16.5 (a) $J\psi|_{t=0} = (N^S(\alpha(0)), \dot\alpha(0) + s(\dot{N}\circ\alpha)(0))$

so $X(S) = J\psi|_{t=0}\cdot\binom{0}{1} = \dot\alpha(0) + s(\dot{N}\circ\alpha)(0)$

$X(0) = \dot\alpha(0) = v$. $\dot X(s) = (\dot N\circ\alpha)(0)$ So $\dot X(0) = (\dot N\circ\alpha)(0) = -L_p(v)$

(b) $\ddot X(s) = 0$. So $X(s) = (\psi(s,0), \dot\alpha(0) + s(\dot N\circ\alpha)(0)) = (\beta(s), v + sw)$

(c) $X(s) = 0 \Leftrightarrow v = -s\,(\dot N\circ\alpha)(0) = sL_p(v)$

So $\frac{1}{s}$ is a principal curvature and v is a principal curvature direction

By Thm 1, $\Leftrightarrow \alpha(0) + \frac{1}{\frac{1}{s}}\cdot N^S(\alpha(t)) = \alpha(0) + sN^S(\alpha(t)) = \beta(s)$ is focal point of S along β.

17.1 $V(\varphi) = \int_0^{2\pi}\int_0^h \left|\det\begin{vmatrix} -r\sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & \sin\theta & 0 \end{vmatrix}\right| dt\,d\theta = \int_0^{2\pi}\int_0^h r\,dt\,d\theta = 2\pi h$

17.2 $E_1 = \frac{\partial\varphi}{\partial\theta} = (-tr\sin\theta, tr\cos\theta, 0)$, $E_2 = \frac{\partial\varphi}{\partial t} = (r\cos\theta, r\sin\theta, -h)$

$E_1\cdot E_1 = t^2r^2$. $E_2\cdot E_2 = r^2+h^2$ $E_1\cdot E_2 = 0$ $V(\varphi) = \int_0^{2\pi}\int_0^1\sqrt{t^2r^2(r^2+h^2)}\,dt\,d\theta = \pi r\sqrt{r^2+h^2}$

17.3 $E_1 = \frac{\partial\varphi}{\partial\theta} = (-(a+b\cos\phi)\sin\theta, (a+b\cos\phi)\cos\theta, 0)$, $E_2 = \frac{\partial\varphi}{\partial\phi} = (-b\sin\phi\cos\theta, -b\sin\phi\sin\theta, +b\cos\phi)$

$E_1\cdot E_1 = (a+b\cos\phi)^2$, $E_2\cdot E_2 = b^2$, $E_1\cdot E_2 = 0$ $V(\varphi) = \int_0^{2\pi}\int_0^{2\pi} b(a+b\cos\phi)\,d\phi\,d\theta = 4\pi^2 ab$

17.4 $E_1 = \frac{\partial\varphi}{\partial\theta} = (-a\sin\theta, a\cos\theta, 0, 0)$, $E_2 = \frac{\partial\varphi}{\partial\phi} = (0, 0, -b\sin\phi, b\cos\phi)$, $E_1\cdot E_1 = a^2$, $E_2\cdot E_2 = b^2$, $E_1\cdot E_2 = 0$

$V(\varphi) = \int_0^{2\pi}\int_0^{2\pi} ab\,d\theta\,d\phi = 4\pi^2 ab$

17.5 $E_1 = \frac{\partial\varphi}{\partial\phi} = (\cos\phi\sin\theta\sin\psi, -\sin\phi\sin\theta\sin\psi, 0, 0)$ $E_2 = \frac{\partial\varphi}{\partial\theta} = (\sin\phi\cos\theta\sin\psi, \cos\phi\cos\theta\sin\psi, -\sin\theta\sin\psi, 0)$

$E_3 = \frac{\partial\varphi}{\partial\psi} = (\sin\phi\sin\theta\cos\psi, \cos\phi\sin\theta\cos\psi, \cos\theta\cos\psi, -\sin\psi)$.

$E_1\cdot E_1 = (\sin\theta\sin\psi)^2$, $E_2\cdot E_2 = (\sin\psi)^2$, $E_3\cdot E_3 = 1$. $E_i\cdot E_j = 0$ $(i\neq j)$.

So $V(\varphi) = \int_0^{2\pi}\int_0^{\pi}\int_0^{\pi}\sin\theta(\sin\psi)^2\,d\phi\,d\theta\,d\psi = 2\pi^2$

17.6 $E_1 = (x'(t), y'(t)\cos\theta, y'(t)\sin\theta)$, $E_2 = (0, -y(t)\sin\theta, y(t)\cos\theta)$

$E_1\cdot E_1 = x'(t)^2 + y'(t)^2$, $E_2\cdot E_2 = y(t)^2$, $E_1\cdot E_2 = 0$.

$V(\varphi) = \int_a^b\int_0^{2\pi}\sqrt{x'(t)^2+y'(t)^2}\,y(t)\,d\theta\,dt = 2\pi\int_a^b y(t)(x'(t)^2+y'(t)^2)^{1/2}\,dt$

17.7 Let $a_i = \frac{\partial g}{\partial u_i}$ then $E_i = \frac{\partial\varphi}{\partial u_i} = (0,\cdots 0, 1, 0, \cdots, 0, a_i)$ $N = \pm(a_1, \cdots, a_n, -1)/(1+\sum_{i=1}^n a_i^2)^{\frac{1}{2}}$

As $\begin{vmatrix} 1 & & & a_1 \\ & \ddots & & \vdots \\ & & 1 & a_n \\ a_1 & \cdots & a_n & -1 \end{vmatrix} = \cancel{\begin{vmatrix} 1+a_1^2 & & 0 \\ & \ddots & \\ & 1+a_n^2 & 0 \\ a_1 & \cdots\ a_n & 1 \end{vmatrix} = \prod_{i=1}^n (1+a_i^2)}$, So $= \begin{vmatrix} 1 & & & 0 \\ & \ddots & & \\ & & 1 & a_n \\ a_1 & & a_n & -1-a_1^2 \end{vmatrix} = \begin{vmatrix} 1 & & & a_2 \\ & \ddots & & \vdots \\ & & 1 & a_n \\ a_2 & \cdots & a_n & -1-a_1^2 \end{vmatrix} = \cdots = -(1+\sum_{i=1}^n a_i^2)$