi'(t)\|^2)$
As $s<\frac{1}{k(t_0)}$. So if $k(t_0)>0$, then $\varphi_s'(t)$ is in the same direction as $\varphi'(t)$, and $N_s(\varphi_s(t))=N(\varphi(t))$. If $k(t_0)<0$, then $N_s(\varphi_s(t))=-N(\varphi(t))$.

1°. $k(t_0)>0$, $k_s(t_0)=\ddot\varphi_s(t_0)N_s(\varphi_s(t_0))/\|\varphi_s'(t_0)\|^2$
$=(\ddot\varphi(t_0)+s(\ddot N\circ\varphi)(t_0))N(\varphi(t_0))/\|\varphi_s'(t_0)\|^2$
$\ddot\varphi(t_0)\cdot N(\varphi(t_0))=k(t_0)$. Besides, as $-(\dot N\circ\varphi)=k\cdot\dot\varphi$,
so $-(\ddot N\circ\varphi)=k'\varphi'+k\cdot\ddot\varphi$, so $-(\ddot N\circ\varphi)(N\circ\varphi)=k\cdot\ddot\varphi(N\circ\varphi)=+k^2$
So $k_s(t_0)=(k(t_0)-sk^2(t_0))/\|\varphi_s'(t_0)\|^2$
$\varphi_s'(t_0)=\varphi'(t_0)+s(\dot N\circ\varphi)(t_0)=\varphi'(t_0)+s\cdot(-k(t_0)\varphi'(t_0))$. So $\|\varphi_s'(t_0)\|=|1-sk(t_0)|$
So $k_s(t_0)=(k(t_0)-sk^2(t_0))/(1-sk(t_0))^2=(\frac{1}{k(t_0)}-s)^{-1}$

2° $k(t_0)<0$. $k_s(t_0)=-\ddot\varphi_s(t_0)N(\varphi(t_0))/\|\varphi_s'(t_0)\|$ similar to above, we have
~~$\ddot\varphi_s(t_0)=$~~ $k_s(t_0)=-(\frac{1}{k(t_0)}-s)^{-1}$ So we suspect that it should be $|s|<|\frac{1}{k(t_0)}|$
or simply assume $k(t_0)>0$. To double check we are correct, see parabola again and $\varphi(t_0)=(0,0)$, $y=\frac{1}{2}x^2$. $k(0)=-1$, $\varphi(t)=\binom{t}{\frac{1}{2}t^2}$
Let $s=-2<1/k(0)$. at P, the curvature should still be negative, while the conclusion in the textbook exercise insists $k_{-2}(0)=\frac{1}{-1-(-2)}=1>0$