As N^φ is orientation vector field along φ. So $|B|>0$.

As $\psi = \varphi\circ h$, $A = (J^\varphi\circ h\cdot J_h\cdot e_1, \cdots, J^\varphi\circ h\cdot J_h\cdot e_n, N^\varphi\circ h) = (J^\varphi\circ h\cdot J_h, N^\varphi\circ h)$ if we assume $N^\psi = N^\varphi\circ h$.

Then $B = (J^\varphi\circ h, N^\varphi\circ h)$, $A^TB = \begin{pmatrix} J_h^T (J^\varphi\circ h)^T (J^\varphi\circ h) & 0 \\ 0 & 1 \end{pmatrix}$ $B^TB = \begin{pmatrix} (J^\varphi\circ h)^T (J^\varphi\circ h) & 0 \\ 0 & 1 \end{pmatrix}$

The zero's are because: $(N^\varphi\circ h)^T\cdot(J^\varphi\circ h)e_i = 0$ by definition of N^φ and $(N^\varphi\circ h)^T (J^\varphi\circ h)\cdot J_h e_i = 0$ by the fact that $\{e_1\cdots e_n\}$ forms a basis of R^n so $J_h e_i$ can be written as a linear combination of $\{e_1\cdots e_n\}$.

So $|A^TB| = |J_h|\cdot|(J^\varphi\circ h)^T(J^\varphi\circ h)|$. $|B^TB| = |(J^\varphi\circ h)^T(J^\varphi\circ h)|$

So $|A|\cdot|B| = |J_h|\cdot|B|^2$. As $|J_h|>0$. $|B|>0$ so $|A| = |J_h||B|>0$

So $N^\psi = N^\varphi\circ h$ satisfies all the conditions to be orientation vector field.

17.12 (a) First prove $w(v_1\cdots v_i\cdots v_j\cdots v_k) = w(v_1\cdots, v_i, \cdots, v_j+\alpha v_i, \cdots v_k)$ where $\alpha\in R$.

This is because the latter $= w(v_1\cdots v_i\cdots v_j\cdots v_k) + \alpha\, w(v_1, \cdots v_i, \cdots, v_i, \cdots v_k)$ and $w(v_1\cdots v_i, \cdots v_i\cdots v_k) = 0$ by skewsymmetry.

If $\{v_1\cdots v_k\}$ is linearly dependent, then exist $\alpha_1\cdots\alpha_n\in R$, s.t. $\sum_{i=1}^{k}\alpha_i v_i = 0$ and $\sum_{i=1}^{n}\alpha_i^2 \neq 0$. So ~~$w(v_1\cdots v_k) = w(v_1$~~ assume $\alpha_i\neq 0$, then

$w(v_1\cdots v_i\cdots v_k) = \frac{1}{\alpha_i} w(v_1\cdots \alpha_i v_i, \cdots v_k) = \frac{1}{\alpha_i} w(v_1, \cdots, \alpha_i v_i + \alpha_1 v_1, \cdots v_k)$

$= \cdots = \frac{1}{\alpha_i} w(v_1, \cdots, \sum_{i=1}^{k}\alpha_i v_i, v_k) = 0$.

(b) If $k>n$, then $\{v_1\cdots v_k\}$ must be linearly dependent, so $w=0$.

17.13 (a) $\xi(v_1\cdots v_n)^2 = \left|\begin{pmatrix} v_1^T \\ \vdots \\ v_n^T \\ N \end{pmatrix}(v_1\cdots v_n N)\right| = \begin{vmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{vmatrix} = 1$ So $\xi(v_1\cdots v_n) = \pm 1$

and $\xi(v_1\cdots v_n) = 1$ iff $\{v_1\cdots v_n\}$ is consistent with N.

(b) We only need to prove $w(u_1, \cdots, u_n) = w(v_1\cdots v_n)\cdot\xi(u_1\cdots u_n)$ for any $\{u_1\cdots u_n\}\in S_p$, and $v_1\cdots v_n$ is arbitrary orthonormal basis for S_p consistent with the orientation N on S. As $\{v_1\cdots v_n\}$ forms a basis of S_p, so there exist $\alpha_{ij}\in R$ s.t. $u_i = \sum_{j=1}^{n}\alpha_{ij} v_j$

So $w(u_1\cdots u_n) = \sum_{i_1\cdots i_n}\alpha_{1i_1}\cdots\alpha_{ni_n} w(v_{i_1}, \cdots, v_{i_n})$. If $i_p = i_q$ $(p\neq q)$ then $w(v_{i_1}\cdots v_{i_n}) = 0$

So $w(u_1\cdots u_n) = \sum_\sigma \alpha_{1\sigma(1)}\cdots\alpha_{n\sigma(n)} w(v_{\sigma(1)}, \cdots, v_{\sigma(n)})$ ~~(sign σ)~~. (1)

Likewise $\xi(u_1\cdots u_n) = \sum_\sigma \alpha_{1\sigma(1)}\cdots\alpha_{n\sigma(n)}\xi(v_{\sigma(1)}, \cdots, v_{\sigma(n)})$ ~~(sign σ)~~ (2)

Notice $w(v_{\sigma(1)}\cdots v_{\sigma(n)}) = (\text{sign}\,\sigma)\, w(v_1\cdots v_n)^{(3)}$, $\xi(v_{\sigma(1)}\cdots v_{\sigma(n)}) = (\text{sign}\,\sigma)\,\xi(v_1\cdots v_n) \overset{\text{by question (a)}}{=} \text{sign}\,\sigma^{(4)}$

So continuing (1) and plugging (3)(4) into (1) we have $w(u_1\cdots u_n) = \sum_\sigma \alpha_{1\sigma(1)}\cdots\alpha_{n\sigma(n)}\left(\xi(v_{\sigma(1)}, \cdots, v_{\sigma(n)})\, w(v_1\cdots v_n)\right) = w(v_1\cdots v_n)\cdot\sum_\sigma\alpha_{1\sigma(1)}\cdots\alpha_{n\sigma(n)}\xi(v_{\sigma(1)}\cdots v_{\sigma(n)})$

$\overset{\text{by (2)}}{=} w(v_1\cdots v_n)\,\xi(u_1\cdots u_n)$