equal to $\frac{1}{k!\,l!\,m!}\sum_\sigma w_1(V_{\sigma(1)}\cdots V_{\sigma(k)})\, w_2(V_{\sigma(k+1)}\cdots V_{\sigma(k+l)})\, w_3(V_{\sigma(k+l+1)}\cdots V_{\sigma(k+l+m)})$ (1)

For the same reason, $w_1\wedge(w_2\wedge w_3)$ is also equal to (1).

Thus $(w_1\wedge w_2)\wedge w_3 = w_1\wedge(w_2\wedge w_3)$.

(e) First prove for $\forall\, k\in[1,n]$ $(w_1\wedge\cdots\wedge w_k)(X_1\cdots X_k)=1$. by induction

If $k=1$, then $w_1(X_1)=X_1(p)\cdot X_1(p)=1$. If it's true for k, then

$(w_1\wedge\cdots\wedge w_{k+1})(X_1\cdots X_{k+1}) = \frac{1}{k!}\sum_\sigma(\text{sign}\,\sigma)(w_1\wedge\cdots\wedge w_k)(X_{\sigma(1)}\cdots X_{\sigma(k)})\, w_{k+1}(X_{\sigma(k+1)})$

If $\sigma(k+1)\neq k+1$, then $w_{k+1}(X_{\sigma(k+1)}) = X_{k+1}\cdot X_{\sigma(k+1)}=0$. So we only look at those σ, s.t. $\sigma(k+1)=k+1$. so $\sigma(1)\cdots\sigma(k)$ is a permutation of $1,2,\cdots,k$ ~~Let $\tilde\sigma(i)=\sigma(i)$ $i=1\cdots k$~~, Then

$(w_1\wedge\cdots\wedge w_{k+1})(X_1\cdots X_{k+1}) = \frac{1}{k!}\sum_\sigma(\text{sign}\,\sigma)^2(w_1\wedge\cdots\wedge w_k)(X_1\cdots X_k)$ as $w_1\wedge\cdots\wedge w_k$ is k-form

$= \frac{1}{k!}\cdot k! = 1$. Implicity using the fact that when a $k+1$ permutation σ satisfies $\sigma(k+1)=k+1$, then its sign is equal to the k permutation $\tilde\sigma$ defined as $\tilde\sigma(i)=\sigma(i)$ $i=1,\cdots,k$.

So $(w_1\wedge\cdots\wedge w_k)(X_1\cdots X_k)=1$ for all $k=1\cdots n$.

This step is not necessary. We just follow the hint on textbook.

Next prove for $\forall\, k\in[1,n]$, $i>k$, $X_i\lrcorner(w_1\wedge\cdots\wedge w_k)=0$. Actually $\forall\, V_1\cdots V_{k-1}\in S_p$, $X_i\lrcorner(w_1\cdots w_k)(V_1\cdots V_{k-1}) = (w_1\wedge\cdots\wedge w_k)(X_i(p),V_1,\cdots,V_{k-1})$. Expanding as in the definition, if $X_i(p)$ appear in $w_k(\cdot)$, then $w_k(X_i(p)) = X_k(p)\cdot X_i(p)=0$. If $X_i(p)$ appear in $w_1\wedge\cdots\wedge w_{k-1}$, then by some induction like proof, it's easy to show $(w_1\wedge\cdots\wedge w_k)(\cdots,X_i(p),\cdots)=0$. So $X_i\lrcorner(w_1\cdots w_k)=0$. $i>k$, $k\in[1,n]$

Finally, as $\{X_1\cdots X_{n+1}\}$ is an orthonormal basis for S_p, by Ex 17.13, $f(p)=1$ because $(w_1\wedge\cdots\wedge w_n)(X_1(p)\cdots X_n(p))=1$. So $w_1\wedge\cdots\wedge w_n=\xi$

17.15 (a) multilinearity is obvious. $f^*w(V_{\sigma(1)},\cdots,V_{\sigma(k)}) = w(df(V_{\sigma(1)}),\cdots,df(V_{\sigma(k)}))$

$= (\text{sign}\,\sigma)\, w(df(V_1),\cdots,df(V_k)) = (\text{sign}\,\sigma) f^*w(V_1,\cdots,V_k)$

As w, df are smooth. f^*w is also smooth.

(b) $\int_\varphi f^*w = \int_U w(df(E_1^\varphi),\cdots,df(E_k^\varphi)) = \int_U w(E_1^{f\circ\varphi},\cdots E_k^{f\circ\varphi}) = \int_{f\circ\varphi} w$

(c) Suppose $\{f_i\}$ is a partition of unity on S subordinate to a collection $\{\varphi_i\}$ of one to one local parametrizations of S. We prove first that $\{f_i\circ f^{-1}\}$ is a partition of unity of $\tilde S$, subordinate to a collection $\{f\circ\varphi_i\}$ of one to one local parametrizations of $\tilde S$.

① $\forall\, q\in\tilde S$, $f^{-1}(q)\in S$ (as f is diffeomorphism), So $f_i(f^{-1}(q))\geq 0$ $i=1\cdots m$

② $\forall\, q\in\tilde S$, $f^{-1}(q)\in S$, thus $\sum_{i=1}^m f_i(f^{-1}(q))=1$

③ as f,φ_i are both one to one, so $f\circ\varphi_i$ is one to one. As φ_i is regular and $m=n$ and f ~~is dif~~ is diffeomorphism, so Jf is invertible and $f\circ\varphi_i$ is regular so $f\circ\varphi_i$ is ~~also~~ local parametrization of $\tilde S$. Besides, f is orientation preserving and $f\circ\varphi_i$ must be open as f^{-1} is smooth ~~and invertible~~

Suppose f_i is identically zero outside the image under φ_i of a compact subset