B_i of U_i. ~~then as f is smooth. $f(B_i)$ is also compact. $\forall x \in f(B_i)$,~~ and $x \in$

let $x \in f(\varphi_i(B_i))$. $(x \in f(\varphi_i(U_i)))$, then if $f_i(f^{-1}(x)) \neq 0$, then $f^{-1}(x) \in \varphi_i(B_i)$, then $x = f(f^{-1}(x)) \in f(\varphi_i(B_i))$, which contradicts with our assumption. so $f_i(f^{-1}(x)) = 0$

So $f_i \circ f^{-1}$ is identically 0 outside the image under $f \circ \varphi_i$ of a compact subset B_i of U_i.

Combining ①-③, we conclude $\{f_i \circ f^{-1}\}$ is a partition of unity of $\tilde{S}$, subordinate to a collection $\{f \circ \varphi_i\}$ of one-to-one local parametrization of $\tilde{S}$.

Finally. $\int_S f^* w = \sum_i \int_{\varphi_i} f_i f^* w = \sum_i \int_{U_i} f_i \circ \varphi_i \cdot w(df(E_1^{\varphi_i}), \dots, df(E_k^{\varphi_i}))$

$$= \sum_i \int_{U_i} f_i \circ f^{-1} \circ f \circ \varphi_i \cdot w(E_1^{f\circ\varphi_i}, \dots, E_k^{f\circ\varphi_i}) = \int_{f_i \circ \varphi_i} f_i \circ f^{-1} w = \int_{\tilde{S}} W$$

~~17.16~~

17.16 For $\forall p \in S^n$. If $v_1 \cdots v_n \in S_p$ is a basis of S_p and $\begin{vmatrix} v_1 \\ \vdots \\ v_n \\ N(p) \end{vmatrix} > 0$. then $df(v_i) = -v_i$, $N(f(p)) = -N(p)$, so $\begin{vmatrix} df(v_1) \\ \vdots \\ df(v_n) \\ N(f(p)) \end{vmatrix} = (-1)^{n+1} \cdot \begin{vmatrix} v_1 \\ \vdots \\ v_n \\ N(p) \end{vmatrix}$, which is positive iff n is odd.

17.17 (a) $\lim_{t\to 0^+} h(t) = 0$, $h'(t) = \frac{1}{t^2} e^{-\frac{1}{t}}$, so $\lim_{t\to 0^+} h'(t) = 0$. Generally, $h^{(n)}(t)$ must be in the form of $h^{(n)}(t) = P(\frac{1}{t}) \cdot e^{-\frac{1}{t}}$, where $P(x)$ is a polynomial function of x with finite degree. ~~Actually~~, ~~$h''(t)$~~ So $\lim_{t\to 0^+} h^{(n)}(t) = 0$. Obviously $\lim_{t\to 0^-} h^{(n)}(t) = 0$ So h is smooth

(b) $h_r(t) = h(u(t))$, where $u(t) = r^2 - t^2$. Since both $u(t)$. $h(u)$ are smooth, so $h_r(t)$ is smooth.

In the proof of Thm 4. φ_p^{-1} is smooth, $u(q) = -\|q - \varphi_p^{-1}(p)\|^2 + r_p^2$ is also smooth wrt $q \in R^{n+1}$

So $g_p(q) = h(u(\varphi_p^{-1}(q)))$ is smooth.

$$\begin{tikzcd}
s \quad U = \varphi^{-1}(W) \arrow[r, "\varphi"] & p \in S \supseteq W \\
t \quad V = \psi^{-1}(W) \arrow[u, "h"] \arrow[ur, "\psi"'] &
\end{tikzcd}$$

17.18 Since φ, ψ are both one to one local parametrization

So $\varphi|_{\varphi^{-1}(W)}$ and $\psi|_{\psi^{-1}(W)}$ are both bijective ~~to W~~ from $\varphi^{-1}(W)$ or $\psi^{-1}(W)$ to W. So φ^{-1} and ψ^{-1} are bijective,

thus $\varphi^{-1} \circ \psi|_{\psi^{-1}(W)}$ is also bijective. φ and ψ are both smooth and regular, so φ^{-1}, ψ^{-1} must be smooth. So $\varphi^{-1} \circ \psi|_{\psi^{-1}(W)}$ is smooth, and its inverse $\psi^{-1} \circ \varphi|_{\varphi^{-1}(W)}$ is also smooth. So $\varphi^{-1} \circ \psi|_{\psi^{-1}(W)}$ is diffeomorphism.

The textbook only defines "orientation preserving" for a map between two oriented n-surfaces in R^{n+1} at a regular point, so we don't know what it means by h being orientation preserving, because h maps from $\psi^{-1}(W)$ (open set) to $\varphi^{-1}(W)$ (open set). However we can still prove that $|J_h| > 0$, thus $\psi|_{\psi^{-1}(W)}$ is reparametrization of $\varphi|_{\varphi^{-1}(W)}$.

For any point $p \in W$, suppose $s = \varphi^{-1}(p)$, $t = \psi^{-1}(p)$. Since both φ and ψ are local parametrizations of S, we have $s = \varphi^{-1}(\psi(t)) = h(t)$. and