

$B_i$  of  $U_i$ . Then  $f$  is smooth.  $f(B_i)$  is also compact.  $\forall x \in f(B_i)$ , and  $x \in f(\varphi_i(B_i))$ . If  $x \in f(\varphi_i(B_i))$ , then if  $f_i(f^{-1}(x)) \neq 0$ , then  $f \circ \varphi_i(B_i) \subset f_i(f^{-1}(x))$ , which contradicts with our assumption. So  $f_i(f^{-1}(x)) = 0$ . So  $f \circ f^{-1}$  is identically 0 outside the image under  $f \circ \varphi_i$  of a compact subset  $B_i$  of  $U_i$ .

Combining ①-③, we conclude  $\{f_i \circ f^{-1}\}$  is a partition of unity of  $\tilde{S}$ , subordinate to a collection  $\{f \circ \varphi_i\}$  of one-to-one local parametrizations of  $\tilde{S}$ .

Finally.  $\int_S f^* w = \sum_i \int_{\varphi_i} f_i^* f^{-1} w = \sum_i \int_{U_i} f_i \circ \varphi_i \cdot w(df(E_1^{U_i}), \dots, df(E_k^{U_i}))$   
 $= \sum_i \int_{U_i} f_i \circ f^{-1} \circ \varphi_i \cdot w(E_1^{f \circ \varphi_i}, \dots, E_k^{f \circ \varphi_i}) = \int_{\tilde{S}} f_i \circ \varphi_i \cdot f_i \circ f^{-1} w = \int_{\tilde{S}} w$

17.16

For  $\forall p \in S^n$ . If  $v_1, \dots, v_n \in S_p$  is a basis of  $S_p$  and  $\begin{vmatrix} v_1 & \dots & v_n \\ N(p) & \dots & N(p) \end{vmatrix} > 0$ . then  $df(v_i) = -v_i$ ,  $N(f(p)) = -N(p)$ , so  $\begin{vmatrix} df(v_1) & \dots & df(v_n) \\ df(v_1) & \dots & df(v_n) \\ \vdots & \ddots & \vdots \\ N(p) & \dots & N(p) \end{vmatrix} = (-1)^{n+1} \begin{vmatrix} v_1 & \dots & v_n \\ N(p) & \dots & N(p) \end{vmatrix}$ , which is positive iff  $n$  is odd.

17.17(a)  $\lim_{t \rightarrow 0} h(t) = 0$ ,  $h'(t) = \frac{1}{t^2} e^{\frac{1}{t}}$ , so  $\lim_{t \rightarrow 0} h'(t) = 0$ . Generally,  $h^{(n)}(t)$  must be in the form of  $h^{(n)}(t) = P(t) \cdot e^{\frac{1}{t}}$ , where  $P(x)$  is a polynomial function of  $x$  with finite degree. So  $\lim_{t \rightarrow 0} h^{(n)}(t) = 0$ . Obviously  $\lim_{t \rightarrow 0} h^{(n)}(t) = 0$ . So  $h$  is smooth.

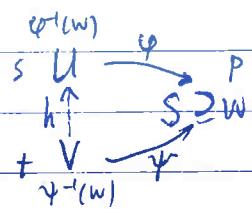
(b)  $h_r(t) = h(u(t))$ , where  $u(t) = t^2 - t^2$ . Since both  $u(t)$ ,  $h(u)$  are smooth, so  $h_r(t)$  is smooth.

In the proof of Thm 4,  $\varphi_p^+$  is smooth,  $\varphi_p^+ - \varphi_p^+(p) \parallel r_p^2$  is also smooth wrt  $q \in R^{n+1}$ . So  $g_p(q) = h(u(\varphi_p^+(q)))$  is smooth.

17.18 Since  $\varphi, \psi$  are both one-to-one local parametrizations

So  $\varphi|\varphi^{-1}(w)$  and  $\psi|\psi^{-1}(w)$  are both bijective from  $\varphi^{-1}(w)$  or  $\psi^{-1}(w)$  to  $W$ . So  $\varphi^{-1} \circ \psi$  and  $\psi^{-1} \circ \varphi$  are bijective,

thus  $\varphi^{-1} \circ \psi|\psi^{-1}(w)$  is also bijective.  $\varphi$  and  $\psi$  are both smooth and regular, so  $\varphi^{-1}, \psi^{-1}$  must be smooth. So  $\varphi^{-1} \circ \psi|\psi^{-1}(w)$  is smooth, and its inverse  $\psi^{-1} \circ \varphi|\varphi^{-1}(w)$  is also smooth. So  $\varphi^{-1} \circ \psi|\psi^{-1}(w)$  is diffeomorphism.



The textbook only defines "orientation preserving" for a map between two oriented  $n$ -surfaces in  $R^{n+1}$  at a regular point, so we don't know what it means by  $h$  being orientation preserving, because  $h$  maps from  $\psi^{-1}(w)$  (open set) to  $\varphi^{-1}(w)$  (open set). However we can still prove that  $|J_h| > 0$ , thus  $\psi|\psi^{-1}(w)$  is reparametrization of  $\varphi|\varphi^{-1}(w)$ .

For any point  $p \in W$ , suppose  $s = \varphi^{-1}(p)$ ,  $t = \psi^{-1}(p)$ . Since both  $\varphi$  and  $\psi$  are local parametrizations of  $S$ , we have  $s = \varphi^{-1}(\psi(t)) = h(t)$ . and